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Best Proximity Points for Generalized Geraghty Quasi-Contraction Type Mappings in Metric Spaces

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Abstract. In this paper, we introduce a new concept of α-ϕ-Geraghty proximal quasi-contraction type mappings and establish best proximity point theorems for those mappings in proximal T-orbitally complete metric spaces. This generalizes and complements the proofs of some known fixed and best proximity point results.

1. Introduction

Let A and B be two nonempty subsets of a metric space (X, d). A best proximity point of a non-self mapping $T : A \to B$, is the point $x \in A$, satisfying $d(x, Tx) = d(A, B)$. Numerous results on best proximity point theory were studied by several authors ([1], [3], [4], [5]) imposing sufficient conditions that would assure the existence and uniqueness of such points. These results are generalizations of the contraction principle and other contractive mappings ([2], [6], [8], [16], [21], [22], [24]) in the case of self-mappings, which reduces to a fixed point if the mapping under consideration is a self-mapping. The notion of best proximity point was introduced in [14], the class of proximal quasi contraction mappings was introduced in [11] and thereafter, several known results were derived ([10], [12], [13]). Best proximity pair theorems analyse the conditions under which the optimization problem, namely $\min_{x \in A} d(x, Tx)$ has a solution and is known to have applications in game theory. For additional information on best proximity point, see [7], [9], [10], [11], [12], [13], [14], [15], [17], [18], [20], [23].

Definition 1.1 [4]. Let $T : X \to X$ be a map on metric space. For each $x \in X$ and for any positive integer n,

$$O_T(x, n) = \{x, Tx, ..., T^n x\} \ \ and \ \ O_T(x, \infty) = \{x, Tx, ..., T^n x, ...\}.$$

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The set $O_T(x,\infty)$ is called the orbit of T at x and the metric space X is called T-orbitally complete if every Cauchy sequence in $O_T(x,\infty)$ is convergent in X.

Quasi contraction mapping is known in literature as one of the most generalized contractive mappings and is defined as follows.

Definition 1.2 [6]. A mapping $T : X \to X$ of a metric space X into itself is said to be a quasi-contraction if and only if there exists a number k, $0 \le k < 1$, such that

$$d(Tx,Ty) \le k \max\{d(x,y); d(x,Tx); d(y,Ty); d(x,Ty); d(y,Tx)\}$$

holds for every $x, y \in X$.

Consider the class F of functions $\beta : [0,\infty) \to [0,1)$ satisfying the condition:

$$\lim_{n\to\infty} \beta(t_n) = 1 \text{ implies } \lim_{n\to\infty} t_n = 0.$$

Recently, using these class of functions, Umudu et al. [22] introduced a new class of quasi-contraction type mappings called generalized α-ϕ-Geraghty quasi-contraction type mappings and proved the existence of its unique fixed point as follows.

Definition 1.3 [22]. Let (X,d) be a metric space and $\alpha : X \times X \to \mathbb{R}^+$. A mapping $T : X \to X$ is called a generalized α-Geraghty quasi-contraction type mapping if there exists $\beta \in F$ such that for all $x, y \in X$,

$$\alpha(x,y)(d(Tx,Ty)) \le \beta(M_T(x,y))(M_T(x,y)), \tag{1}$$

where $M_T(x,y) = \max\{d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\}$.

Let Φ denote the class of the functions $\phi : [0,\infty) \to [0,\infty)$ which satisfies the following conditions:

(i) ϕ is nondecreasing;
(ii) ϕ is continuous;
(iii) $\phi(t) = 0 \iff t = 0$.

Definition 1.4 [22]. Let (X,d) be a metric space and $\alpha : X \times X \to \mathbb{R}^+$. A self mapping $T : X \to X$ is called a generalized α-ϕ-Geraghty quasi-contraction type mapping if there exists $\beta \in F$ such that for all $x, y \in X$,

$$\alpha(x,y)\phi(d(Tx,Ty)) \le \beta(\phi(M_T(x,y)))\phi(M_T(x,y)), \tag{2}$$

where $M_T(x,y) = \max\{d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\}$, and $\phi \in \Phi$.

If $\phi(t) = t$, inequality (2) reduces to inequality (1). The generalized α-ϕ-Geraghty quasi-contraction type self mapping is a generalization of other quasi-contraction type self mappings in literature.

The following mappings introduced by Popescu [19] and used by Umudu et al. [22] to establish the existence of a fixed point will also be needed in this paper.

Definition 1.5 [19]. Let $T : X \to X$ be a self-mapping and $\alpha : X \times X \to \mathbb{R}^+$ be a function. Then T is said to be α-orbital admissible if $\alpha(x, Tx) \geq 1$ implies $\alpha(Tx, T^2x) \geq 1$.

Definition 1.6 [19]. Let $T : X \to X$ be a self-mapping and $\alpha : X \times X \to \mathbb{R}^+$ be a function. Then T is said to be triangular α-orbital admissible if T is α-orbital admissible, $\alpha(x, y) \geq 1$ and $\alpha(y, Ty) \geq 1$ imply $\alpha(x, Ty) \geq 1$.

The main result obtained in [22] is the following.

Theorem 1.7. Let (X, d) be a T orbitally complete metric space, $\alpha : X \times X \to \mathbb{R}^+$ be a function, and let $T : X \to X$ be a self-mapping. Suppose that the following conditions are satisfied:

(i) T is a generalized α-ϕ-Geraghty quasi-contraction type mapping;

(ii) T is triangular α-orbital admissible mapping;

(iii) there exists $x_1 \in X$ such that $\alpha(x_1, Tx_1) \geq 1$;

Then T has a fixed point $x^* \in X$ and $\{T^n x_1\}$ converges to x^*.

In this paper, we extend the concept of generalized α-ϕ-Geraghty quasi-contraction type mapping to generalized α-ϕ-Geraghty proximal quasi-contraction type mapping in the case of non-self mappings. More precisely, we study the existence and uniqueness of best proximity points for generalized α-ϕ-Geraghty proximal quasi-contraction for non-self mappings.

2. Preliminaries

We start this section with the following definitions.

Let A and B be non-empty subsets of a metric space (X, d). We denote by A_0 and B_0 the following sets:

$$\begin{aligned} d(A, B) &= \inf\{d(a, b) : a \in A,\ b \in B\}. \\ A_0 &= \{x \in A : d(x, y) = d(A, B) \text{ for some } y \in B\}. \\ B_0 &= \{y \in B : d(x, y) = d(A, B) \text{ for some } x \in A\}. \end{aligned}$$

Definition 2.1 [14]. An element $x \in A$ is said to be a best proximity point of the non-self-mapping $T : A \to B$ if it satisfies the condition that $d(x, Tx) = d(A, B)$.
We denote the set of all best proximity points of T by $P_T(A)$, that is,
$P_T(A) := \{x \in A : d(x, Tx) = d(A, B)\}$.

The following were introduced by [11].

Definition 2.2 [11]. A non-self mapping $T : A \to B$ is said to be a proximal quasi-contraction if and only if there exists a number q, $0 \leq q < 1$, such that

$$\begin{cases} d(u, Tx) = d(A, B) \\ d(v, Ty) = d(A, B) \end{cases} \Longrightarrow d(u, v) \leq q \max\{d(x, y); d(x, u); d(y, v); d(x, v); d(y, u)\},$$

where $x, y, u, v \in A$.

If T is a self mapping on A, then Definition 2.2 reduces to Definition 1.2.

Lemma 2.3 [11]. Let $T : A \to B$ be a non-self mapping. Suppose that the following conditions hold:

(i) $A_0 \neq \emptyset$;

(ii) $T(A_0) \subseteq B_0$.

Then, for all $a \in A_0$, there exists a sequence $\{x_n\} \subset A_0$ such that

$$\begin{cases} x_0 = a, \\ d(x_{n+1}, Tx_n) = d(A, B), \quad \forall n \in \mathbb{N}. \end{cases}$$

Any sequence $\{x_n\} \subset A_0$ satisfying the equation in Lemma 2.3 is called a proximal Picard sequence associated to $a \in A_0$ and we denote by $PP(a)$ the set of all proximal Picard sequences associated to a.

Suppose $a \in A_0$ and $\{x_n\} \in PP(a)$. For all $(i, j) \in \mathbb{N}^2$, the following sets are defined by:
$O_T(x_i, j) := \{x_l : i \leq l \leq j + i\}$ and $O_T(x_i, \infty) := \{x_l : l \geq i\}$.

Definition 2.4 [11] A_0 is said to be proximal T-orbitally complete if and only if every Cauchy sequence $\{x_n\} \in PP(a)$ for some $a \in A_0$, converges to an element in A_0.

If T is a self mapping on A, then the preceding definition reduces to the condition that A is T-orbitally complete.

The concepts of α-orbital proximal admissible mapping and triangular α-orbital proximal admissible mapping are hereby introduced as follows.

Definition 2.5 Let $T : A \to B$ be a non-self mapping and $\alpha : A \times A \to [0, \infty)$ be a function. The mapping T is said to be α-orbital proximal admissible if

$$\begin{cases} \alpha(x, u) \geq 1 \\ d(u, Tx) = d(A, B) \\ d(v, Tu) = d(A, B) \end{cases} \Longrightarrow \alpha(u, v) \geq 1,$$

for all $x, u, v \in A$.

Definition 2.6 Let $T : A \to B$ be a non-self mapping and $\alpha : A \times A \to [0,\infty)$ be a function. The mapping T is said to be triangular α-orbital proximal admissible if it is α-orbital proximal admissible and

$$\begin{cases} \alpha(x,y) \geq 1 \\ \alpha(y,u) \geq 1 \\ d(u,Ty) = d(A,B) \end{cases} \Longrightarrow \alpha(x,u) \geq 1,$$

for all $x, y, u \in A$.

Remark 2.7. If T is a self mapping, that is, if $A = B$, α-orbital proximal admissible mapping reduces to α-orbital admissible mapping while triangular α-orbital proximal admissible mapping reduces to triangular α-orbital admissible mapping defined in [19] .

Example 2.8. Let X be the Euclidean plane $\mathbb{R}^2$ and consider the two subsets:

$$A = \{(0,0),(0,1),(0,2),(0,3)\}$$

$$B = \{(1,0),(2,1),(2,2),(1,3)\}$$

Define a mapping $T : A \to B$ such that $T(0,0) = (1,0)$, $T(0,1) = (2,2)$, $T(0,2) = (2,1)$ and $T(0,3) = (1,3)$.
Also define a mapping $\alpha : A \times A \to [0,\infty)$ such that

$$\alpha(x,y) = \begin{cases} 1, & \text{if } x = y \in \{(0,0),(0,3)\} \\ \\ 0 & \text{elsewhere.} \end{cases}$$

for all $x, y \in A$.

One can see that $d(A,B) = 1$.

Let $u, v, x \in A$. One can check that

$$\begin{cases} \alpha(x,u) \geq 1 \\ d(u,Tx) = 1 \\ d(v,Tu) = 1 \end{cases} \Longrightarrow x = u = v \in \{(0,0),(0,3)\} \Longrightarrow \alpha(u,v) = 1.$$

Hence, T is α-orbital proximal admissible.

Let $u, x, y \in A$. One can check that

$$\begin{cases} \alpha(x,u) \geq 1 \\ \alpha(y,u) \geq 1 \\ d(u,Ty) = 1 \end{cases} \Longrightarrow x = y = u \in \{(0,0),(0,3)\} \Longrightarrow \alpha(x,u) = 1.$$

Thus, T is also triangular α-orbital proximal admissible.

We introduce the following new classes of non-self mappings.

Definition 2.9 Let A and B be two nonempty subsets of a metric space (X, d) and $\alpha : A\times A \to \mathbb{R}^+$ be a function. A non-self mapping $T : A \to B$ is called a generalized α-ϕ-Geraghty proximal quasi-contraction type mapping if there exists $\beta \in F$ such that for all $x, y, u, v \in A$,

$$\begin{cases} d(u, Tx) = d(A, B) \\ d(v, Ty) = d(A, B) \end{cases} \implies \alpha(x, y)\phi(d(u, v)) \leq \beta(\phi(M_T(x, y)))\phi(M_T(x, y)), \quad (3)$$

where $M_T(x, y) = \max\{d(x, y), d(x, u), d(y, v), d(x, v), d(y, u)\}$, for all $x, y, u, v \in A$ and $\phi \in \Phi$.

If $\phi(t) = t$, then definition 2.9 reduces to the following.

Definition 2.10 Let A and B be two nonempty subsets of a metric space (X, d) and $\alpha : A\times A \to \mathbb{R}^+$ be a function. A non-self mapping $T : A \to B$ is called an α-Geraghty proximal quasi-contraction type mapping if there exists $\beta \in F$ such that for all $x, y, u, v \in A$,

$$\begin{cases} d(u, Tx) = d(A, B) \\ d(v, Ty) = d(A, B) \end{cases} \implies \alpha(x, y)d(u, v) \leq \beta(M_T(x, y))(M_T(x, y)), \quad (4)$$

for all $x, y, u, v \in A$.

where $M_T(x, y) = \max\{d(x, y), d(x, u), d(y, v), d(x, v), d(y, u)\}$ for all $x, y, u, v \in A$.

3. Main results

Now we state and prove our main results.

Theorem 3.1. Let A and B be two nonempty subsets of a metric space such that A_0 is proximal T-orbitally complete, where $T : A \to B$ is a non-self mapping, $\alpha : A \times A \to \mathbb{R}^+$ is a function and the following conditions are satisfied:

(i) T is a generalized α-ϕ-Geraghty proximal quasi-contraction type mapping;
(ii) $T(A_0) \subseteq B_0$ and T is a triangular α-orbital proximal admissible mapping;
(iii) there exists $x_0, x_1 \in A_0$ such that $d(x_1, Tx_0) = d(A, B)$ and $\alpha(x_0, x_1) \geq 1$.

Then there exists an element $x^* \in A_0$ such that

$$d(x^*, Tx^*) = d(A, B).$$

Moreover, if $\alpha(x, y) \geq 1$ for all $x, y \in P_T(A)$, then x^* is the unique best proximity point of T.

Proof.
Let $x_0, x_1 \in A_0$ be such that $d(x_1, Tx_0) = d(A, B)$ and $\alpha(x_0, x_1) \geq 1$.

$T(A_0) \subseteq B_0$ and there exists $x_2 \in A_0$ such that $d(x_2, Tx_1) = d(A, B)$. Now, we have

$$\begin{cases} \alpha(x_0, x_1) \geq 1 \\ d(x_1, Tx_0) = d(A, B), \\ d(x_2, Tx_1) = d(A, B). \end{cases}$$

Since T is α-orbital proximal admissible, $\alpha(x_1, x_2) \geq 1$. Thus, we have

$d(x_2, Tx_1) = d(A, B)$ and $\alpha(x_1, x_2) \geq 1$.

By induction, we can construct a sequence $\{x_i\} \subseteq A_0$ such that

$$d(x_{i+1}, Tx_i) = d(A, B) \;\; and \;\; \alpha(x_i, x_{i+1}) \geq 1, \;\; for \;\; all \;\; i \in \mathbb{N}. \tag{5}$$

For all $i \geq 0$

$$\begin{cases} \alpha(x_i, x_{i+1}) \geq 1 \\ \alpha(x_{i+1}, x_{i+2}) \geq 1 \\ d(x_{i+2}, Tx_{i-1}) = d(A, B), \end{cases} \implies \alpha(x_i, x_{i+2}) \geq 1,$$

Since T is triangular α-orbital proximal admissible. Thus by induction, $\alpha(x_i, x_j) \geq 1$ for all i, j such that $0 \leq i < j$.

Therefore for any $i \in \mathbb{N}$, we have

$$\begin{cases} \alpha(x_{i-1}, x_{j-1}) \geq 1 \\ d(x_i, Tx_{i-1}) = d(A, B), \\ d(x_j, Tx_{j-1}) = d(A, B) \end{cases}$$

for all i, j such that $1 \leq i < j$.

Clearly, if $x_{i+1} = x_i$ for some $i \in \mathbb{N}$ from inequality (5), x_i will be a best proximity point, so henceforth, in this proof, we assume $d(x_i, x_{i+1}) > 0, \;\; \forall \;\; i \in \mathbb{N}$.

From inequality (3), we have

$$\begin{aligned} \phi(d(x_i, x_j)) &\leq \alpha(x_{i-1}, x_{j-1})\phi(d(x_i, x_j)) \\ &\leq \beta(\phi(M_T(x_{i-1}, x_{j-1})))\phi(M_T(x_{i-1}, x_{j-1})) \end{aligned} \tag{6}$$

$1 \leq i < j$ where

$$\begin{aligned} \phi(M_T(x_{i-1}, x_{j-1})) &\leq \phi(\max\{d(x_{i-1}, x_{j-1}), d(x_{i-1}, x_i), d(x_{j-1}, x_j), \\ &\quad d(x_{i-1}, x_j), d(x_{j-1}, x_i)\}) \\ &\leq \phi(\delta[O_T(x_{i-1}, n)]), \;\; for \;\; i \leq j \leq n + i. \end{aligned}$$

Note that the case $\phi(M_T(x_{i-1}, x_{j-1})) = \phi(d(x_i, x_j))$ is impossible. Indeed, by inequality (6),

$$\begin{aligned} \phi(d(x_i, x_j)) &\leq \beta(\phi(M_T(x_{i-1}, x_{j-1})))\phi(M_T(x_{i-1}, x_{j-1})) \\ &\leq \beta(\phi(d(x_i, x_j)))\phi(d(x_i, x_j)) \\ &< \phi(d(x_i, x_j)), \end{aligned}$$

is a contradiction. Thus, we conclude that $\phi(d(x_i, x_j)) < \phi(d(x_{i-1}, x_{j-1}))$ for all $0 < i < j$ and so the sequence $\{\phi(d(x_i, x_j))\}$ is positive and decreasing. Consequently, there exists $r \geq 0$ such that

$$\lim_{i,j\to\infty} \phi(d(x_i, x_j)) = r.$$

We claim that $r = 0$. Suppose, on the contrary, that $r > 0$. Then we have

$$\frac{\phi(d(x_i, x_j))}{\phi(d(x_{i-1}, x_{j-1}))} \leq \beta(\phi(M_T(x_{i-1}, x_{j-1}))) \leq 1 \;\; for \;\; each \;\; i, j \in \mathbb{N} \;\; such \;\; that \;\; i < j.$$

Then, since $\beta \in F$,

$$\lim_{i,j\to\infty} \beta(\phi(M_T(x_{i-1}, x_{j-1}))) = 1,$$

implying that

$$\lim_{i,j\to\infty} \phi(M_T(x_{i-1}, x_{j-1})) = 0, \tag{7}$$

and so by inequality (6)

$$\lim_{i,j\to\infty} \phi(d(x_i, x_j)) = 0,$$

which is a contradiction.

Now, by the continuity property of ϕ,

$$\phi\left(\lim_{i,j\to\infty}(d(x_i, x_j))\right) = \phi(0). \tag{8}$$

But $\phi(t) = 0$ if and only if $t = 0$ and so (8) gives

$$\lim_{i,j\to\infty}(d(x_i, x_j)) = 0.$$

Therefore, $\{x_n\}$ is a Cauchy sequence in A_0 and since A_0 is proximal T-orbitally complete, there exists $x^* \in A_0$ such that $\lim_{i\to\infty} x_i = x^*$. Also, since $T(A_0) \subseteq B_0$, then there exists $y \in A_0$ such that

$$d(y, Tx^*) = d(x_i, Tx_{i-1}) = d(A, B) \;\; \forall n \in \mathbb{N}, \;\; \forall i \geq 0.$$

T being a generalized α-ϕ-Geraghty proximal quasi-contraction type mapping gives

$$\begin{aligned} \phi(d(y, x_i)) &\leq \alpha(x^*, x_{i-1})\phi(d(y, x_i)) \\ &\leq \beta(\phi(M_T(x^*, x_{i-1})\phi(M_T(x^*, x_{i-1})) \end{aligned}$$

provided that $\alpha(x^*, x_{i-1}) \geq 1$ where

$$\phi(M_T(x^*, x_{i-1})) = \phi(\max\{d(x^*, x_{i-1}), d(x^*, x_i), d(x_{i-1}, x_i), d(x^*, y), d(x_{i-1}, y)\}).$$

But taking the limit,

$$\phi(d(y, x^*)) \leq \lim_{i\to\infty} \beta(\phi(M_T(x^*, x_{i-1})))\phi(d(x^*, y)),$$

which gives, $1 \leq \lim_{i\to\infty} \beta(\phi(M_T(x^*, x_{i-1}))) = \beta(\phi(d(y, x^*))) = 1$ implying $\phi(d(y, x^*)) = 0$ and $d(y, x^*) = 0$ i.e $y = x^*$. We have $d(x^*, Tx^*) = d(y, Tx^*) = d(A, B)$ and $x^* \in A_0$ is a best proximity point of T.

For uniqueness, suppose the best proximity point of T is not unique. Let x^*, y^* be two best proximity points of T with $x^* \neq y^*$. Then,

$$\left\{\begin{array}{l} \alpha(x^*, y^*) \geq 1 \\ d(x^*, Tx^*) = d(A, B) \\ d(y^*, Ty^*) = d(A, B) \end{array}\right\}$$

Since T is a generalized α-ϕ-Geraghty proximal quasi-contraction type mapping,

$$\begin{aligned} \phi(d(x^*, y^*)) &\leq \alpha(x^*, y^*)\phi(d(x^*, y^*)) \\ &\leq \beta(M_T(x^*, y^*))\phi(M_T(x^*, y^*)) \\ &< \phi(M_T(x^*, y^*)) \end{aligned}$$

where

$$\begin{aligned} M_T(x^*, y^*) &= \max\{d(x^*, y^*), d(x^*, x^*), d(y^*, y^*), d(x^*, y^*), d(y^*, x^*)\} \\ &= d(x^*, y^*). \end{aligned}$$

This gives $d(x^*, y^*) < d(x^*, y^*)$, which is a contradiction. Therefore $x^* = y^*$, and the best proximity point of T is unique.

Corollary 3.2. Let A and B be two nonempty subsets of a metric space such that A_0 is proximal T-orbitally complete, where $T : A \to B$ is a non-self mapping, $\alpha : A \times A \to \mathbb{R}^+$ is a function and the following conditions are satisfied:

(i) T is a generalized α-Geraghty proximal quasi-contraction type mapping;

(ii) $T(A_0) \subseteq B_0$ and T is a triangular α-orbital proximal admissible mapping;

(iii) there exists $x_0, x_1 \in A_0$ such that $d(x_1, Tx_0) = d(A, B)$ and $\alpha(x_0, x_1) \geq 1$.

Then there exists an element $x^* \in A_0$ such that

$$d(x^*, Tx^*) = d(A, B).$$

Moreover, if $\alpha(x, y) \geq 1$ for all $x, y \in P_T(A)$, then x^* is the unique best proximity point of T.

4. Conclusion

In this paper, we introduced the notion of generalized α-ϕ-Geraghty proximal quasi-contraction type mappings which, for a self mapping, reduces to that in Umudu *et al.* [22]. Equipped with an example, we also introduced α-orbital proximal admissible mappings and triangular α-orbital proximal admissible mappings which include the admissible mappings defined by Popescu [19]. The existence of best proximity point was investigated for the class of mappings in a proximal T-orbitally complete metric space.

Competing interests:

The authors declare that they have no competing interests.

Authors' contributions:

All authors contributed equally in the preparation of the paper. The authors read and approved the final manuscript.

References

[1] A. Abkar, M. Gabeleh, Best proximity points for cyclic mappings in ordered metric spaces, J. Optim. Theory Appl. 150 (2011), 188-193. https://doi.org/10.1007/s10957-011-9810-x.

[2] S. Banach, Sur les operations dans les ensembles abstraits et leur applications aux equations integrales, Fund. Math. 3 (1922), 133-181.

[3] S. Sadiq Basha, Best proximity points: optimal solutions, J. Optim. Theory Appl. 151 (2011), 210-216. https://doi.org/10.1007/s10957-011-9869-4.

[4] N. Bilgili, E. Karapınar, K. Sadarangani, A generalization for the best proximity point of Geraghty-contractions, J. Inequal. Appl. 2013 (2013), 286. https://doi.org/10.1186/1029-242x-2013-286.

[5] J. Caballero, J. Harjani, K. Sadarangani, A best proximity point theorem for Geraghty-contractions, Fixed Point Theory Appl. 2012 (2012), 231. https://doi.org/10.1186/1687-1812-2012-231.

[6] L.B. Ciric, A generalization of Banach's contraction principle, Proc. Amer. Math. Soc. 45 (1974), 267-273.

[7] A.A. Eldred, P. Veeramani, Existence and convergence of best proximity points, J. Math. Anal. Appl. 323 (2006), 1001-1006. https://doi.org/10.1016/j.jmaa.2005.10.081.

[8] M. Geraghty, On contractive mappings, Proc. Amer. Math. Soc. 40 (1973), 604-608.

[9] J. Hamzehnejadi, R. Lashkaripour, Best proximity points for generalized α-ϕ-Geraghty proximal contraction mappings and its applications, Fixed Point Theory Appl. 2016 (2016), 72. https://doi.org/10.1186/s13663-016-0561-0.

[10] M. Jleli, E. Karapinar, B. Samet, Best proximity point for generalized α-ψ-proximal contraction type mapping, J. Appl. Math. 2013 (2013), 534127. https://doi.org/10.1155/2013/534127.

[11] M. Jleli, B. Samet, An optimization problem involving proximal quasi-contraction mappings, Fixed Point Theory Appl. 2014 (2014), 141. https://doi.org/10.1186/1687-1812-2014-141.

[12] E. Karapinar, I.M. Erhan, Best proximity point on different type of contractions, Appl. Math. Inf. Sci. 5 (2011), 558-569.

[13] E. Karapinar, On best proximity point of ψ-Geraghty contractions, Fixed Point Theory Appl. 2013 (2013), 200. https://doi.org/10.1186/1687-1812-2013-200.

[14] W.A. Kirk, P.S. Srinavasan, P. Veeramani, Fixed points for mapping satisfying cyclical contractive conditions, Fixed Point Theory. 4 (2003), 79-89.

[15] C. Mongkolkeha, Y.J. Cho, P. Kumam, Best proximity points for Geraghty's proximal contraction mappings, Fixed Point Theory Appl. 2013 (2013), 180. https://doi.org/10.1186/1687-1812-2013-180.

[16] J. Olaleru, A comparison of Picard and Mann iterations for quasi-contraction maps, Fixed Point Theory. 8 (2007), 87-95.

[17] J. Olaleru, V. Olisama, M. Abbas, Coupled best proximity points for generalised Hardy-Rogers type cyclic (ω)-contraction, Int. J. Math. Anal. Optim.: Theory Appl. 1 (2015), 33-54.

[18] V. Olisama, J. Olaleru, H. Akewe, Best proximity point results for some contractive mappings in uniform spaces, Int. J. Anal. 2017 (2017), 6173468. https://doi.org/10.1155/2017/6173468.

[19] O. Popescu, Some new fixed point theorems for α-Geraghty contraction type maps in metric spaces, Fixed Point Theory Appl. 2014 (2014), 190. https://doi.org/10.1186/1687-1812-2014-190.

[20] V. Sankar Raj, A best proximity point theorem for weakly contractive non-self-mappings, Nonlinear Anal.: Theory Meth. Appl. 74 (2011), 4804-4808. https://doi.org/10.1016/j.na.2011.04.052.

[21] B.E. Rhoades, A comparison of various definitions of contractive maps, Trans. Amer. Math. Soc. 226 (1977), 257-290.

[22] J.C. Umudu, J.O. Olaleru, A.A. Mogbademu, Fixed point results for Geraghty quasi-contraction type mappings in dislocated quasi-metric spaces, Fixed Point Theory Appl. 2020 (2020), 16. https://doi.org/10.1186/s13663-020-00683-z.

[23] J.C. Umudu, J.O. Olaleru, A.A. Mogbademu, Best proximity point results for Geraghty p-proximal cyclic quasi-contraction in uniform spaces, Divulgaciones Mat. 21 (2020), 21-31.

[24] J. Umudu, A. Mogbademu, J. Olaleru, Fixed point results for Geraghty contractive type operators in uniform spaces, Caspian J. Math. Sci. 11 (2022), 191-202. https://doi.org/10.22080/cjms.2021.3052.