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On Degree-Based Topological Indices of Petersen Subdivision Graph

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Abstract. In this paper, we adequately describe the generalised petersen graph, expanding to the categories of graphs. We created a petersen graph, which is cyclic and has vertices that are arranged in the centre and nine gons plus one vertex, leading to the factorization of regular graphs. Petersen graph is still shown in graph theory literature, nevertheless.

1. Introduction

Named after Julius Petersen, a Danish mathematician, the graph of Petersen is(from 1839 to 1910). Petersen researched factorizations of normal factorizations during the 1890s. In 1891, a significant paper of graphs was published which is commemorated in that volume. Petersen proved that any graph of 3-regular with at a *I*-factor includes much of the two bridges. Tait had written a few years ago that he had shown I-factorable for each 3-regular graph,but that this outcome It was not valid without restriction. But Tait's comment in 1898 was interpreted by petersen to imply that each 3-regular bridge less graph is L-factorable. If this outcome were valid, then it would have been stronger than Theorem for Petersen. The key characteristics of the petersen graph were examined in detail in 1985. The graph of petersen continously to express in the entire graph-theory education. We update our previous analysis in the present article by denoting extra recently findings concerning the petersen graph.
Julius Petersen's 'Die Theorie der regulken graphs' is an exceptional paper that developed a new

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theory in graph theory, based on the exchange property of trees spanning and the cyclomatic number of trees spanningresently resently Zaib Hassan Niazi et.al[15].

1.1. **The graph of petersen.** Every petersen graph is cyclic graph and the graph G' in general form consists V having set of vertex and E having set of edge, if the natural number, there exist n the graph with vertices are $V(G') = 4n$, edges are $E(G') = 6n$, and the specific of this graph is that about degree of every each vertex is $P_{(k,t)} = [d(x_1), d(x_2)] = 3$. Then this graphic which is said to be petersen graphic. Then petersen graphic is denoted by $P_{[V(G'),E(G')]} = (4n, 6n)$ petersen map explored by 1985, updated by new analysis.

Sylvester's association to graphs of invariants and covariants requires interpretation of principle of invariants in 1880s.

1.2. **Graphical idea of petersen graph.** If the set of natural number is $T_n = \{1, 2, 3, ...\}$, if there exists n then graph with vertices are $V(G') = 4n$, and edges $E(G') = 6n$, in general form of petersen expressed by $P_{[V(G'),E(G')]} = (4n, 6n)$. This graph having a specification, that degree of every each vertex is $P_(k, t) = [d(x_1), d(x_2)] = 3$.
Now we write;
$T_n = 1, 2, 3, ...$

$$V(G') = 4n \to [1]$$

$$E(G') = 6n \to [2]$$

$$k = d(x_1) = 3$$

$$t = d(x_2) = 3$$

Then

$$P_{(k,t)} = [d(x_1), d(x_2)] = 3$$

$$P_{[V(G'),E(G')]} = (4n, 6n).$$

Next we discuss the topological indices

Zagreb indices of the group were recognized in the early 1980s and are now known as the first and second zagreb indices. They are important molecular descriptors and have been closely correlated with chemical properties.[Degree based topological indices]
The first zagreb index $M_1(G)$ is equal to the sum of the squares of the degrees of the vertices for

the (molecular) graph G[1]. It can also be considered as the sum over the edges of G, and $M_1(G)$ is defined as:[The first and second zagreb indices of some graph operations]

$$M_1(G,x) = \sum_{[x_1,x_2 \in E(G)]} [d(x_1) + d(x_2)] \tag{1}$$

The second zagreb index $M_2(G)$ is equal to the sum of the products of the degrees of the adjacent vertices for the pair of vertices for the (molecular) graph G, and $M_2(G)$ is defined as:[the first and second zagreb indices of some graph operations]

$$M_2(G,x) = \sum_{[x_1,x_2 \in E(G)]} [d(x_1)d(x_2)] \tag{2}$$

In 1972, the first zagreb index, a very old topological index, was launched and several variants of the zagreb index were subsequently proposed, e.g. Shirdel et al. described a novel index in 2013 under the title of 'hyper-zagreb index' and then it was identified as[2]: [A note on hyper-zagreb index of graph operations]

$$HM_1(G) = \sum_{[x_1,x_2 \in E(G)]} [d(x_1) + d(x_2)]^2 \tag{3}$$

E. Deutshi and S. Klavzar,in 2015, defined a new polynomial, M-polynomial in the following way, based on the degree of the vertex[3]:[COMPUTING HYPER ZAGREB INDEX AND M-POLYNOMIALS]

$$M_1(G,y,z) = \sum_{[x_1,x_2 \in E(G)]} y^{[d(x_1)]} z^{[d(x_2)]} \tag{4}$$

In Shuxian defined two polynomials related to the first zagreb index as in the form:

$$M_1^*(G,x) = \sum_{[x_i \in V(G)]} [d(x_i)][x^{(x_i)}] \tag{5}$$

$$M_0(G,x) = \sum_{[x_i \in V(G)]} (x)^{[d(x_i)]} \tag{6}$$

Two zagreb type polynomials are defined as follow:

$$M_{a,b}(G,x) = \sum_{[x_i,x_j \in E(G)]} (x)^{[a\{d(x_i)\}+b\{d(x_i)\}]} \tag{7}$$

$$M'_{a,b}(G,x) = \sum_{[x_i,x_j \in E(G)]} (x)^{([a+\{d(x_i)\}][b+\{d(x_i)\}])} \tag{8}$$

Todeshine et al. introduced two updated models of the zagreb index for molecular graphs[4]:[MULTIPLICATIVE ZAGREB INDICES OF TREES] First multiplicative zagreb index for molecular graph G defined as follows:

$$PM_1(G) = \prod_{[x_1,x_2 \in E(G)]} [d(x_1) + d(x_2)] \tag{9}$$

Second multiplicative zagreb index for molecular graph G defined as follows:

$$PM_2(G) = \prod_{[x_1, x_2 \in E(G)]} [d(x_1) \times d(x_2)] \tag{10}$$

First multiplicative zagreb polynomial for molecular graph G defined as follows:

$$PM_1(G, x) = \prod_{[x_1, x_2 \in E(G)]} X^{[d(x_1) + d(x_2)]} \tag{11}$$

Second multiplicative zagreb polynomial for molecular graph G defined as follows:

$$PM_2(G, x) = \prod_{[x_1, x_2 \in E(G)]} X^{[d(x_1) d(x_2)]} \tag{12}$$

The first degree-based topological index was proposed by Milan Randic in 1975[5]:[Degree-Based Topological Indices]

$$R_{1(\alpha)}(G) = \sum_{[x_1, x_2 \in E(G)]} [d(x_1) + d(x_2)]^{\alpha} \tag{13}$$

Atom-bond connectivity index (ABC) is a topological index used in chemistry, environmental sciences and pharmacology[6]: [Estrada, Torres, Rodriguez, and Gutman, 1998b]

$$ABC(G) = \sum_{[x_1, x_2 \in E(G)]} \sqrt{\frac{[d(x_1) + d(x_2)] - 2}{d(x_1) \times d(x_2)}} \tag{14}$$

First, second and third reduced zagreb indices[7] are described as follow:

$$MR_1(G) = \sum_{[x_1, x_2 \in E(G)]} |(d(x_1) - 1) + (d(x_2) - 1)| \tag{15}$$

$$MR_2(G) = \sum_{[x_1, x_2 \in E(G)]} [(d(x_1) - 1)(d(x_2) - 1)] \tag{16}$$

$$MR_3(G) = \sum_{[x_1, x_2 \in E(G)]} |(d(x_1) - 1) - (d(x_2) - 1)| \tag{17}$$

$$RR(G) = \sum_{[x_1, x_2 \in E(G)]} \sqrt{d(x_1) \times d(x_2)} \tag{18}$$

The reduced reciprocal randic index is defined as[8]:

$$RRR(G) = \sum_{[x_1, x_2 \in E(G)]} \sqrt{[d(x_1) - 1] \times [d(x_2) - 1]} \tag{19}$$

Recently in 2015 Furtula and Gutman [8] introduced another topological index known as forgotten index or $F - index$. For more detail on the $F - index$, we refer to the articles [9].The forgotten index of a graph G is defined as[10, 11, 12].

$$F(G) = \sum_{[x_1, x_2 \in E(G)]} [(d_{x_1})^2 + (d_{x_2})^2] \tag{20}$$

The forgotten polynomial of a graph G is defined as:

$$F(G,x)=\sum_{[x_1,x_2\in E(G)]}(x)^{[(d_{x_1})^2+(d_{x_2})^2]} \tag{21}$$

The symmetric division degree index of a connected graph G is defined as:

$$SDD(G)=\sum_{[x_1,x_2\in E(G)]}\frac{mini(d(x_1),d(x_2))}{max(d(x_1),d(x_2))}+\frac{maxi(d(x_1),d(x_2))}{mini(d(x_1),d(x_2))} \tag{22}$$

There are two types of general connectivity index. The general randic index (or product-connectivity index) was proposed by Bolloba and Erdos and is defined as follows:

$$M_1(G)=\sum_{[x_2\in V(G)]}[d_G(x_2)]^2 \tag{23}$$

where α is a real number. If $\alpha=-\frac{1}{2}$, then it becomes the randic index and if $\alpha=1$ then it becomes the second zagreb index. Zhou and Trinajstic developed the general sum-connectivity index: [On the general sum-connectivity index of trees]

$$M_1(G)=\sum_{[x_1,x_2\in E(G)]}[d(x_1)+d(x_2)]^{\alpha} \tag{24}$$

where α is a real number. If $\alpha=1$, then the general sum connectivity index becomes the first zagreb index resently Asghar et.al[14].

2. Main Results

In this section, we established some results on degree based topological indices of Petersen graph.

Theorem 2.1 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n=\{1,2,3,...\}$, first zagreb polynomials indices are,

$M_1(G,x)=$[$|6n|$]$(x)^{(6)}$

Proof: The petersen graph $T_n=\{1,2,3,...\}$ appears in figure(graph). The petersen graph T_n $=\{1,2,3,...\}$ contains $V(G')=4n$ no of vertices and $E(G')=6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now first zagreb polynomials indices are i.e. ,

$\Rightarrow GA(R)=\sum_{y_1,y_2\in E(R)}\frac{2\sqrt{d_{y_1}d_{y_2}}}{d_{y_1}+d_{y_2}}$

Now we suppose vertices are $V(G)=4n$, edges are $E(G)=6n$ and degree of petersen graph about every each vertices is $P_{(k,t)}$ =$[d(x_1),d(x_2)]=3$. Now putting the values in first zagreb topological index of the general form,

$\Rightarrow M_1(G,x) = [|E(G)|](x)^{[(3)+(3)]}$
$\Rightarrow M_1(G,x) = (6n)(x)^{(6)}$
$\Rightarrow M_1(G,x) = (6n)(x)^6.$
$M_1(G,x) = (x)^6 \times$ [General edges of Petersen graph]

Theorem 2.2 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1,2,3,...\}$, second zagreb polynomials indices are,
$M_2(G,x) = (6n)(x)^9$

Proof: The petersen graph $T_n = \{1,2,3,...\}$ appears in figure(graph). The petersen graph $T_n = \{1,2,3,...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now second zagreb polynomials indices are i.e. ,

$$M_2(G,x) = \sum_{[x_1,x_2 \in E(G)]}(x)^{[d(x_1)\times d(x_2)]} \rightarrow [1]$$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)} = [d(x_1), d(x_2)] = 3$. Now putting the values in second zagreb topological index of the general form,

$\Rightarrow M_2(G,x) = \sum_{x_1,x_2 \in E(G)}(x)^{[(3)(3)]}$
$\Rightarrow M_2(G,x) = [|E(G)|](x)^9$
$\Rightarrow M_2(G,x) = (6n)(x)^9.$
$M_2(G,x) = (x)^9 \times$ [General edges of petersen graph]

Theorem 2.3 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1,2,3,...\}$, randic indices are,

$$R_{1(\alpha)}(G) = (6n)[6]^\alpha$$

Proof: The petersen graph $T_n = \{1,2,3,...\}$ appears in figure(graph). The petersen graph $T_n = \{1,2,3,...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now randic indices are i.e. ,

$$R_{1(\alpha)}(G) = \sum_{[x_1,x_2 \in E(G)]}[d(x_1) + d(x_2)]^\alpha$$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)} = [d(x_1), d(x_2)] = 3$. Now putting the values in randic indices topological index of the general form,

$R_{1(\alpha)}(G) = \sum_{[x_1,x_2 \in E(G)]}[d(x_1) + d(x_2)]^{\alpha}$

In general form of topological index becomes;

$\Rightarrow R_{1(\alpha)}(G) = \sum_{[x_1,x_2 \in E(G)]}[d(x_1) + d(x_2)]^{\alpha}$

Now putting values in above equation,

$\Rightarrow R_{1(\alpha)}(G) =[|E(G)|][(3) + (3)]^{\alpha}$

$\Rightarrow R_{1(\alpha)}(G) =[|E(G)|][6]^{\alpha}$

$\Rightarrow R_{1(\alpha)}(G) =[|6n|][6]^{\alpha}$

$\Rightarrow R_{1(\alpha)}(G) = (6n)[6]^{\alpha}.$

$R_{1(\alpha)}(G) = [6^{\alpha}]\times$[The general edges of petersen graph]

Theorem 2.4 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1, 2, 3, ...\}$, reduced reciprocal randic are,

$RRR(G) = 12n.$

Proof: The petersen graph $T_n = \{1, 2, 3, ...\}$ appears in figure(graph). The petersen graph T_n $= \{1, 2, 3, ...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now reduced reciprocal randic are i.e. ,

$RR(G) = \sum_{[x_1,x_2 \in E(G)]} \sqrt{d(x_1) \times d(x_2)}$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)}$ $=[d(x_1), d(x_2)] = 3$. Now putting the values in reduced reciprocal randic topological index of the general form,

$\Rightarrow RRR(G) = \sum_{[x_1,x_2 \in E(G)]} \sqrt{[d(x_1) - 1] \times [d(x_2) - 1]}$

Now puttings the values then;

$\Rightarrow RRR(G) = \sum_{[x_1,x_2 \in E(G)]} \sqrt{(3 - 1) \times (3 - 1)}$

$\Rightarrow RRR(G) = [|E(G)|]\sqrt{(4)}$

$\Rightarrow RRR(G) = [|6n|]\sqrt{(4)}$

$\Rightarrow RRR(G) = (6n)\sqrt{(4)}$

$\Rightarrow RRR(G) = 6n(2)$

$\Rightarrow RRR(G) = 12n.$

Theorem 2.5 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1, 2, 3, ...\}$, hyper zagreb index are,

$HM_1(G) = 216n$

Proof: The petersen graph $T_n = \{1, 2, 3, ...\}$ appears in figure(graph). The petersen graph T_n $= \{1, 2, 3, ...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now hyper zagreb index are i.e. ,

$$HM_1(G) = \sum_{[x_1,x_2 \in E(G)]} [d(x_1) + d(x_2)]^2 \rightarrow [1]$$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)} = [d(x_1), d(x_2)] = 3$. Now putting the values in hyper zagreb index topological index of the general form,

$\Rightarrow HM_1(G) = [|E(G)|][(3 + 3)]^2$

$\Rightarrow HM_1(G) = [|6n|](6)^2$

$\Rightarrow HM_1(G) = (6n)(6)^2$

$\Rightarrow HM_1(G) = (6n)(36)$

$\Rightarrow HM_1(G) = 216n.$

$HM_1(G) =$ Thirty six times to general edges of petersen graph.

Theorem 2.6 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1, 2, 3, ...\}$, two polynomial related to the first zagreb index are,

$M_1^*(G, x) = (12n)x^{4n}$

$M_0(G, x) = 4nx^3$

Proof: The petersen graph $T_n = \{1, 2, 3, ...\}$ appears in figure(graph). The petersen graph $T_n = \{1, 2, 3, ...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now two polynomial related to the first zagreb index are i.e. ,

$$M_1^*(G, x) = \sum_{[x_i \in V(G)]} [d(x_i)][x^{[x_i]}]$$

$$M_0(G, x) = \sum_{[x_i \in V(G)]} (x)^{[d(x_i)]}$$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)} = [d(x_1), d(x_2)] = 3$. Now putting the values in two polynomial related to the first zagreb index topological index of the general form,

$\Rightarrow M_1^*(G,x) = \sum_{[x_i \in V(G)]}(3)[x^{[x_i]}]$

$\Rightarrow M_1^*(G,x) = \sum_{[x_i \in V(G)]}(3)x^{[4n]}$

$\Rightarrow M_1^*(G,x) = [|V(G)|](3)x^{4n}$

$\Rightarrow M_1^*(G,x) = 4n(3)x^{4n}$

$M_1^*(G,x) = (12n)x^{4n}$.

$M_1^*(G,x) = [3x^{4n}]\times$ [General vertices of petersen graph]

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)} = [d(x_1), d(x_2)] = 3$. Now putting the values in two polynomial related to the first zagreb index topological index of the general form,

$\Rightarrow M_0(G,x) = \sum_{[x_i \in V(G)]}(x)^3$

$\Rightarrow M_0(G,x) = [|V(G)|](x)^3$

$M_0(G,x) = 4nx^3$.

$M_0(G,x) = [x^3]\times$ [General vertices of petersen graph]

Theorem 2.7 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1,2,3,...\}$, zagreb type polynomials are,

$M_{a,b}(G,x) = 6nx^{[3(a+b)]}$

$M'_{a,b}(G,x) = (6n)(x)^{[(a+3)(b+3)]}$

Proof: The petersen graph $T_n = \{1,2,3,...\}$ appears in figure(graph). The petersen graph $T_n = \{1,2,3,...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now zagreb type polynomials are i.e. ,

$M_{a,b}(G,x) = \sum_{[x_i,x_j \in E(G)]}(x)^{[a\{d(x_i)\}+b\{d(x_i)\}]}$

$M'_{a,b}(G,x) = \sum_{[x_i,x_j \in E(G)]}(x)^{([a+\{d(x_i)\}][b+\{d(x_i)\}])}$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)} = [d(x_1), d(x_2)] = 3$. Now putting the values in zagreb type polynomials topological index of the general form,

$\Rightarrow M_{a,b}(G,x) = \sum_{[x_i,x_j \in E(G)]}(x)^{[a(3)+b(3)]}$

$\Rightarrow M_{a,b}(G,x) = \sum_{[x_i,y_j \in E(G)]}(x)^{[3(a+b)]}$

$\Rightarrow M_{a,b}(G,x) = [|E(G)|](x)^{[3(a+b)]}$

$M_{a,b}(G,x) = 6nx^{[3(a+b)]}$.

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)} = [d(x_1), d(x_2)] = 3$. Now putting the values in zagreb type polynomials topological index of the general form,

$\Rightarrow M'_{a,b}(G,x) = \sum_{[x_i,x_j \in E(G)]} (x)^{([(a+(3))][(b+(3))])}$
$\Rightarrow M'_{a,b}(G,x) = [|E(G)|](x)^{[(a+(3))(b+(3))]}$
$\Rightarrow M'_{a,b}(G,x) = [|6n|](x)^{[(a+3)(b+3)]}$
$M'_{a,b}(G,x) = (6n)(x)^{[(a+3)(b+3)]}$.
$M'_{a,b}(G,x) = [(x)^{[(a+3)(b+3)]}] \times$ [General edges of petersen graph]

Theorem 2.8 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1, 2, 3, ...\}$, atomic-bond-connectivity (ABC) index are,

$ABC(G) = (4n)$.

Proof: The petersen graph $T_n = \{1, 2, 3, ...\}$ appears in figure(graph). The petersen graph $T_n = \{1, 2, 3, ...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now atomic-bond-connectivity (ABC) index are i.e. ,

$$ABC(G) = \sum_{[x_1,x_2 \in E(G)]} \sqrt{\frac{[d(x_1)+d(x_2)]-2}{d(x_1)\times d(x_2)}} \rightarrow [1]$$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)} = [d(x_1), d(x_2)] = 3$. Now putting the values in atomic-bond-connectivity (ABC) index topological index of the general form,

$\Rightarrow ABC(G) = \sum_{[x_1,x_2 \in E(G)]} \sqrt{\frac{[d(x_1)+d(x_2)]-2}{d(x_1)\times d(x_2)}}$
Putting values in above equation;
$\Rightarrow ABC(G) = [|E(G)|]\sqrt{\frac{[(3)+(3)-2]}{(3)\times(3)}}$
$\Rightarrow ABC(G) = [|6n|]\sqrt{\frac{(4)}{(3)^2}}$
$\Rightarrow ABC(G) = (6n)\frac{\sqrt{4}}{(3)}$
$\Rightarrow ABC(G) = (4n)$.
$ABC(G) =$ General vertices of petersen graph

Theorem 2.9 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1, 2, 3, ...\}$, geometric arithmetic(GA) index are,

$GA(G) = 6n$

Proof: The petersen graph $T_n = \{1, 2, 3, ...\}$ appears in figure(graph). The petersen graph $T_n = \{1, 2, 3, ...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now geometric arithmetic(GA) index are i.e. ,

$$GA(G) = \sum_{[x_1,x_2 \in E(G)]} \frac{2\sqrt{d(x_1)\times d(x_2)}}{d(x_1)+d(x_2)}$$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)}$ =$[d(x_1), d(x_2)] = 3$. Now putting the values in geometric arithmetic(GA) index topological index of the general form,

$$\Rightarrow GA(G) = [|E(G)|]\frac{2\sqrt{(3)\times(3)}}{(3)+(3)}$$
$$\Rightarrow GA(G) = [|6n|]\frac{2\sqrt{(3)^2}}{(6)}$$
$$\Rightarrow GA(G) = (6n)\frac{2\sqrt{(3)^2}}{6}$$
$$\Rightarrow GA(G) = (6n)\frac{2(3)}{(6)}$$
$$\Rightarrow GA(G) = 6n.$$

$GA(R)$ = General edges of petersen graph.

Theorem 2.10 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1, 2, 3, ...\}$, first multiple zagreb index are,

$PM_1(G) = (6)^{6n}$

Proof: The petersen graph $T_n = \{1, 2, 3, ...\}$ appears in figure(graph). The petersen graph T_n $= \{1, 2, 3, ...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now first multiple zagreb index are i.e. ,

$$PM_1(G) = \prod_{[x_1,x_2\in E(G)]}[d(x_1) + d(x_2)]$$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)}$ =$[d(x_1), d(x_2)] = 3$. Now putting the values in first multiple zagreb index topological index of the general form,

$$\Rightarrow PM_1(G) = \prod_{[x_1,x_2\in E(G)]}[(3 + 3)]$$
$$\Rightarrow PM_1(G) = (6)^{[|E(G)|]}$$
$$\Rightarrow PM_1(G) = (6)^{[|6n|]}$$
$$\Rightarrow PM_1(G) = (6)^{6n}.$$

$PM_1(G)$ = General edges of petersen graph to the power of six.

Theorem 2.11 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1, 2, 3, ...\}$, second multiple zagreb index are,

$PM_2(G) = (9)^{[6n]}$

Proof: The petersen graph $T_n = \{1, 2, 3, ...\}$ appears in figure(graph). The petersen graph T_n $= \{1, 2, 3, ...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now second multiple zagreb index are i.e. ,

$$PM_2(G) = \prod_{[x_1,x_2\in E(G)]}[d(x_1) \times d(x_2)]$$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)}$ =$[d(x_1), d(x_2)] = 3$. Now putting the values in second multiple zagreb index topological index of the general form,

$\Rightarrow PM_2(G) = \prod_{[x_1,x_2 \in E(R)]}[(3) \times (3)]$
$\Rightarrow PM_2(G) = (9)^{[|E(R)|]}$
$\Rightarrow PM_2(G) = (9)^{[6n]}$.
$PM_2(G) = (9)^{6n}$ $PM_2(G)$ = General edges of petersen graph to the power of nine.

Theorem 2.12 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1, 2, 3, ...\}$, forgotten polynomial are,

$F(R) = (6n)x^{18}$

Proof: The petersen graph $T_n = \{1, 2, 3, ...\}$ appears in figure(graph). The petersen graph $T_n = \{1, 2, 3, ...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now forgotten polynomial are i.e. ,

$F(G, x) = \sum_{[x_1,x_2 \in E(G)]}(x)^{[(d_{x_1})^2+(d_{x_2})^2]}$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)}$ =$[d(x_1), d(x_2)] = 3$. Now putting the values in forgotten polynomial topological index of the general form,

$\Rightarrow F(G, x) = \sum_{[x_1,x_2 \in E(G)]}(x)^{[(3)^2+(3)^2]}$
$\Rightarrow F(G, x) = [|E(G)|](x)^{[9+9]}$
$\Rightarrow F(G) = [6n](x)^{18}$
$\Rightarrow F(R) = (6n)x^{18}$.
$F(R) = [x^{18}] \times$ [General edges of petersen graph]

Theorem 2.13 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1, 2, 3, ...\}$, symmetric division deg. index are,

$SDD(G) = 12n$

Proof: The petersen graph $T_n = \{1, 2, 3, ...\}$ appears in figure(graph). The petersen graph $T_n = \{1, 2, 3, ...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now symmetric division deg. index are i.e. ,

$SDD(G) = \sum_{[x_1,x_2 \in E(G)]}[\frac{d(x_1)^2+d(x_2)^2}{d(x_1)d(x_2)}]$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)}$ =$[d(x_1), d(x_2)] = 3$. Now putting the values in symmetric division deg. index topological index of the general form,

$\Rightarrow SDD(G) = \sum_{[x_1,x_2 \in E(G)]} \frac{mini(3,3)}{max(3,3)} + \frac{maxi(3,3)}{mini(3,3)}$
$\Rightarrow SDD(G) = [|E(G)|][\frac{3}{3} + \frac{3}{3}]$
$\Rightarrow SDD(G) = [|E(G)|][\frac{(3)+(3)}{3}]$
$\Rightarrow SDD(G) = [|6n|][\frac{(6)}{3}]$
$\Rightarrow SDD(G) = (6n)[2]$
$\Rightarrow SDD(G) = 12n.$
$SDD(G) =$ Two times of general edges of petersen graph.

Theorem 2.14 Let $P_{(k,t)}$ be petersen subdivision graph. Then, for $T_n = \{1, 2, 3, ...\}$, general connectivity index are,

$SDD(G) = 12n$

Proof: The petersen graph $T_n = \{1, 2, 3, ...\}$ appears in figure(graph). The petersen graph $T_n = \{1, 2, 3, ...\}$ contains $V(G') = 4n$ no of vertices and $E(G') = 6n$ no of edges. The degree of each vertex in $P_{(k,t)}$ is 3 and now general connectivity index are i.e. ,

$M_1(G) = \sum_{[x_2 \in V(G)]} [d_G(x_2)]^2$
$M_2(G) = \sum_{[x_1,x_2 \in E(G)]} [(d_G(x_1)) \times (d_G(x_2))] \rightarrow [1]$

Now we suppose vertices are $V(G) = 4n$, edges are $E(G) = 6n$ and degree of petersen graph about every each vertices is $P_{(k,t)} = [d(x_1), d(x_2)] = 3$. Now putting the values in general connectivity index topological index of the general form,

$\Rightarrow M_1(G) = \sum_{[x_2 \in V(G)]} [(3)^2]$
$\Rightarrow M_1(G) = [|V(G)|](3)^2$
$\Rightarrow M_1(G) = (4n)(9)$
$\Rightarrow M_1(G) = 36n.$
$M_1(G) =$ Nine times to vertices of petersen graph.

In general form of real number index is, in equation [1] becomes;
$\Rightarrow M_2(G) = \sum_{[x_1,x_2 \in E(G)]} [d_G(x_1) \times d_G(x_2)]$
Now putting values in above equation.
$\Rightarrow M_2(G) = \sum_{[x_1,x_2 \in E(G)]} (3)(3)$
$\Rightarrow M_2(G) = [|E(G)|](3)(3)$
$\Rightarrow M_2(G) = (6n)(9)$

$\Rightarrow M_2(G) = 54n.$

$M_2(G) =$ Nine times to edges of petersen graph.

3. Numerical Examples

Every petersen graph is cyclic graph and the graph G' in general form consists V having set of vertex and E having set of edge, if the natural number, there exist n the graph with vertices are $V(G') = 4n$, edges are $E(G') = 6n$, and the specific of this graph is that about degree of every each vertex is $P_{(k,t)} = [d(x_1), d(x_2)] = 3$. Then this graphic which is said to be petersen graphic. Then petersen graphic is denoted by $P_{[V(G'),E(G')]} = (4n, 6n)$

The core features of petersen map explored by length in 1985. However, the petersen line continuously arise in literature of the theoretical graphing. By this article, we update previous analysis to introduce additionally fresh findings on the petersen mapping.

Sylvester's association to graphs of invariants and covariants requires interpretation of principle of invariants in 1880s.

Example 3.1. If there exists n is positive natural number then $T_n = \{1, 2, 3, ...\}$, so graph with vertices are $V(G') = 4n$, and edges $E(G') = 6n$, in general form of petersen expressed by $P_{[V(G'),E(G')]} = (4n, 6n)$. This graph having a specification, that degree of every each vertex is $P_(k, t) = [d(x_1), d(x_2)] = 3$.

Now we write;

$T_n = 1, 2, 3, ...$

$V(G') = 4n$ ....(1)

$E(G') = 6n$ ....(2)

$k = d(x_1) = 3$

$t = d(x_2) = 3$

Put $n = 3$ in equations (1) and (2) and these equations become;

$T_3 = 3$

$V(G') = 12$

$E(G') = 18$

$P_{(k,t)} = [d(x_1), d(x_2)] = 3$

$P_{[V(G'),E(G')]} = (12, 18)$

Now figure is;

Example 3.2. If there exists n is positive natural number then $T_n = \{1, 2, 3, ...\}$, so graph with vertices are $V(G') = 4n$, and edges $E(G') = 6n$, in general form of petersen expressed by $P_{[V(G'),E(G')]} = (4n, 6n)$. This graph having a specification, that degree of every each vertex is $P_(k, t) = [d(x_1), d(x_2)] = 3$.

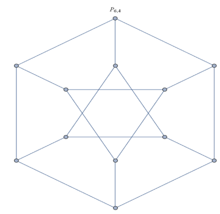

Figure 1. Petersen graph

Now we write;

$T_n = 1, 2, 3, \ldots.$

$V(G') = 4n$ ....(1)

$E(G') = 6n$ ....(2)

$k = d(x_1) = 3$

$t = d(x_2) = 3$

Put $n = 4$ in equations (1) and (2) and these equations become;

$T_4 = 4$

$V(G') = 16$

$E(G') = 24$

$P_{(k,t)} = [d(x_1), d(x_2)] = 3$

$P_{[V(G'),E(G')]} = (16, 24)$

Now figure is;

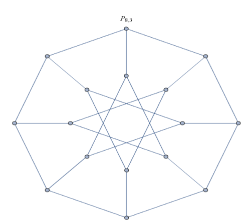

Figure 2. Petersen graph

Example 3.3. If there exists n is positive natural number then $T_n = \{1, 2, 3, \ldots\}$, so graph with vertices are $V(G') = 4n$, and edges $E(G') = 6n$, in general form of petersen expressed by $P_{[V(G'),E(G')]} = (4n, 6n)$. This graph having a specification, that degree of every each vertex is $P_(k, t) = [d(x_1), d(x_2)] = 3$.

Now we write;

$T_n = 1, 2, 3, \ldots.$

$V(G') = 4n$ ....(1)

$E(G') = 6n$ ....(2)

$k = d(x_1) = 3$

$t = d(x_2) = 3$

Put $n = 5$ in equations (1) and (2) and these equations become;

$T_5 = 5$

$V(G') = 20$

$E(G') = 30$

$P_{(k,t)} = [d(x_1), d(x_2)] = 3$

$P_{[V(G'),E(G')]} = (20, 30)$

Now figure is;

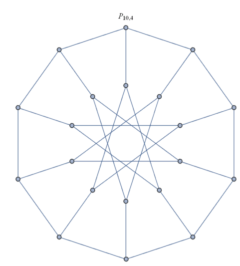

Figure 3. Petersen graph

Example 3.4. If there exists n is positive natural number then $T_n = \{1, 2, 3, \ldots\}$, so graph with vertices are $V(G') = 4n$, and edges $E(G') = 6n$, in general form of petersen expressed by $P_{[V(G'),E(G')]} = (4n, 6n)$. This graph having a specification, that degree of every each vertex is $P_(k, t) = [d(x_1), d(x_2)] = 3$.

Now we write;

$T_n = 1, 2, 3, \ldots.$

$V(G') = 4n$ ....(1)

$E(G') = 6n$ ....(2)

$k = d(x_1) = 3$

$t = d(x_2) = 3$

Put $n = 10$ in equations (1) and (2) and these equations become;

$T_{10} = 10$

$V(G') = 40$

$E(G') = 60$

$P_{(k,t)} = [d(x_1), d(x_2)] = 3$

$P_{[V(G'),E(G')]} = (40, 60)$

Now figures are;

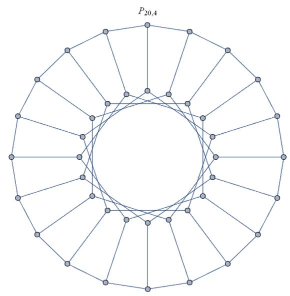

Figure 4. Petersen graph

4. Conclusion and Future Studies

Frequently, graph theory is refuted using the petersen graph. In this paper, the general petersen graph was constructed, and the exact expressions of the first and second zagreb indices, the forgotten topological index, the hyper zagreb index, the reduced second zagreb index and the petersen graph in terms of cyclic graph were then examined. The future work will concentrate on topological indeces, then generalised petersen via graph operations.

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