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Global Stability Analysis of Onchocerciasis Transmission Dynamics with Vigilant Compartment in Two Interacting Populations

K. M. Adeyemo

Department of Mathematics, Hallmark University Ijebu-Itele, Ogun State, Nigeria
mikyade2019@gmail.com

Abstract. A deterministic compartmental model for the transmission dynamics of onchocerciasis with vigilant compartment in two interacting populations is studied. The model is qualitatively analyzed to investigate its global asymptotic behavior with respect to disease-free and endemic equilibria. It is shown, using a linear Lyapunov function, that the disease-free equilibrium is globally asymptotically stable when the associated basic reproduction number, $\mathcal{R}_0 < 1$. When the basic reproduction number $\mathcal{R}_0 > 1$, under some certain conditions on the model parameters, we prove that the endemic equilibrium is globally asymptotically stable with the aid of a suitable nonlinear Lyapunov function.

1. Introduction

Onchocerciasis is one of the neglected tropical diseases caused by the parasite Onchocerca Volvulus, a filarial nematode [3]. The disease is transmitted from one person to another by repeated bites of black flies. The disease is endemic in Sub-saharan Africa. Many researchers have worked on many ways to reduce the spread of the disease. For instance, Remme et al. [14] used skin snip survey in West Africa to investigate the impact of controlling black flies by larviciding. Plaisier et al. [13] used micro simulation model to determine the period required for combining annual ivermectin treatment and vector control in the onchocerciasis Control Programme in West Africa. Alley et al. [3] used a computer simulation model to study prevention of onchocerciasis by using macrofilaricide which kills the adult worms. Asha Hassan & Nyimvua Shaban [5] investigated the effects of four control strategies on the spread of the disease.

In this paper, we consider global stability analysis of onchocerciasis transmission dynamics with vigilant compartment. The human population is sub-divided into four compartments and the vector population is sub-divided into three compartments. We show global asymptotic behaviour in disease-free and endemic equilibria. This is an extension of the work done in [1] where the author worked on the local stability of the model without the vigilant compartment.

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The case of onchocerciasis model presented in this paper incorporates a new class of human compartment called vigilant individuals denoted by $V_h(t, x_i)$. The individuals in the compartment are assumed to be tired of onchocerciasis and guide against it by strictly adhering to the vector control measures such as: regular indoor residual spraying (IRS), insecticide-treated bed-nets (ITNs), clearing of stagnant water bodies and drainages and the use of head-nets in the outdoor. The rest of the paper is organized as follows: the description of the model and theorems on positivity of solutions and reproduction number are given in section 2 while section 3, we explored the global asymptotic stability of the disease-free equilibrium and endemic equilibrium with a concluding remark.

2. Model Description

Two interacting populations are considered; the humans and the black-flies populations. The human population is partitioned into four compartments: the susceptible human compartment; S_h, the exposed compartment; E_h, the infectious human compartment; I_h and the vigilant compartment; V_h. The black-fly population is partitioned into three compartments: susceptible vector; S_v, the exposed vector compartment; E_v and the infective vector compartment. The total human and vector populations at any given time, t, are respectively given by; $N = S_h(t)+E_h(t)+I_h(t)+V_h(t)$ and $Ve = S_v(t)+E_v(t)+I_v(t)$. We assume that the transmission of onchocerciaisis in susceptible hosts is only through contact with infectious vector. We also assume that susceptible vector becomes infectious as a result of contact with infectious hosts during blood meal. The population under study is assumed to be large enough to be modelled deterministically. The following system of non-linear ordinary differential equations, with non-negative initial conditions, describes the dynamics of onchocerciaisis epidemics.

$$\left.\begin{aligned}
\frac{dS_h(t,x_i)}{dt} &= \sum_{i=0}^{L}(1-\tau)\Psi_h(x_i) - \frac{\delta\lambda_h(x_i)\sigma_h(t,x_i)I_v(t)}{N_h(t,x_i)} - \mu_h(x_i)S_h(t,x_i)\\
\frac{dE_h(t,x_i)}{dt} &= \sum_{i=0}^{L}\frac{\delta\lambda_h(x_i)\sigma S_h(t,x_i)I_v(t)}{N_h(t,x_i)} - (\alpha_h(x_i)+\mu_h(x_i))E_h(t,x_i)\\
\frac{dI_h(t,x_i)}{dt} &= \sum_{i=0}^{L}(1-\theta)\alpha_h(x_i)E_h - (\gamma(x_i)+\mu_h(x_i))I_h(t,x_i)\\
\frac{dV_h(t,x_i)}{dt} &= \tau\Psi_h(x_i)N(t,x_i) + \theta\alpha_h(x_i)E_h(t,x_i) + \gamma(x_i)I_h(t,x_i) - \mu_h(x_i)V_h(t,x_i)\\
\frac{dS_v}{dt} &= \Psi_v - \frac{\delta\lambda_v(x_i)S_v(t)I_h(t,x_i)}{N_h(t,x_i)} - \mu_v S_v(t)\\
\frac{dE_v}{dt} &= \frac{\delta\lambda_v(x_i)S_v(t)I_h(t,x_i)}{N_(t,x_i)} - (\alpha_v+\mu_v)E_v(t)\\
\frac{dI_v}{dt} &= \alpha_v E_v(t) - \mu_v I_v(t)
\end{aligned}\right\} \tag{2.1}$$

subject to the following initial conditions:

$$\begin{aligned}
S_h(0,x_i) = S_{0h}(x_i), E_h(0,x_i) = E_{0h}(x_i),\\
I_h(0,x_i) = I_{0h}(x_i), V_h(0,x_i) = V_{0h}(x_i)\\
S_v(0) = S_{0v}, E_v(0) = E_{0v}, I_v(0) = I_{0v}
\end{aligned} \tag{2.2}$$

Symbols	Definitionss
$S_h(t, x_i)$	Number of susceptible humans at time t and discrete age x_i
$E_h(t, x_i)$	Number of exposed humans at time t and discrete age x_i
$I_h(t, x_i)$	Number of infectious humans at time t and discrete age x_i
$V_h(t, ai)$	Number of vigilant host humans at time t and discrete age x_i
$S_v(t)$	Number of susceptible black-flies at time t
$E_v(t)$	Number of exposed black-flies at time t
$I_v(t)$	Number of infectious black-flies at time t
$\Psi_h(x_i)$	Recruitment term of the susceptible humans at discrete age x_i
Ψ_v	Recruitment term of the susceptible vectors
δ	Biting rate of the vector
$\lambda_h(x_i)$	Probability that a bite by an infectious vector results in transmission
of disease to human	
at discrete age x_i	
λ_v	Probability that a bite results in transmission of parasite
to a susceptible vector	
$\mu_h(x_i)$	Per capita death rate of humans at discrete age x_i
μ_v	Per capita death rate of vector
$\gamma_h(x_i)$	Disease-induced death rate of humans at discrete age x_i
γ_v	Disease-induced death rate of vectors
$\alpha_h(x_i)$	Per capita rate of progression of humans from the exposed state to the
infectious state	
at discrete age x_i	
α_v	Per capita rate of progression of vectors from the exposed state to the
infectious state	
$\nu_h(x_i)$	Humans disease-inhibiting factor at discrete age x_i
ν_v	Vectors disease-inhibiting factor
$\tau(x_i)$	Proportion of human population that is born vigilant at discrete age x_i
$\theta(x_i)$	Proportion of exposed humans that becomes vigilant at discrete age x_i
$\gamma(x_i)$	Per capita recovery rate of infectious humans to the vigilant state at discrete age x_i

Model assumptions

The formulation of the compartmental model is based on the following assumptions:

1. That only humans are vigilant.
2. That humans are born either susceptible or vigilant.
3. That exposed humans progress to either become infectious or vigilant. The assumption that exposed humans can become vigilant is motivated by the possibility of treating Plasmodium vivax infection which is at the dormant liver stage
4. That all infectious humans become vigilant upon recovery due to treatment
5. That strict adherence to vector control measures by the vigilant humans does not result into re-infection.

6. All black-flies are born susceptible.
7. That the susceptible black-flies, when infected, becomes exposed black-flies who are not yet infectious.
8. That the exposed black-flies progress to become infectious only.
9. That the infectious black-flies remain infectious for life. That is, there is no recovered class for black-fly population.
10. That a proportion of susceptible humans is infected by infectious mosquitoes and that susceptible mosquitoes become infected when in contact with a proportion of infectious humans

To carry out the analysis of the formulated model (2.1), it is convenient to rescale the variables by dividing the number of the individuals in the subpopulations by their respective total number of populations $Nh(t, x_i)$ and $N_v(t)$. This process is achieved by making the following change of variables:

$\bar{S}_h(t, x_i) = \frac{S_h(t,x_i)}{N_h(t,x_i)}$, $\bar{E}_h(t, x_i) = \frac{E_h(t,x_i)}{N_h(t,x_i)}$, $\bar{I}_h(t, x_i) = \frac{I_h(t,x_i)}{N_h(t,x_i)}$, $\bar{V}_h(t, x_i) = \frac{V_h(t,x_i)}{N_h(t,x_i)}$,
$\bar{S}_v(t, x_i) = \frac{S_v(t,x_i)}{N_v(t,x_i)}$, $\bar{E}_v(t, x_i) = \frac{E_v(t,x_i)}{N_v(t,x_i)}$, $\bar{I}_v(t, x_i) = \frac{I_v(t,x_i)}{N_v(t,x_i)}$
so that

$$\bar{S}_h(t, x_i) + \bar{E}_h(t, x_i) + \bar{I}_h(t, x_i) + \bar{V}_h(t, x_i) = 1 \text{ and } \bar{S}_v(t, x_i) + \bar{E}_v(t, x_i) + \bar{I}_v(t, x_i) = 1$$

The consequence of this, we have $\Psi_h(x_i) = \mu_h(x_i)$, $\Psi_v(x_i) = \mu_v$ and $\sigma = \frac{N_v(t)}{N_h(t,x_i)}$. After dropping of bars (), model (2.1) gives rise to the following system of equations:

$$\left.\begin{array}{l} \frac{dS_h(t,x_i)}{dt} = (1-\tau)\Psi_h(x_i) - \sum_{i=0}^{L} \delta\lambda_h(x_i)\sigma_h(t, x_i)I_v(t) - \mu_h(x_i)S_h(t, x_i) \\ \frac{dE_h(t,x_i)}{dt} = \sum_{i=0}^{L} \delta\lambda_h(x_i)\sigma S_h(t, x_i)I_v(t) - (\alpha_h(x_i) + \mu_h(x_i))E_h(t, x_i) \\ \frac{dI_h(t,x_i)}{dt} = \sum_{i=0}^{L}(1-\theta)\alpha_h(x_i)E_h - (\gamma(x_i) + \mu_h(x_i))I_h(t, x_i) \\ \frac{dV_h(t,x_i)}{dt} = \tau\Psi_h(x_i) + \theta\alpha_h(x_i)E_h(t, x_i) + \gamma(x_i)I_h(t, x_i) - \mu_h(x_i)V_h(t, x_i) \\ \frac{dS_v}{dt} = \Psi_v - \delta\lambda_v(x_i)S_v(t)I_h(t, x_i) - \mu_v S_v(t) \\ \frac{dE_v}{dt} = \delta\lambda_v(x_i)S_v(t)I_h(t, x_i) - (\alpha_v + \mu_v)E_v(t) \\ \frac{dI_v}{dt} = \alpha_v E_v(t) - \mu_v I_v(t) \end{array}\right\} \quad (2.3)$$

subject to the following initial conditions:

$$\begin{aligned} S_h(0, x_i) = S_{0h}(x_i), E_h(0, x_i) = E_{0h}(x_i), \\ I_h(0, x_i) = I_{0h}(x_i), V_h(0, x_i) = V_{0h}(x_i) \\ S_v(0) = S_{0v}, E_v(0) = E_{0v}, I_v(0) = I_{0v} \end{aligned} \quad (2.4)$$

3. Global Stability Analysis

Here, we explore the global asymptotic stability of the DFE and EE for the special case with no loss of immunity acquired by the recovered individuals. We use the concept of Lyapunov functions to analyze the global stability

3.1. **Global Stability of Disease-free Equilibrium.** The following result establishes the global asymptotic behavior of system (2.1) around E_0 which is determined by the basic reproduction number $\mathcal{R}_0$.

Theorem 3:

The disease-free equilibrium (2.12) of model (2.1) is globally asymptotically stable in Ω whenever $\mathcal{R}_0 \leq 1$

Proof:

Consider the linear Lyapunov function of the form

$$\mathcal{M} = d_1 E_h(t, x_i) + d_2 I_h(t, x_i) + d_3 E_v(t) + d_4 I_v(t) \tag{3.1}$$

where

$$d_1 = \frac{\alpha_h(x_i)(1-\theta)}{(\alpha_h(x_i)+\mu_h(x_i))(\gamma(x_i)+\mu_h(x_i))}$$

$$d_2 = \frac{1}{(\gamma(x_i)+\mu_h(x_i))}$$

$$d_3 = \frac{1}{\delta\lambda_v}$$

$$d_4 = \frac{\alpha_v+\mu_v}{\delta\lambda_v\alpha_v}$$

In what follows, the time derivative of $\mathcal{M}$ given by (3.1) along the solutions of the model (2.3) yields

$$\begin{aligned}
\dot{\mathcal{M}} = {} & \frac{\alpha_h(x_i)(1-\theta)[\delta\lambda_h(x_i)\sigma_h(t,x_i)I_v - (\alpha_h(x_i)I_v + \mu_h(x_i))E_h(t,x_i)]}{(\alpha_h(x_i)+\mu_h(x_i))(\gamma(x_i)+\mu_h(x_i))} \\
& + \sum_{i=0}^{L}(\gamma(x_i)+\mu_h(x_i))[(1-\theta)\alpha_h(x_i)E_h(t,x_i) - (\gamma(x_i)+\mu_h(x_i))I_h(t,x_i)] \\
& + \frac{1}{\delta\lambda_v}[\delta\lambda_v S_v I_h(t,x_i) - (\alpha_v+\mu_v)E_v] + \frac{\alpha_v+\mu_v}{\delta\lambda_v\alpha_v}[\alpha_v E_v - \mu_v I_v] \\
& = \sum_{i=0}^{L}\frac{\delta\lambda_h(x_i)\sigma\alpha_h(x_i)(1-\theta)S_h(t,x_i)I_v}{(\alpha_h(x_i))(\gamma(x_i)+\mu_h(x_i))} - \frac{\alpha_h(x_i)(1-\theta)E_h(t,x_i)}{\gamma(x_i)+\mu_h(x_i)} \\
& + \sum_{i=0}^{L}\frac{(1-\theta)E_h(t,x_i)}{(\gamma(x_i)+\mu_h(x_i))} - I_h(t,x_i) + S_v I_h(t,x_i) - \frac{(\alpha_v+\mu_v)\mu_v I_v}{\delta\lambda_v\alpha_v} \\
& \leq \sum_{i=0}^{L}\frac{\delta\lambda_h(x_i)\sigma\alpha_h(x_i)(1-\theta)(1-\tau)I_v}{(\alpha_h(x_i)+\mu_h(x_i))(\gamma(x_i)+\mu_h(x_i))} - \frac{(\alpha_v+\mu_v)\mu_v I_v}{\delta\lambda_v\alpha_v} \\
& = \left[\sum_{i=0}^{L}\frac{\delta\lambda_h(x_i)\sigma\alpha_h(x_i)(1-\theta)(1-\tau)}{(\alpha_h(x_i)+\mu_h(x_i))(\gamma(x_i)+\mu_h(x_i))} - \frac{(\alpha_v+\mu_v)\mu_v}{\delta\lambda_v\alpha_v}\right]I_v \\
& = \frac{(\alpha_v+\mu_v)\mu_v}{\delta\lambda_v\alpha_v}[\mathcal{R}_0^2 - 1]I_v
\end{aligned}$$

We have that $\dot{\mathcal{M}} \leq 0$ whenever $\mathcal{R}_0 \leq 1$ with $\dot{\mathcal{M}} = 0$ if and only if $I_v = 0$. We also see that $(S_h(t,x_i), E_h(t,x_i), I_h(t,x_i), V_h(t,x_i), S_v(t), E_v(t))$ tends to $((1-\tau), 0, 0, 0, 1, 0)$ as $t \to \infty$ since $I_v(t) \to 0$ as $t \to \infty$. By LaSalle's principle [7], one concludes that every solution of the model (2.3) in Ω approaches the disease-free equilibrium, E_0, as $t \to \infty$.

We have that $\dot{\mathcal{M}} \leq 0$ whenever $\mathcal{R}_0 \leq 1$ with $\dot{\mathcal{M}} = 0$ if and only if $I_v = 0$. We also see that $(S_h(t,x_i)), E_h(t,x_i), I_h(t,x_i), V_h(t,x_i), S_v(t), E_v())$ Hence E_0 is globally asymptotically stable in Ω if $\mathcal{R}_0 \leq 1$ $\square$

The global asymptotic stability analysis of the endemic equilibrium is considered next for the special case with $\tau = \theta = 0$. The disease-present (endemic) equilibrium of the model (2.3) is referred to the steady-state solution where at least one of the infected compartments is nonzero. Let the arbitrary endemic equilibrium of the model (2.1) be represented by $E_e = (S_h^{**}(x_i), E_h^{**}(x_i), I_h^{**}(x_i), V_h^{**}(x_i), S_m^{**}, E_m^{**}, I_m^{**})$ In order to do this, nonlinear Lyapunov function is used of Goh-Volterra type [6, 15].

Theorem 4: The unique endemic equilibrium,E_e, of the model (2.3) is globally asymptotically stable if $\mathcal{R}_0 > 1$.

Proof: Let $\mathcal{R}_0 > 1$ so that there exists a unique endemic equilibrium and consider the nonlinear Lyapunov function defined by

$$\mathcal{M} = \left(S_h(t,x_i) - S_h^{**}(x_i) - S_h^{**}(x_i)\ln\frac{S_h(t,x_i)}{S_h^{**}(x_i)}\right) + \left(E_h(t,x_i) - E_h^{**}(x_i) - E_h^{**}(x_i)\ln\frac{E_h(t,x_i)}{E_h^{**}(x_i)}\right)$$
$$+ \sum_{i=0}^{L}\frac{\alpha_h(x_i)+\mu_h(x_i)}{\alpha_h(x_i)}\left[I_h(t,x_i) - I_h^{**}(x_i) - I_h^{**}(x_i)\ln\frac{I_h(t,x_i)}{I_h^{**}(x_i)}\right] + \left(S_v - S_v^{**} - S_v^{**}\ln\frac{S_v}{S_v^{**}}\right)$$
$$+ \left(E_v - E_v^{**} - E_v^{**}\ln\frac{E_v}{E_v^{**}}\right) + \frac{\alpha_v+\mu_v}{\alpha_v}\left[I_v - I_v^{**} - I_v^{**}\ln\frac{I_v}{I_v^{**}}\right]$$

With Lyapunov time-derivative given as

$$\dot{\mathcal{M}} = \dot{S}_h(t,x_i) - \frac{S_h^{**}(x_i)}{S_h(x_i)}\dot{S}_h(t,x_i) + \dot{E}_h(t,x_i) - \frac{E_h^{**}(x_i)}{E_h(x_i)}\dot{E}_h(t,x_i)$$
$$+ \sum_{i=0}^{L}\frac{\alpha_h(x_i)+\mu_h(x_i)}{\alpha_h(x_i)}\left(\dot{I}_h(t,x_i) - \frac{I_h^{**}(x_i)}{I_h(x_i)}\dot{I}_h(t,x_i)\right)$$
$$+ \dot{S}_v - \frac{S_v^{**}}{S_v}\dot{S}_v + \dot{E}_v - \frac{E_v^{**}}{E_v}\dot{E}_v + \frac{\alpha_v+\mu_v}{\alpha_m}\left(\dot{I}_v - \frac{I_v^{**}}{I_v}\dot{I}_v\right) \quad (3.2)$$

Using equations of the model (2.3) in (3.3) we obtain

$$\dot{\mathcal{M}} = (1-\tau)\Psi_h(x_i) - \sum_{i=0}^{L} \delta\lambda_h(x_i)\sigma S_h(t,x_i)I_v - \mu_h(x_i)S_h(t,x_i) \tag{3.3}$$
$$- \sum_{i=0}^{L} \frac{S_h^{**}(x_i)}{S_h(t,x_i)} \left(\Psi_h(x_i) - \delta\lambda_h(x_i)\sigma S_h(t,x_i)I_v - \mu_h(x_i)S_h(t,x_i)\right)$$
$$+ \sum_{i=0}^{L} \delta\lambda_h(x_i)S_h(t,x_i)I_v + [\alpha_h + \mu_h]E_h(t,x_i) - \sum_{i=0}^{L} \frac{E_h^{**}(x_i)}{E_h(t,x_i)} \left(\delta\lambda_h(x_i)\sigma S_h(t,x_i)I_v + [\alpha_h + \mu_h]E_h(t,x_i)\right)$$
$$+ \sum_{i=0}^{L} \frac{\alpha_h(x_i) + \mu_h(x_i)}{\alpha_h(x_i)} \times ((1-\theta)\alpha_h(x_i)E_h(t,x_i) - [r(x_i) + \mu_h(x_i) + \gamma_h(x_i)])I_h(t,x_i)$$
$$- \sum_{i=0}^{L} \frac{I_h^{**}(x_i)(\alpha_h(x_i) + \mu_h(x_i))}{I_h(t,x_i)\alpha_h(x_i)} \times ((1-\theta)\alpha_h(x_i)E_h(t,x_i) - [r(x_i) + \mu_h(x_i) + \gamma_h(x_i)])I_h(t,x_i)$$
$$+ \Psi_v - \delta\lambda_v S_v I_h(t,x_i) - \mu_v S_v - \frac{S_v^{**}}{S_v} \left(\Psi_v - \delta\lambda_v S_v I_h(t,x_i) - \mu_v S_v\right) + \delta\lambda_v S_v I_h(t,x_i) + [\alpha_v + \mu_v]E_v$$
$$- \frac{E_v^{**}}{E_h(t,x_i)} \left(\delta\lambda_h(x_i)S_h(t,x_i)I_v + [\alpha_h + \mu_h]E_v\right) + \frac{\alpha_v + \mu_v}{\alpha_v} \left[\alpha_v E_v - [\mu_v + \gamma_v]I_v - \frac{I_v^{**}}{I_v}(\alpha_v E_v - [\mu_v + \alpha_v]I_v)\right]$$

Simplifying $\dot{\mathcal{M}}$ gives

$$\dot{\mathcal{M}} = \sum_{i=0}^{L} \Psi_h(x_i) \left(1 - \frac{S_h^{**}(x_i)}{S_h(t,x_i)}\right) - \sum_{i=0}^{L} \mu_h(x_i)S_h(t,x_i) \left(1 - \frac{S_h^{**}(x_i)}{S_h(t,x_i)}\right) + \sum_{i=0}^{L} \delta\lambda_h(x_i)S_h^{**}(x_i)I_v \tag{3.4}$$
$$- \sum_{i=0}^{L} \frac{E_h^{**}(x_i)\delta\lambda_h(x_i)S_h(t,x_i)I_v}{E_h(t,x_i)} + \sum_{i=0}^{L} (\alpha_h(x_i) + \mu_h(x_i))E_h^{**}(x_i) \tag{3.5}$$
$$- \sum_{i=0}^{L} \frac{(\alpha_h(x_i) + \mu_h(x_i))}{\alpha_h(x_i)} (r(x_i) + \mu_h(x_i)\gamma_h(x_i))I_h(t,x_i) \tag{3.6}$$
$$- \sum_{i=0}^{L} \frac{(\alpha_h(x_i) + \mu_h(x_i))I_h^{**}(x_i)E_h(t,x_i)}{I_h(t,x_i)} + \sum_{i=0}^{L} \frac{\alpha_h(x_i) + \mu_h(x_i)}{\alpha_h(x_i)} (r(x_i) + \mu_h(x_i) + \gamma_h(x_i))I_h^{**}(x_i) \tag{3.7}$$
$$+ \mu_v \left(1 - \frac{S_v^{**}}{S_v}\right) - \mu_v S_v \left(1 - \frac{S_v^{**}}{S_v}\right) + \delta\lambda_v S_v^{**}(x_i)I_h - \frac{E_v^{**}\delta\lambda_v S_v I_h}{E_v} + (\alpha_v + \mu_v)E_v^{**} \tag{3.8}$$
$$- \frac{(\alpha_v + \mu_v)(\mu_v + \gamma_v)I_v}{\alpha_v} - \frac{(\alpha_v + \mu_v)I_v^{**}E_v}{I_v} + \frac{(\alpha_v + \mu_v)(\mu_v + \gamma_v)I_v}{\alpha_v} \tag{3.9}$$

At the endemic equilibrium E_e, we get from model (2.4) that

$$\left.\begin{aligned}
&\Psi_h(x_i) = \textstyle\sum_{i=0}^{L} \delta\lambda_h(x_i)\sigma_h^*(x_i)I_v^* + \sum_{i=0}^{L} \mu_h(x_i)S_h^*(x_i)\\
&\alpha_h(x_i) + \mu_h(x_i) = \textstyle\sum_{i=0}^{L} \frac{\delta\lambda_h(x_i)\sigma_h^* I_v^*}{E_h^*(x_i)}\\
&\mu_h(x_i) + \gamma_h(x_i) = \textstyle\sum_{i=0}^{L} \frac{\alpha_h(x_i)E_h^*(x_i)}{I_h^*(x_i)}\\
&\Psi_v = \delta\lambda_v S_v^* I_h^* + \mu_v S_v^*\\
&\alpha_v + \mu_v = \frac{\delta\lambda_v S_v^* I_h^*}{E_v^*}\\
&\mu_v + \gamma_v = \frac{\alpha_v E_v^*}{I_v^*}
\end{aligned}\right\} \tag{3.10}$$

Using (3.6) in (3.5), we have

$$\dot{\mathcal{M}} = \sum_{i=0}^{L} \mu_h(x_i)\sigma_h^* \left(2 - \frac{S_h^*(x_i)}{S_h(t,x_i)} - \frac{S_h(t,x_i)}{S_h^*(x_i)}\right) + \sum_{i=0}^{L} \delta\lambda_h(x_i)\sigma S_h^*(x_i)I_v^* \tag{3.11}$$
$$- \sum_{i=0}^{L} \frac{\delta\lambda_h(x_i)(S_h^*)^2 I_v^*}{S_h(x_i)} + \delta\lambda_h(x_i)\sigma S_h^*(x_i)I_v - \sum_{i=0}^{L} \frac{E_h^*(x_i)\delta\lambda_h(x_i)\sigma S_h(t,x_i)I_v}{E_h(t,x_i)} + \delta\lambda_h(x_i)\sigma S_h^*(x_i)I_v$$
$$- \sum_{i=0}^{L} \frac{\delta\lambda_h(x_i)\sigma S_h^*(x_i)I_h(t,x_i)I_v^*}{I_h^*(x_i)} - \sum_{i=0}^{L} \frac{\delta\lambda_h(x_i)I_h^*(x_i)E_h(t,x_i)I_v^*}{E_h^*(x_i)I_h(t,x_i)} + \sum_{i=0}^{L} \delta\lambda_h(x_i)\sigma S_h^* I_v^* + \mu_v S_v^* \left(2 - \frac{S_v^{**}}{S_v} - \frac{S_v}{S_v^*}\right)$$
$$- \delta\lambda_v S_v^* I_h^* - \frac{\delta\lambda_v (S_v^*)^2 I_h^*}{S_v} + \delta\lambda_v S_v^* I_h - \frac{E_v^*\delta\lambda_v S_v I_h}{E_v} + \delta\lambda_v S_v^* I_h$$
$$- \frac{\delta\lambda_v S_v^* I_v I_h^*}{I_v^*} - \frac{\delta\lambda_v I_v^* E_v I_h^*}{E_v^* I_v} + \delta\lambda_v S_v^* I_h^*$$

Simplifying further, we have

$$\dot{\mathcal{M}} = \sum_{i=0}^{L} \mu_h(x_i)S_h^* \left(2 - \frac{S_h^*(x_i)}{S_h(t,x_i)} - \frac{S_h(t,x_i)}{S_h^*(x_i)}\right) + \sum_{i=0}^{L} \delta\lambda_h(x_i)\sigma S_h^{**} I_v^* \tag{3.12}$$
$$\times \left[4 - \frac{S_h^*(x_i)}{S_h(t,x_i)} - \frac{E_h^*(x_i)\sigma S_h(t,x_i)I_v}{E_h(t,x_i)\sigma S_h^* I_v^*} - \frac{I_h^*(x_i)E_h(t,x_i)}{I_h(t,x_i)E_h^*(x_i)} - \frac{I_h(t,x_i)I_v^*}{I_h^*(x_i)I_v}\right]$$
$$+ \sum_{i=0}^{L} \delta\lambda_h(x_i)\sigma S_h^* I_v^* - \frac{\delta\lambda(x_i)\sigma S_h^*(x_i)I_h(t,x_i)I_v^*}{I_h^*(x_i} + \sum_{i=0}^{L} \frac{\delta\lambda_h(x_i)\sigma S_h^*(x_i)I_h(t,x_i)(I_v^*}{I_h^*(x_i)I_v} - \delta\lambda_h(x_i)\sigma S_h^* I_v^{**}$$
$$+ \mu_v S_v^{**} \left(2 - \frac{S_v^{**}}{S_v} - \frac{S_v}{S_v^{**}}\right) + \delta\lambda_v S_v^* I_h^* \times \left[4 - \frac{S_v^*}{S_v} - \frac{E_v^* S_h(t,x_i)g(I_h)}{E_v S_v^{**} g(I_h^{**})} - \frac{I_v^{**}E_v}{I_v E_v^{**}} - \frac{I_v g(I_h^{**})}{I_v^{**} g(I_h)}\right]$$
$$+ \delta\lambda_v S_v^* I_h^*) - \frac{\delta\lambda_v S_v^* I_v I_h^*)}{I_v^*} + \frac{\delta\lambda_v S_v^* I_v (I_h^*(x_i)^2}{)} I_v^* I_h - \delta\lambda_v S_v^* I_h^*$$

Further simplification yields

$$\dot{\mathcal{M}} = -\dot{\mathcal{M}}_1 - \dot{\mathcal{M}}_2 - \sum_{i=0}^{L} \delta\lambda_h(x_i)S_h^*(x_i)I_v^* \left[1 - \frac{I_v}{I_v^*} + \frac{I_h(t,x_i)}{I_h^*(x_i)} + \frac{I_h(t,x_i)I_v^*}{I_h^*(x_i)I_v}\right] - \dot{\mathcal{M}}_3 - \dot{\mathcal{M}}_4$$
$$- \sum_{i=0}^{L} \delta\lambda_v S_v^{**} I_h^* \left[1 - \frac{I_h}{I_h^{**}} + \frac{I_v}{I_v^*} + \frac{I_v I_h^*}{I_v^* I_h}\right] \tag{3.13}$$

Where

$$\mathcal{M}_1 = \sum_{i=0}^{L} \mu_h(x_i)S_h^*(x_i) \left(\frac{S_h^*(x_i)}{S_h(t,x_i)} + \frac{S_h(t,x_i)}{S_h^*(x_i)} - 2\right),$$

$$\mathcal{M}_2 = \sum_{i=0}^{L} \delta\lambda_h(x_i)S_h^*(x_i)I_v \times \left[\frac{S_h^*(x_i)}{S_h(t,x_i)} + \frac{E_h^*(x_i)S_h(t,x_i)I_v}{E_h(t,x_i)S_h^* I_v^*} + \frac{I_h^*(x_i)E_h(t,x_i)}{I_h(t,x_i)E_h^*(x_i)} + \frac{I_h(t,x_i)I_v^*}{I_h^*(x_i)I_v} - 4\right],$$

$$\mathcal{M}_3 = \mu_v S_v^* \left(\frac{S_v^*}{S_v} + \frac{S_v}{S_v^*} - 2\right)$$

$$\mathcal{M}_4 = \delta\lambda_v S^* \left[\frac{S_v^*}{S_v} + \frac{E_v^* S_v(t,x_i)g(I_h)}{E_v S_v^* I_h^*} + \frac{I_v^* E_v}{I_v E_v^*} + \frac{I_v I_h^*}{I_v^* I_h} - 4\right]$$

Conclusion: In this article, a onchocerciasis transmission dynamics with vigilant compartment governed by system of differential equations has been theoretically analyzed. The analysis is centered on the global asymptotic behavior of solutions of the system (2.3) around the disease-free and endemic equilibria using Lyapunov functions. The system has a globally asymptotically stable disease-free equilibrium whenever the basic reproduction $\mathcal{R}_0 < 1$. Moreover, the endemic equilibrium of the system, when it exists, is shown to be globally asymptotically stable whenever the associated basic reproduction number $\mathcal{R}_0 > 1$.

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