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A Unified Kantorovich-type Convergence Analysis of Newton-like Methods for Solving Generalized Equations under the Aubin Property

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Abstract. Numerous applications from diverse disciplines reduce to solving generalized equations in a Banach space setting. These equations are solved mostly iteratively, when a sequence is generated approximating a solution provided that certain conditions are valid on the starting point and the operators appearing on the method. In particular, Newton-like methods are developed whose specializations reduce to well known methods such as Newton, modified Newton, Secant, Kurchatov and Steffensen to mention a few. A unified semi-local analysis of these methods is presented using the contraction mapping principle under the Aubin property of a set valued operator, and generalized continuity assumption on the operators on these methods.

1. Introduction

Let B_1 and B_2 stand for complete normed spaces; D be an open and convex subset of B_1; operator $F : D \longrightarrow B_2$ be continuous and $G : B_1 \rightrightarrows B_2$ be a set-valued operator with closed graph, which is a nonempty set [15].

We are concerned with the problem of finding a solution $x^* \in B_1$ of the generalized equation iteratively in the form:

$$\text{Find} \quad x \in B_1 \text{ so that } F(x) + G(x) \ni 0. \tag{1.1}$$

Many applications from diverse disciplines, especially in Mathematical programming can be formulated like the generalized equation (1.1) [1–15, 23–25, 34]. S. M. Robinson inaugurated the

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study of generalized equations in [23–25]. A solution $x^* \in B_1$ in analytical form or closed form is computationally hard or impossible to find. Thus, researchers and practitioners generate iterative methods approximating x^* if certain conditions related to the starting point and the operators on the methods are fulfilled. N, H. Josephy introduced the Newton method for solving the generalized equation (1.1) in [18]. Later, numerous other authors worked on various other iterative methods under diverse convergence conditions (see [1–3,7–15] and references there in).

All these iterative methods are useful and provide insight in the solutions of generalized equations. But as far as we know there is not a unified convergence analysis for the existing iterative methods. That is very useful, since this way under the same set of conditions the convergence and comparison of numerous iterative methods becomes possible. This is our motivation for the present article. In particular, consider the Newton-like iterative method (NLM) for solving the generalized equation in the following form

$$F(x_n) + L(x_n)(x_{n+1} - x_n) + G(x_{n+1}) \ni 0, \ n = 0, 1, 2, \dots, \tag{1.2}$$

where $L(.) : B_1 \longrightarrow L(B_1, B_2)$ which stands for the space of linear operators which are bounded mapping from B_1 into B_2. By specializing the linear operator L, many iterative methods can be obtained such as:

Newton's Method [4,5,21,22]: Select $L(x) = F'(x)$, $x \in B_1$ to obtain

$$F(x_n) + F'(x_n)(x_{n+1} - x_n) + G(x_{n+1}) \ni 0, \ n = 0, 1, 2, \dots,$$

where F' denotes the derivative according to Fréchet of the operator F.

Modified Newton's Method [4,5,20]: Set $L(x) = F'(x_0), x \in B_1$ to obtain

$$F(x_n) + F'(x_0)(x_{n+1} - x_n) + G(x_{n+1}) \ni 0, \ n = 0, 1, 2, \dots.$$

Secant Method [20]: Let $L(x_n) = [x_{n-1}, x_n; F], n = 0, 1, 2, \dots$ a divided difference of order one [20]. Then, iterative method (1.1) becomes

$$F(x_n) + [x_{n-1}, x_n; F](x_{n+1} - x_n) + G(x_{n+1}).$$

Modified Secant Method [4,5]: Take $L(x_n) = [x_{-1}, x_0; F], x_{-1}, x_0 \in B-1, n = 0, 1, 2, \dots$ to obtain

$$F(x_n) + [x_{-1}, x_0; F](x_{n+1} - x_n) + G(x_{n+1}).$$

Kurchatov Method [29,30]: Set $L(x_n) = [2x_n - x_{n-1}, x_{n-1}; F]$ to obtain

$$F(x_n) + [2x_n - x_{n-1}, x_{n-1}; F](x_{n+1} - x_n) + G(x_{n+1}).$$

Modified Kurchatov Method [29,30]: Let $L(x) = [2x_0 - x_{-1}, x_{-1}; F]$ to get

$$F(x_n) + [2x_0 - x_{-1}, x_{-1}; F](x_{n+1} - x_n) + G(x_{n+1}).$$

Picard Method [20]: Pick $L(x) = I, x \in B_1$ for $B_1 = B_2$ to obtain

$$F(x_n) + x_{n+1} - x_n + G(x_{n+1}),\ n = 0, 1, 2, \ldots.$$

Steffensen's Method [4, 5]: Define $L(x) = [x + F(x), x - F(x); F]$ for $B_1 = B_2$ to get

$$F(x_n) + [x_n + F(x_n), x_n - F(x_n); F](x_{n+1} - x_n) + G(x_{n+1}),\ n = 0, 1, 2, \ldots.$$

Stirling's Method [20, 31]: Define $L(x) = I - F'(x),\ x \in B_1$ to obtain

$$F(x_n) + (I - F'(x_n))(x_{n+1} - x_n) + G(x_{n+1}),\ n = 0, 1, 2, \ldots.$$

Traub and Other Multi-point and Multi-step Methods [4, 5, 17, 31, 32]: Therefore, it is important to develop unifying conditions for the convergence of (1.2). There are two popular convergence approaches in the literature. We develop, the semi-local analysis of convergence. In the local case information is used to produce usually a ball centered at x^*, so that if one picks a point inside of it the convergence of the iterative method is assured. Note that , in the semi-local case the convergence ball is centered at the starting point x_0. The convergence conditions usually involve Lipschitz [4, 5] and Hölder-type conditions [20]. The new convergence analysis in both cases involves generalized continuity conditions, majorant functions, majorizing sequences (in the semi-local case) in combination under the Aubin property of the set valued operator on the method and the celebrated contraction mapping principle [15]. Upper error estimates on $\|x^* - x_n\|$ for the solution are developed which are computable.

The rest of the article contains: The Mathematical background necessary to make this article as self contained as possible appears in Section 2; semi-local convergence results appear is Section 3. The article ends with concluding remarks in Section 4.

2. Mathematical Bachground

Certain standard concepts are restated in order to make the article as self-contained as possible. More detailed information can be found in [15].

The graph of a set-valued operator $G : B_1 \rightrightarrows B_2$ is

$$gpG = \{(v_1, v_2) \in B_1 \times B_2 : v_2 \in G(v_1)\}$$

the domain $Dom(G) = \{v \in B_1 : G(v) \neq \emptyset\}$; the $Rge(G) = \{v_2 \in B_2 : \text{for some } \ v_1 \in B_1, v_2 \in G(v_1)\}$.

Moreover, the inverse $G^{-1} : B_2 \rightrightarrows B_1$ is

$$G^{-1}(v_2) = \{v_1 \in B - 1 : v_2 \in G(v_1)\}.$$

Furthermore, for sets C_1 and C_2 in B_1, define

$$d(v, C_1) = \inf_{v_1 \in C_1} d(v, v_1) \text{ and } e(C_1, C_2) = \sup_{v_1 \in C_1} d(v, C_2),$$

where d, e are standard symbols for the distance from v to C_2 and the excess of C_1 to C_2. Recall that $e(\emptyset, \emptyset) = +\infty$, $d(v, C_2) = +\infty$, if $C_2 = \emptyset$ and $e(\emptyset, C_2) = 0$, if $C_2 \neq \emptyset$ (by convention), where $\emptyset$ is the symbol for the empty set.

Next, some more definitions and standard results are stated.

Definition 2.1. *The inverse operator G^{-1} of G has the Aubin property for $v_1 \in B_1$ at $v_2 \in B_2$ of modulus $\lambda \geq 0$, if when $v_1 \in G^{-1}(v_2)$, there exist $\alpha > 0$ and $\beta > 0$ so that*

$$e(G^{-1}(v_4) \cap H[v_1, \alpha], G^{-1}(v_3)) \leq \lambda \|v_4 - v_3\| \text{ for each } v_3, v_4 \in H[v_2, \beta] \tag{2.1}$$

and the inverse operator G^{-1} is locally closed at the pair (v_2, v_1), where $H(v, \alpha)$, $H[v, \alpha]$ denote open and closed balls, respectively of center $v \in B_1$ with radius $\alpha > 0$.

It is useful to recall that there is a relationship between the Aubin property and the metric regularity (see e.g. [15, Theorem 3.7]). In particular, $G^{-1} : B_2 \rightrightarrows B_1$ has the Aubin property at $v_1, v_2)$ with modulus $\lambda > 0$ if and only if $G : B_1 \rightrightarrows B_2$ is metrically regular at (v_1, v_2) with the same constant λ. Therefore, the results that follows are given equivalently in terms of metric regularity.

The celebrated contraction Mapping Principle [15,21,22] plays a vital role in our investigations.

THEOREM 2.2. *Let us consider a set-valued operator $\Psi : B_1 \rightrightarrows B_2$ and $v \in B_1$. Assume that there exist constants $\gamma_0 > 0$ and $\delta_0 \in (0, 1)$ so that $gph\Psi \cap (H[v, \gamma_0] \times H[v, \delta_0])$ is a closed set:*
(i) $d(v, \Psi(v)) \leq \gamma_0(1 - \delta_0)$ and
(ii) $e(\Psi(v_1) \cap H[v, \gamma_0], \Psi(v_2)) \leq \delta_0 m(v_1, v_2)$ for each $v_1, v_2 \in H[v, \gamma_0]$, where m is some metric.
Then, the operator Ψ admit a fixed point in the closed ball $H[v, \gamma_0]$.

Majorizing sequences play an important role in the study of iterative methods.

Definition 2.3. *Let $\{s_n\}$ stand for a nonnegative sequence of numbers and let $\{z_n\}$ be a sequence in a Banach space. Assume:*

$$\|z_{n+1} - z_n\| \leq s_{n+1} - s_n \text{ for each } n = 0, 1, 2 \dots.$$

Then, the sequence $\{s_n\}$ is said to be majorizing for the sequence $\{y_n\}$. In the case of convergence of the sequence $\{s_n\}$, the sequence $\{z_n\}$ is Cauchy in the Banach space and as such it is convergent to some z^*, i.e., $\lim_{n \longrightarrow \infty} z_n = z^*$.

3. Convergence

Let $T = [0, +\infty)$. The following conditions are used in the semi-local convergence analysis of the NLM.

Assume:

(A_1) There exists a continuous and nondecreasing function $w_0 : T \longrightarrow \mathbb{R}$ such that the equation $w_0(t) - 1 = 0$ has a smallest solution $\rho_0 \in T - \{0\}$. Set $T_0 = [0, \rho_0)$.

(A_2) There exist CNF $w : T_0 \longrightarrow \mathbb{R}$, and $w_1 : T \longrightarrow \mathbb{R}$. Let $\lambda > 0$. Define the sequence $\{s_n\}$ for $s_0 = 0$, some $s_1 \in [0, \rho_0)$, and each $n = 0, 1, 2, \ldots$ by

$$\begin{aligned} & s_{n+1} \\ & = s_n + \frac{[\int_0^1 w((1-\theta)(s_n - s_{n-1}))d\theta + w_0(s_{n-1}) + w_1(s_{n-1})](s_n - s_{n-1})}{1 - w_0(s_n)} \end{aligned} \tag{3.1}$$

It is shown in Theorem 3.1 that $\{s_n\}$ is a majorizing sequence for $\{x_n\}$. But let us first present a general convergence criterion for it.

(A_3) There exists a parameter $\rho \in [0, \rho_0)$ such that for each $n = 0, 1, 2, \ldots$
$w_0(s_n) < 1$ and $s_n \leq \rho$.
It follows by (3.1) and (A_3) that $0 \leq s_n \leq s_{n+1} \leq \rho$
and there exists $s \in [0, \rho)$ such that $\lim_{n \to +\infty} s_n = s$.
The functions "w" and sequence $\{s_n\}$ are connected to the operators on NLM.

(A_4) There exists a linear operator M such that
$\lambda \|L(x) - M\| \leq w_0(\|x - x_0\|)$ for each $x \in D$.
Set $D_0 = D \cap S(x_0, \rho_0)$.

(A_5) $\lambda \|F'(x) - F'(y)\| \leq w(\|x - y\|)$ for each $x, y \in D_0$ and
$\lambda \|L(x) - M\| \leq w_1(\|x - x_0\|)$ for each $x \in D_0$.

(A_6) There exist $x_1 \in D$ generated by NLM so that $\|x_1 - x_0\| \leq s_1$, and the multi-operator $(F(x_0) + M(. - x_0) + G(.))^{-1}$ is Aubin continuous at $(0, x_1)$ with corresponding parameters α and β.

(A_7) For $\rho > s_1$
$2\rho - s_1 < \alpha$,

$$\frac{1}{\lambda}\left[\int_0^1 w_0((1-\theta)\rho)d\theta + w_0(\rho) + \int_0^1 w((1-\theta)\rho)d\theta + w_1(\rho)\right]\rho \leq \beta,$$

and $w_0(\rho) < 1$.
and

(A_8) $S[x_0, s] \subset D$.

Next, the semi-local convergence analysis of NLM is developed using the conditions $(A_1) - (A_8)$.

THEOREM 3.1. *Assume that the conditions $(A_1) - (A_8)$ are valid. Then, the sequence $\{x_n\}$ generated by NLM is well defined in $S(x_0, s)$, remains in $S(x_0, s)$ for each $n = 0, 1, 2, ..$ and is convergent to some $x^* \in S[x_0, s]$ solving the generalized equation (1.1). Moreover, the following error estimates hold for each $n = 0, 1, 2, \ldots$*

$$\|x^* - x_n\| \leq s - s_n. \tag{3.2}$$

Proof. Mathematical induction is employed to show the assertion for each $n = 0, 1, 2, \ldots$

$$\|x_{n+1} - x_n\| \leq s_{n+1} - s_n < s \tag{3.3}$$

The assertion (3.3) holds for $n = 0$ by (A_2) and the definition of s_1 in (A_6). Let us assume that there exist $x_1, \ldots, x_m$ generated by NLM satisfying for all integers $m = 0, 1, 2, \ldots, n-1$

$$\|x_m - x_{m-1}\| \leq s_m - s_{m-1}.$$

Then,

$$\begin{aligned}
\|x_m - x_0\| &\leq \|x_m - x_{m-1}\| + \|x_{m-1} - x_{m-2}\| + \ldots + \|x_1 - x_0\| \\
&\leq s_m - s_{m-1} + s_{m-1} - s_{m-2} + \ldots + s_1 - s_0 = s_m < s,
\end{aligned}$$

and

$$\|x_m - x_1\| \leq \|x_m - x_0\| + \|x_0 - x_1\| \leq s_m - s_1 \leq s - s_1.$$

Pic $x \in U(x_m, \|x_m - x_0\|)$ to be arbitrary.

Define the operator

$$Q = F(x_0) + M(x - x_0) + G(x),$$

and the multi-operator

$$\psi_m(x) = Q^{-1}[F(x_0) + M(x - x_0) - F(x_m) - L(x_m)(x - x_m)].$$

The conditions of the Theorem 2.2 are validated in turn next. By applying the conditions (A_5) and (A_7), we get

$$\begin{aligned}
& \|F(x_0) + M(x - x_0) - F(x_m) - L(x_m)(x - x_m)\| \\
\leq\; & \|F(x) - F(x_0) - M(x - x_0)\| \\
& + \|F(x) - F(x_m) - F'(x_m)(x - x_m)\| \\
& + \|F'(x_m) - M\|\|x - x_m\| + \|M - L(x_m)\|\|x - x_m\| \\
& \frac{1}{\lambda}\left[\int_0^1 w_0((1-\theta)\|x - x_0\|)d\theta\|x - x_0\|\right. \\
& + \int_0^1 w((1-\theta)\|x - x_m\|)d\theta\|x - x_m\| \\
& + w_0(\|x_m - x_0\|)\|x - x_m\| + w_1)\|x_m - x_0\|)\|x - x_m\|] \\
\leq\; & \frac{1}{\lambda}\left[\int_0^1 w_0((1-\theta)\rho)d\theta + w_0(\rho) + \int_0^1 w((1-\theta)\rho)d\theta + w_1(\rho)\right]\rho \leq \beta.
\end{aligned}$$

Notice that $x_m \in Q^{-1}[F(x_0) + M(x_m - x_0) - F(x_{m-1} - L(x_{m-1}(x_m - x_{m-1})]$.

By Aubin property of $Q^{-1}(.)$ at $(0, x_1)$ with modulus λ and parameters α, β we have in turn

$$
\begin{aligned}
& d(x_m, \psi_m(x_m)) \\
\leq\ & e\{Q^{-1}[F(x_0) + M(x_m - x_0) - F(x_{m-1}) - L(x_{m-1})(x_m - x_{m-1})] \\
& \cap S(x_1, \alpha), \psi_m(x_m)\} \\
\leq\ & \lambda \|F(x_m) - F(x_{m-1}) - L(x_{m-1})(x_m - x_{m-1})\| \\
\leq\ & \lambda \|F(x_m) - F(x_{m-1}) - F'(x_{m-1})(x_m - x_{m-1})\| \\
& + \lambda \|(L(x_{m-1}) - F'(x_{m-1}))(x_m - x_{m-1})\| \\
\leq\ & \lambda \|F(x_m) - F(x_{m-1}) - F'(x_{m-1})(x_m - x_{m-1})\| \\
& + \lambda \|(L(x_{m-1}) - M)(x_m - x_{m-1})\| \\
& + \lambda \|(F'(x_{m-1}) - M)(x_m - x_{m-1})\| \\
\leq\ & \Big[\int_0^1 w((1-\theta)\|x_m - x_{m-1}\|) d\theta \|x_m - x_{m-1}\| \\
& + w_0(\|x_{m-1} - x_0\|)\|x_m - x_{m-1}\| + w_1(\|x_{m-1} - x_0\|)\|x_m - x_{m-1}\|\Big] \\
=\ & \gamma(1 - w_0(\|x_m - x_0\|)),
\end{aligned}
$$

where

$$
\begin{aligned}
\gamma \ = \ & \frac{\left[\int_0^1 w((1-\theta)\|x_m - x_{m-1}\|) d\theta + w_0(\|x_{m-1} - x_0\|) + w_1(\|x_{m-1} - x_0\|)\right]}{1 - w_0(\|x_m - x_0\|)} \\
& \times \|x_m - x_{m-1}\|
\end{aligned} \tag{3.4}
$$

Pick $v_1, v_2 \in S(x_m, \|x_m - x_0\|)$. Then, we get

$$
\begin{aligned}
& e\{\psi_m(v_1) \cap S(x_m, \|x_m - x_0\|), \psi_m(v_2)\} \\
\leq\ & e\{\psi_m(v_1) \cap S(x_1, \alpha), \psi_m(v_2)\} \\
\leq\ & \lambda \|M - L(x_m)\| \|v_1 - v_2\| \\
\leq\ & w_0(\|x_m - x_0\|)\|v_1 - v_2\| \leq w_0(\rho)\|v_1 - v_2\|,
\end{aligned}
$$

where $w_0(\rho) < 1$, by the definition of ρ. Thus, the Theorem 2.2 is applicable if we take $\psi = \psi_m, \gamma_0 = \gamma$ and $\delta_0 = \delta = w_0(\|x_m - x_0\|)$. So, there exists $x_{m+1} \in S[x^*, \rho]$ satisfying

$$
x_{m+1} \in Q^{-1}[F(x_0) + M(x_{m+1} - x_0) - F(x_m) - L(x_m)(x_{m+1} - x_m)]
$$

leading to

$$
\begin{aligned}
\|x_{m+1} - x_m\| \ \leq \ & \frac{\left[\int_0^1 w((1-\theta)(s_m - s_{m-1})) d\theta + w_0(\|s_{m-1}\|) + w_1(s_{m-1})\right]}{1 - w_0(s_m)} \\
& \times (s_m - s_{m-1}) \qquad (3.5) \\
\leq \ & s_{m+1} - s_m,
\end{aligned}
$$

where we used (A_2), (3.4) and the induction hypothesis (3.3).
By (A_3) and (3.5), we obtain

$$\begin{aligned}\sum_{m=m_0}^{\infty}\|x_{m+1}-x_m\| &\leq \sum_{m=m_0}^{\infty}(s_{m+1}-s_m)\\ &\leq s-s_{m_0}<+\infty.\end{aligned}$$

if follows that the sequence $\{x_m\}$ is complete in a Banach space B_1, and as such it is convergent to some $x^* \in S[x_0, s]$. Then, by (3.3), we can write

$$\begin{aligned}\|x_{m+j}-x_n\| &\leq \|x_{m+j}-x_{m+j-1}\|+\|x_{m+j-1}-x_{m+j-2}\|+\|x_{m+1}-x_n\|\\ &\leq s_{m+j}-s_{m+j-1}+s_{m+j-1}-s_{m+j-2}+\ldots+s_{m+1}-s_n\\ &= s_{m+j}-s_m. \qquad (3.6)\end{aligned}$$

By letting $j \longrightarrow +\infty$ in (3.6), we conclude that (3.2) is valid. In view of the definition the sequence $\{x_m\}$, $0 \in F(x_m)+L(x_m)(x_{m+1}-x_m)+G(x_{n+1})$ for each $m = 0, 1, \ldots$ Then, by letting $m \longrightarrow +\infty$, we deduce that $0 \in F(x^*) + G(x^*)$. □

REMARK 3.2. *A popular choice for $M = F'(x_0)$. But this is not necessarily the most flexible choice. The two conditions in (A_3) are very general. By specializing the functions w_0, w and w_1, we can provide other stronger conditions that imply the ones in (A_3). Let us consider the interesting Lipchitz case, i.e. when $w_0(t) = l_0t, w(t) = lt$ and $w_1(t) = l_1t$. Then, the sequence $\{s_n\}$ in (A_2) reduces for $l_2 = l_0 + l_1$ to*

$$s_{n+1} = s_n + \frac{\left(\frac{l}{2}(s_n-s_{n-1})+s_{n-1}\right)(s_n-s_{n-1})}{1-l_0(s_n)} \qquad (3.7)$$

Such sequences appear as majorant of Newton-like methods for solving nonlinear equations (i.e. when $G = \{0\}$). The Kantorovich-type convergence conditions in such studies imply the ones in (A_3) but not necessarily vice versa [20,35]. Our approach for the study of majorizing sequence $\{s_n\}$ has provided even weaker convergence conditions than the Kantorovich-type [4,5,6].

4. Conclusion

A very general theory for studying the convergence of Newton-like methods is developed for generating sequences approximating a solution of a generalized equation involving set-valued operators. The semi-local analysis of convergence depend on the Aubin property and the concept of generalized continuity. The error analysis includes, computable upper error bounds on the norms $\|x_{n+1} - x_n\|$ and $\|x^* - x_n\|$. In particular, the semi-local analysis of convergence is based on majorizing sequences for $\{x_n\}$ generated by NLM. It is shown that even specializations of the operators involved lead to better results when compared to existing ones (see Remark 3.2). The future direction of our research involves the application of the developed theory on other methods [1–3, 9, 14, 17, 18, 21–35].

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