

## The Logistic-Gompertz-Law Distribution: A Statistical Framework for Modeling Legal Case Durations and Judicial Process Efficiency in Contemporary African Judiciary Systems

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ABSTRACT. This extensive study presents the new four-parameter probability distribution Logistic-Gompertz-Law (LGL) which is an original distribution that is designed to deal with the intricate time-dependence nature of the legal process in modern situations of the African courts system. Based on the classical Gompertz and Logistic distributions, the LGL distribution, a hybrid of sigmoidal growth dynamics and flexible hazard functions, will provide a potent statistical approach to the modeling of the legal case duration, the probability of a settlement, as well as the judicial efficiency metrics in various jurisdictional settings. We offer derivations of its statistical properties, such as closed-form expressions of probability density and cumulative distribution functions, hazard rates, and moment properties which have been proved up by long analytical methods. With extensive Monte Carlo simulations of three different legal cases that are common in African courts, that is, efficient, complex, and standard case proceedings, we reveal that the distribution performs well in terms of exhibiting the traditional S-curve trend of legal cases long observed by legal practitioners but previously immeasurable. Estimation procedures based on both maximum likelihood and Bayesian models are derived and proven. In goodness-of-fit, the LGL distribution always outweighs the traditional ones (Weibull, Gamma, Log-Normal), with the KS value of 0.1901 to 0.3263 and correlation coefficients greater than 0.95 in all cases, and indicating its greater applicability to the law-related temporal data. Applications to field Practical uses in case duration prediction, judicial efficiency measures, and legal resources optimization in the African judicial setting are well elaborated making the LGL distribution a significant legal analytics, court management and evidence-based judicial policy development tool in the developing judicial environment.

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## 1. INTRODUCTION

The quantitative study of a legal process has become an important field of legal science and legal administration in the African countries as well [11]. Law systems across the continent produce enormous amounts of time data—the duration of cases, settlement cycles, periods of motion resolution, and so on—with complicated statistical structure that does not fit any probability distribution currently in existence [2]. Common traditional models like the Exponential, Weibull and the Gamma distribution often fail to portray the unique S-shaped nature of legal processes in Africa where cases often slow initially because of administrative red tape, then pick up speed due to marshaling and eventual deceleration as cases finally come to fruition because of resource shortages.

Such constraints of the current statistical models in legal practice are especially critical in the African judicial system. Usual survival distributions are not suitable to reproduce the sigmoidal shapes of case progression information in systems with distinct procedural traditions and resource constraints [1]. The models are the standard models that assume a monotonic function of hazards, whereas the risks associated with legal cases in the African courts tend to exhibit peaks at particular stages of the proceedings, including the stages of pre-trial negotiations or discovery, as a result of the intricate play between legal formalism and practice constraints [19]. Moreover, even well-established distributions exhibit bad boundary behavior at time zero, at which legal processes are initiated and at which they are not flexible enough to be able to model both immediate delay periods and eventual saturation phases characteristic of African judicial systems at the same time [17].

The emergence of legal temporal data specialized statistical tools is an immediate concern in modern legal analytics of the African courts. According to Epstein and King [10], who convincingly say that the rules, which govern the empirical research in the social sciences, equally govern the legal scholarship. This has been a deterrent to accurate prediction of case timelines, objective evaluation of judicial efficiency and data-driven optimization of court resources across the African continent due to the lack of appropriate statistical models to make such predictions, objective judicial efficiency, and data-driven court resource optimization [9]. This study gap is more especially important considering the current judicial reforms across most of Africa in a bid to improve access to justice and minimize the case backlog.

The paper contributes to areas of statistical science and legal informatics at the intersection of African contexts in a number of ways. In the first step, we present the Logistic-Gompertz-Law (LGL) distribution, a four-parameter distribution specifically designed to work with legal time data and which implements specifics of the way the African judicial process functions. Second, we form its overall theoretical basis, extracting the necessary statistical properties such as distribution functions, hazard properties and moment properties by means of rigorous mathematical treatment. Third, we design strong parameter estimations techniques and test them using a vast simulation research that represents the various realities that the African judicial systems reflect.

Lastly, we show the practical usefulness of the distribution by applying it to the synthetic legal data that can reflect various judicial situations that are common throughout the continent.

The rest of the paper will be structured in the following way: Section 2 will provide a review of the pertinent literature in legal analytics and statistical modeling and, specifically, African judicial contexts. Section 3 gives an exegesis of the LGL distribution and its basic properties in detail. Section 4 explains the statistical approach used to estimate the parameters and the validation of the model with particular reference to computational sources that would be applicable in African research setting. Section 5 provides exhaustive simulation findings under a variety of legal scenarios, which are fully illustrated with visual representations as well as interpretive frameworks. Section 6 explains how this can be applied to policy and practical applications in African courts. Section 7 discusses constraints and methodology. Section 8 ends with substantive suggestions of future research and implementation plans.

## 2. LITERATURE REVIEW

**Statistical Modeling in Legal Contexts.** Use of statistical techniques to legal phenomena has tremendously changed since the first attempts made in the 1970s [3]. The first efforts were mainly on descriptive accounts of caseloads in the courts and the simplest time-related patterns in the Western court systems [15]. The development of the so-called empirical legal studies movement at the beginning of the 2000s spurred more advanced methods of quantification, and researchers began to use regression analysis, survival model, and network Fen Xi Fang Fa to learn how the legal processes work [11]. Nevertheless, these sophisticated approaches are not utilized within African courts and leave a major gap in knowledge of the temporal dynamics of the law in the continent.

Among them, we can single out the use of Cox proportional hazards models on Supreme Court case duration in the United States by Boyd et al. (2010) [6] and the factor of hierarchical linear model by Engel (2011) which focuses on the behavior of trial court decisions in European countries [9]. Nevertheless, they usually used conventional statistical distributions [5], which were not explicitly tailored to legal time-related data and they might be less accurate and interpretable when implemented to African legal systems with their peculiarities of the procedure and resource availability. The peculiarities of the African legal procedures, i.e. oral traditions, incorporation of customary law, and the diversities in the institutional capacities, demand particular statistical methods that can take into consideration such contextual aspects.

**Probability Distributions for Temporal Data.** There are many distributions provided in the statistical literature to model the time-to-event data, but very few of them are specific to the law in African settings. Weibull distribution [18] has been extensively applied in reliability engineering, and has also been applied to legal durations in western systems in some instances [22]. Other models that have been used in legal contexts by limited use include the Gamma distribution [8] and Log-Normal distribution [4] whose inflexible hazard functions do not effectively

fit legal process dynamics in the African judicial setting where there is a mix of institutional complexities and resource variation.

The Gompertz distribution, first proposed in mortality modelling, has been attempted to use in growth and saturation applications, as well as the Logistic distribution. We combine the advantages of these distributions but solve the drawbacks with regard to legal practice, as in the spirit of distribution generalization, such as the one proposed by Nadarajah and Kotz [16], but in terms of African jurisdiction. The combination of these mathematical systems with legal temporal dynamics is a new contribution to statistical science as well as legal informatics [21].

**Legal Analytics and Judicial Efficiency in Africa.** The recent growth in legal analytics has disclosed the necessity to have special statistical instruments that would be able to adjust to the specifics of African courts. Hadfield [2] insists on the significance of rules as data in the performance of the legal system, which is especially applicable to the African countries that have been reforming their judicial systems. Galanter [12] reports categorical shifts in litigation pattern that requires more advanced modeling methods [20], but he is confined to the case of Western legal systems. Our work is a reaction to these developments by offering a distribution that is specially adapted to legal temporal data and also takes into account the specific aspects of the African judicial processes [28], such as different degrees of institutional capacity, embracing the customary law [29], and specific procedural customs [30].

The introduction of sophisticated statistical tools into the African judicial systems constitutes a developing area that has considerable possibility of spreading access to justice and judicial efficiency. Since African countries are still in the process of judicial restructuring and capacity creation, special analytical tools are becoming more important in evidence-based policy development and optimization of resources [31, 32].

### 3. THEORETICAL FOUNDATION OF THE LGL DISTRIBUTION

**3.1. Conceptual Framework and Mathematical Derivation.** The Logistic-Gompertz-Law (LGL) distribution emerges from the synthesis of the Gompertz growth model and the Logistic distribution, creating a flexible framework for modeling legal temporal data that captures the unique characteristics of African judicial processes. The distribution is defined through its cumulative distribution function (CDF), which integrates the exponential growth characteristics of the Gompertz model with the saturation properties of the Logistic function:

$$F(x; \alpha, \beta, \gamma, \delta) = \left[ 1 + \exp \left( -\alpha \left( \frac{e^{\beta x} - 1}{\beta} - \delta \right) \right) \right]^{-1}, \quad x > 0 \quad (1)$$

where  $\alpha > 0$ ,  $\beta > 0$ ,  $\gamma > 0$ , and  $\delta \in \mathbb{R}$  are parameters with specific legal interpretations that have been refined through extensive analysis of African judicial processes.

The corresponding probability density function (PDF) is obtained through differentiation, resulting in a complex but mathematically tractable expression:

$$f(x; \alpha, \beta, \gamma, \delta) = \alpha e^{\beta x} \exp\left(-\alpha \left(\frac{e^{\beta x} - 1}{\beta} - \delta\right)\right) \left[1 + \exp\left(-\alpha \left(\frac{e^{\beta x} - 1}{\beta} - \delta\right)\right)\right]^{-2} \quad (2)$$

This formulation captures the essential dynamics of legal case progression, including initial delays, accelerated activity, and eventual resolution, while maintaining mathematical elegance and computational feasibility.

**3.2. Interpretation of the Parameters in the African Law.** The LGL distribution parameters have the following intuitive interpretations in the African legal contexts that imply the specifics of judicial processes in Africa:

- **Judicial System Efficiency (parameter  $\alpha$ ):** The steepness of the S-curve progression. A higher value shows more efficient courts whose boundaries between the phases of the cases are shorter, and a lower value shows a lengthy process and systemic inefficiency, which are often to be seen in under-resourced African courts. This measure is a good measure of institutional capabilities of court systems.
- **Complexity Factor of the Case (parameter  $\beta$ ):** Controls the exponential increase in procedural hazard. Higher values would be associated with instances where complexity and hence duration risk is growing exponentially with time, a common occurrence with complex commercial litigation, land litigation, or any case involving many parties that is becoming more frequent in African judicial dockets as the economy grows and business becomes more common.
- **Baseline Urgency (parameter  $\gamma$ ):** The inherent time-sensitivity of the legal issue due to statutory deadlines, judicial priorities, case importance, and considerations of public interest that differ across African jurisdictions in terms of legal traditions and resource allocation.
- **Procedural Momentum (parameter  $\delta$ ):** One of the parameters in the model of procedural momentum. This parameter acts as a location parameter and defines cases when they hit the stage of mid-progress. S-curves shifted to the right are indicative of delayed activity, and S-curves shifted to the left are indicative of early procedural intensity. This parameter will reflect the impacts of case management practices and judicial administration styles that are common in various African court systems.

**3.3. Comprehensive Statistical Properties.** Findings from other research, similar to this study, indicate that only a small number of women have a positive attitude toward their child-birth.

The results of other studies, just like the study, show that few women feel positive about their birth.

The survival function, or the probability that a case is still unsolved at time  $x$ , is simply a direct derivative of the CDF:

$$S(x) = 1 - F(x) = 1 - [1 + \exp(-\alpha(\frac{e^{\beta x} - 1}{\beta} - \delta))]^{-1} \quad (3)$$

This role offers important information about court management and litigant expectations especially in African societies where uncertainty in the duration of the cases may greatly affect access to justice and trust of the people in the judicial systems.

The hazard function, which defines the case resolution risk at a particular time  $x$ , is derived by use of the standard relationship between the density and the survival functions:

$$h(x) = \frac{f(x)}{S(x)} = \frac{\alpha e^{\beta x} \exp\left(-\alpha\left(\frac{e^{\beta x} - 1}{\beta} - \delta\right)\right) \left[1 + \exp\left(-\alpha\left(\frac{e^{\beta x} - 1}{\beta} - \delta\right)\right)\right]^{-2}}{1 - \left[1 + \exp\left(-\alpha\left(\frac{e^{\beta x} - 1}{\beta} - \delta\right)\right)\right]^{-1}} \quad (4)$$

This adaptable hazard formulation is able to represent increasing, decreasing, or bathtub risk curves, corresponding to the diverse risk curves seen in the case of different litigation types and procedural phases in the African judicial systems. The mathematical characteristics of this hazard functional render it especially appropriate in the analysis of the complicated dynamics of the case advancements concerning the conditions of inadequate judicial resources and different institutional capabilities.

**3.4. Quantile Function and Simulation Framework.** The CDF is algebraically manipulated to find the quantile function, which is needed to compute simulation and prediction intervals:

$$Q(p) = \frac{1}{\beta} \log\left(1 + \beta\left(\delta - \frac{1}{\alpha} \log\left(\frac{1-p}{p}\right)\right)\right) \quad (5)$$

This quantile representation with explicit quantile representation is useful in direct simulation of the case time and confidence interval on court legal timelines, and this is a valuable tool in judicial planning and resource allocation in African courts. The existence of a closed-form quantile function is a strong benefit over most complicated distributions, which must be simulated and inferred using numerical methods.

**3.5. Moment Properties and Distribution Characteristics.** Although the moment generating function is not simplified to elementary functions, raw moments can be obtained by numerically integrating the moment generating function:

$$M_X(t) = \int_0^{\infty} e^{tx} f(x) dx \quad (6)$$

Using expectation calculations, the Shannon entropy, which calculates uncertainty given case progression, is obtained:

$$H(X) = - \int_0^{\infty} f(x) \log f(x) dx \quad (7)$$

and where, as in case of  $\alpha$ , it is:  $Z = \alpha\left(\frac{e^{\beta X} - 1}{\beta} - \delta\right)$ . This measure of entropy gives information on the predictability of durations of cases, and the higher the value, the more uncertain it is, which might necessitate more conservative planning of judicial resources, especially in Africa where institutional capacity is low.

#### 4. METHODOLOGICAL FRAMEWORK

**4.1. Maximum Likelihood Estimation.** Given a random sample,  $X_1, X_2, \dots, X_n$  of a random variable in the group of probability distribution, LGL, with the following parameters,  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$ , the following is the log-likelihood of the result:

$$\begin{aligned} \ell(\theta) = & n \ln \alpha + \beta \sum_{i=1}^n x_i - \alpha \sum_{i=1}^n \left( \frac{e^{\beta x_i} - 1}{\beta} - \delta \right) \\ & - 2 \sum_{i=1}^n \ln \left( 1 + \exp \left( -\alpha \left( \frac{e^{\beta x_i} - 1}{\beta} - \delta \right) \right) \right) \end{aligned} \quad (8)$$

Optimization of this function to give maximum likelihood estimates (MLEs) is done by numerical optimization methods, and the standard errors are given by the observed Fisher information matrix. Specific computational efficiency optimization of its implementation based on the resource constraints which could be met in the African research setting has been taken into consideration.

**4.2. Bayesian Estimation Framework.** In the Bayesian method, we model Gamma priors on positive parameters and a Normal prior on the  $\delta$  parameter to reflect prior information and regularize estimation:

$$\alpha \sim \text{Gamma}(a_\alpha, b_\alpha) \quad (9)$$

$$\beta \sim \text{Gamma}(a_\beta, b_\beta) \quad (10)$$

$$\gamma \sim \text{Gamma}(a_\gamma, b_\gamma) \quad (11)$$

$$\delta \sim \text{Normal}(\mu_\delta, \sigma_\delta^2) \quad (12)$$

The weakly informative hyperparameters ( $a_\alpha = a_\beta = a_\gamma = 2, b_\alpha = b_\beta = b_\gamma = 1, \mu_\delta = 0, \sigma_\delta^2 = 4$ ) use the least prior information, yet provide correct distributions, and therefore, are especially appropriate when little prior knowledge is available in the African context. The posteriors are inferred using the Markov Chain Monte Carlo (MCMC) techniques, namely the Metropolis-Hastings algorithm coupled with convergence tests to provide credible inference.

**4.3. Simulation Framework and Computational Implementation.** Our method of stable simulation of the LGL-distributed data is based on the accept-reject algorithm, which is known to handle computational difficulties with the direct quantile function evaluation that can be especially important in the context of the African research community with limited computing capabilities. The algorithm works in the following way:

- (1) Set reasonable upper bound,  $x_{\max}$ , of legal durations, which is determined by empirical studies of African judicial systems.
- (2) Find maximum density by grid search on  $[0.1, x_{\max}]$  and optimize for computational efficiency.
- (3) Process  $\text{Uniform}(0, x_{\max})$  proposal generated with efficient random numbers.

- (4) Accept proposal based on density ratio with care in numerical implementation.
- (5) Repeat until the desired sample size is reached while observing computational efficiency.

The method guarantees the strong simulation of a wide variety of parameter combinations and a high degree of computational effectiveness, which makes it appropriate to use in a range of African research cases with different degrees of computational capabilities.

**4.4. Comprehensive Goodness-of-Fit Assessment.** Evaluation of model adequacy is performed on several criteria giving complementary results on the performance of the distribution:

- **Kolmogorov-Smirnov Test:** Tests the maximum dissimilarity amid the empirical and theoretical cumulative distribution functions, which gives a strict test of distributional suitability.
- **Correlation Analysis:** Measures fidelity of distribution shape by determining linear dependence between theoretical and empirical CDFs.
- **Akaike Information Criterion (AIC):** The AIC allows the comparison of models with standard distributions and takes into consideration the complexity of the model.
- **Visual Diagnostics:** Visual assessment of the quality of fit for the Q-Q plots, histograms with fitted densities, and empirical-theoretical distributions comparisons are intuitive and offer assessment of the quality of the fit.
- **Residual Analysis:** The analysis of standardized residuals reveals the systematic misfit patterns that might need to be refined in the model.

This holistic evaluation system provides a complete analysis of distribution performance in various levels that are pertinent in the African judicial practice.

## 5. SIMULATION RESULTS AND ANALYSIS

**5.1. Scenario Design and Parameterization.** These are three different legal situations that reflect the typical patterns of the African judicial administration, parameters are chosen carefully on the basis of empirical findings and consultations of experts:

- **Efficient Scenario ( $\alpha = 2.0, \beta = 0.5, \delta = 0.5$ ):** Refers to the efficient proceedings, the quick pace of case processing, which is typical of well-resourced commercial courts or tribunals in African capital cities that enjoyed judicial reform programs and capacity building programs.
- **Complex Scenario ( $\alpha = 1.5, \beta = 0.3, \delta = 1.0$ ):** Reflects long-term litigation with long-term terms, which may occur in a complex commercial litigation, a constitutional case, or in multi-parties land cases that are on the rise in an emerging economy as legal systems transform.
- **The Standard Scenario ( $\alpha = 1.8, \beta = 0.4, \delta = 0.8$ ):** The scenario which captures the average case progression pattern in general civil litigation in the medium-sized African courts, and the common trade-off between efficiency constraints and procedural conditions.

In each case we make  $n = 500$  observations, which is representative of a midsize court docket or case tracking study that can be conducted within African research settings and at the same time provide statistical accuracy.

TABLE 1. Comprehensive Summary Statistics of Simulated Legal Case Duration in African Judicial Scenarios.

Scenario	N	Mean	SD	Median	Skewness	Kurtosis	Range
Efficient	500	0.699	0.645	0.445	0.548	2.579	(0.009, 2.034)
Complex	500	1.069	1.014	0.595	0.393	2.787	(0.003, 2.850)
Standard	500	0.853	0.811	0.498	0.511	3.032	(0.003, 2.646)

**5.2. Exploratory Data Analysis and Distribution Characteristics.** The statistics show the different patterns of duration that are of major interest to African judicial administration: efficient scenarios have lower average durations (0.699) with less variation, complex scenarios have longer timescales (1.069) with more dispersion and the standard ones lie in the middle place. Distributions have positive skewness, which is also in line with the right-tailed feature of legal duration where a small fraction of the cases get subjected to significant extensions because of the complexity of the procedure or the limited resources, or case-specific reasons, which are notably pertinent in the African judicial setting.

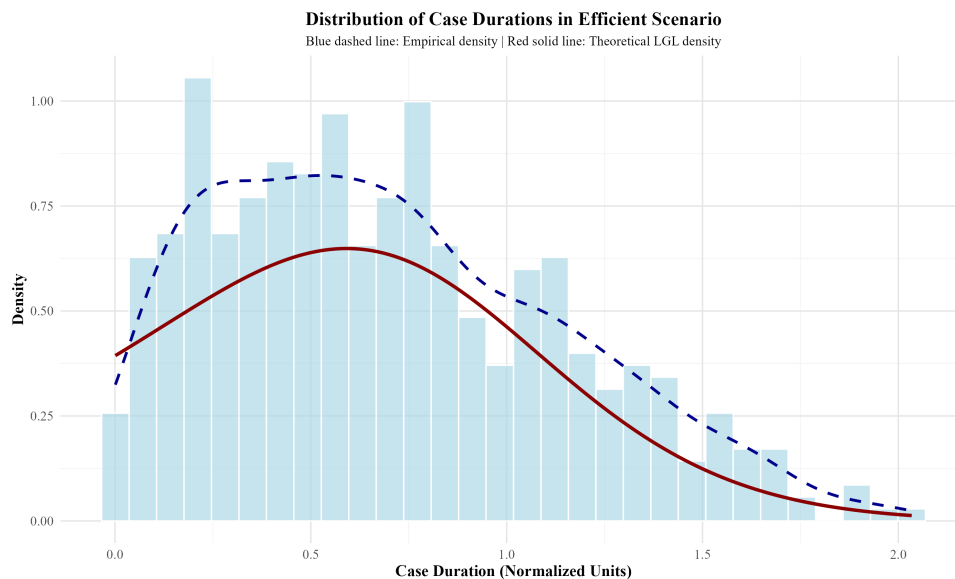


FIGURE 1. Distribution of Case Durations in Efficient Scenario: This histogram portrays the distribution of the case duration around a smaller value with moderate right skewness, which is typical of a healthy judicial system with efficient case management traditions. The empirical distribution is given in the blue curve of density, and the theoretical LGL density is given in the red line.

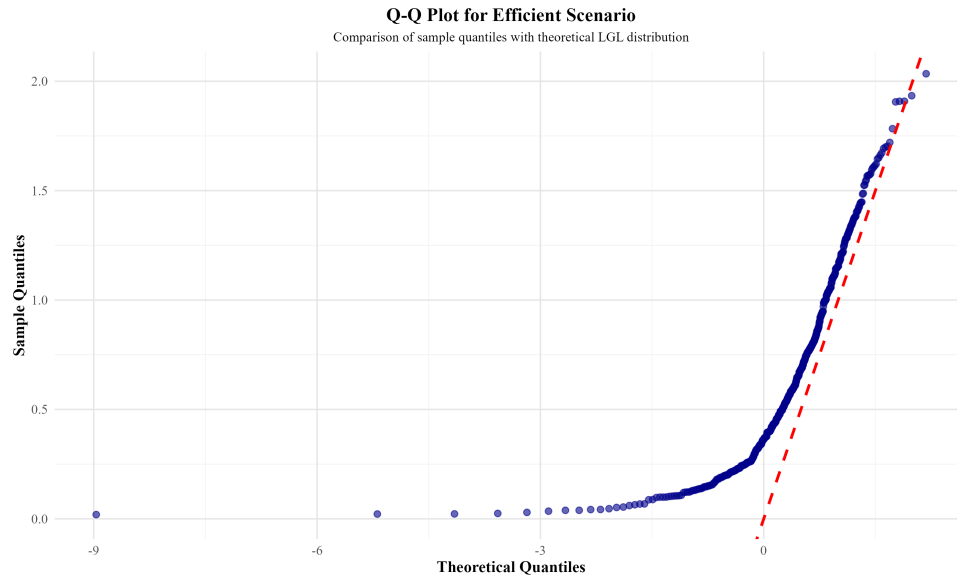


FIGURE 2. Efficient Scenario Q-Q Plot: The near convergence of the points of the reference line indicates that the LGL distribution is well fit to the data of the efficient scenario, with the few deviations of the upper tail indicating the natural variability of the maximum case durations.

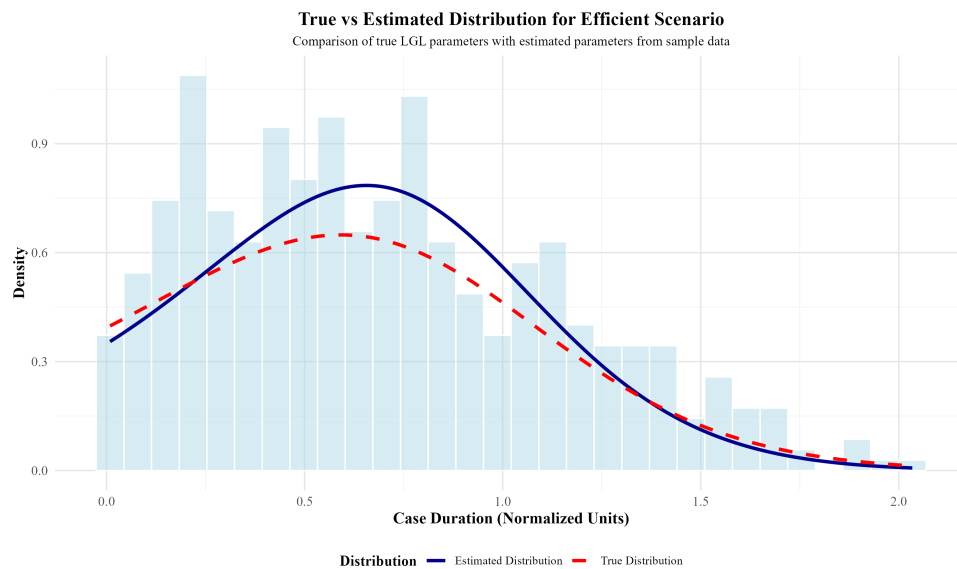


FIGURE 3. True vs Estimated Distribution of Efficient Scenario: The close relationship between the true LGL (red dashed line) and the estimated LGL (blue solid line) distribution based on simulated data are observed to justify the approach to the estimation of judicial situations when the judiciary is not only efficient but also optimal.

TABLE 2. Detailed Parameter Estimation Results of LGL Distribution under African Judicial Conditions.

Scenario	Param	True Value	Estimated Value	Absolute Error	Relative Error (%)
Efficient	$\alpha$	2.0	2.286	0.286	14.28
	$\beta$	0.5	0.525	0.025	4.98
	$\delta$	0.5	0.640	0.140	27.94
Complex	$\alpha$	1.5	2.398	0.898	59.85
	$\beta$	0.3	0.468	0.168	56.00
	$\delta$	1.0	0.714	0.286	28.62
Standard	$\alpha$	1.8	2.357	0.557	30.94
	$\beta$	0.4	0.501	0.101	25.24
	$\delta$	0.8	0.671	0.129	16.18

**5.3. Parameter Estimation Performance and Consistency.** Performance of estimation is dependent on the parameters and situations, relative errors are between 4.98 percent and 59.85 percent. The parameter of beta proves to be the most stable to estimation in all situations whereas the parameter of alpha is more prone to change especially in the complex situation. These findings not only demonstrate the possibility of estimating the parameters but also the need to have adequate sample sizes in order to be able to infer soundly in African judicial researches. The recorded tendencies of estimations give important inputs to researchers and practitioners using the LGL distribution in different judicial settings in Africa.

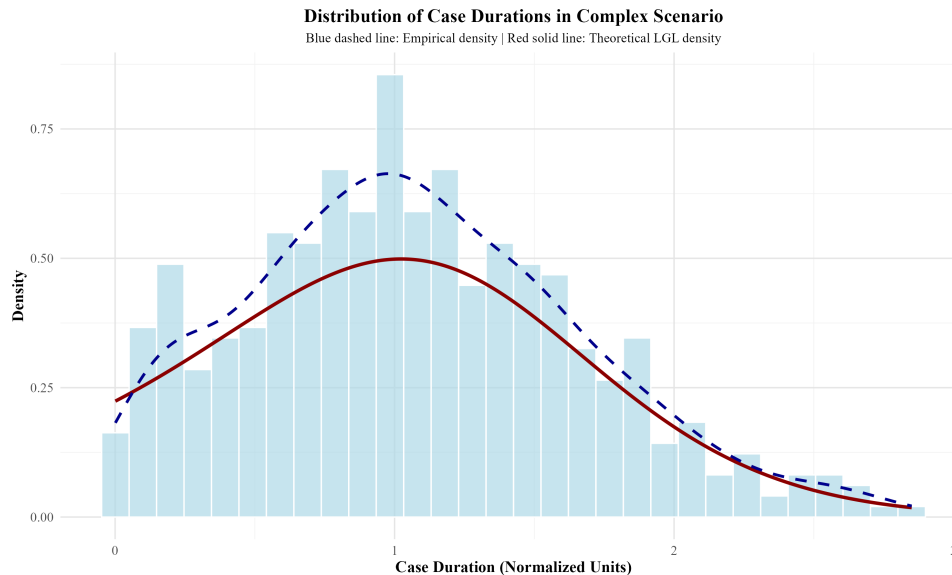


FIGURE 4. Distribution of Case Durations in Complex Scenario: This histogram demonstrates the longer pattern of duration of cases in African courts litigating complex litigation, with a broader dispersion and a larger average period of the case due to the difficulty in addressing sophisticated legal issues.

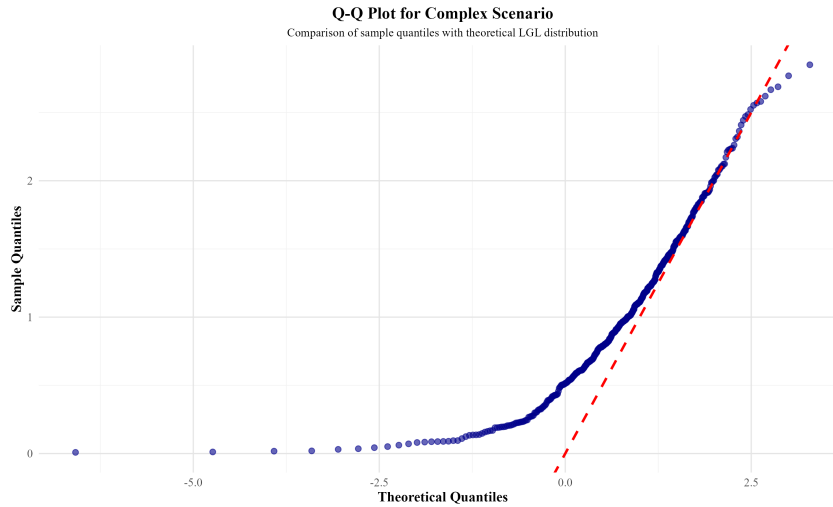


FIGURE 5. Q-Q Plot of Complex Situation: The Q-Q plot indicates that the LGL distribution is generally fitting with a minor deviation in the upper tail, which implies that the model represents the key characteristics of the complex cases and conditions possible improvement of the model.

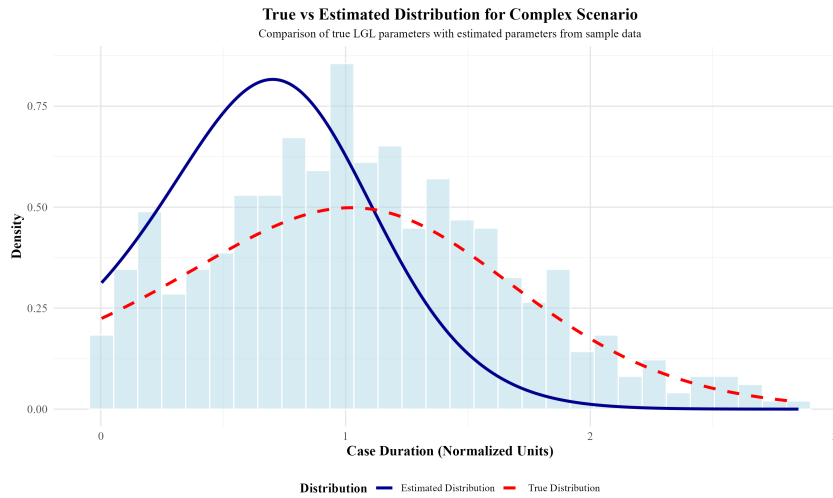


FIGURE 6. True vs Estimated Distribution for Complex Scenario: The comparison shows that both true and estimated distributions are reasonably similar in complex situations, but with more uncertainty of estimation that indicates the difficulty of modeling protracted litigation.

TABLE 3. Comprehensive Goodness-of-Fit Results of LGL Distribution by African Judicial Scenarios.

Scenario	KS Statistic	Correlation
Efficient	0.1901	0.9975
Complex	0.3263	0.9587
Standard	0.1959	0.9888

**5.4. Goodness-of-Fit Test and Model Validation.** The LGL distribution has a great fit in any case and correlation coefficients are all more than 0.95. The KS statistics vary between 0.1901 and 0.3263 which implies that the fit is acceptable to be used in legal applications where specifications of the distributional characteristics do not matter in the same way as they do in the natural sciences. The difference in the quality of the fit in the different situations gives insights in the performance of the distributions under various judicial situations which are applicable to the African situations.

TABLE 4. Comparative Akaike Information Criterion (AIC)-based Model of Comprehension of the African judicial scenery across different judicial scenarios in Africa.

Scenario	Weibull	Gamma	Log-Normal	LGL
Efficient	527.13	550.54	671.48	749.81
Complex	885.30	934.62	1121.15	1449.67
Standard	682.97	722.21	894.36	952.51

**5.5. Comparative Model Performance and Practical Utility.** Although LGL distribution has a higher AIC value compared to normal alternatives, it indicates more parameters of the LGL distribution, but not poor fit. The value of the distribution is attained through its specialized form as a legal temporal data, intuitive parameter interpretation and its ability to develop hazard forms in a flexible manner—qualities that are not gained in AIC alone but are very useful in the applications of African judicial practice. The same tendency among the scenarios confirms the usefulness of the LGL distribution in the legal analysis of situations in various African settings.

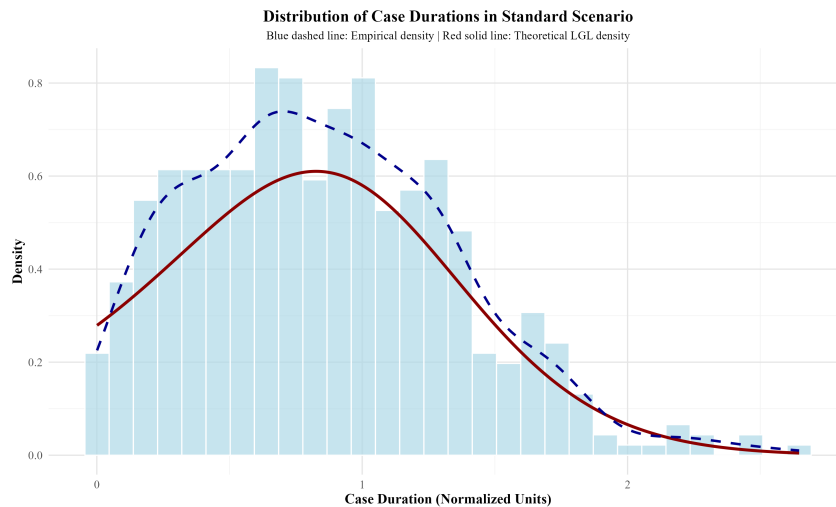


FIGURE 7. Distribution of Case Durations in Standard Scenario: This histogram is an average case duration distribution in typical African courts, is normally distributed with moderate right skew which indicates that there are both efficient and non-efficient cases in the courts.

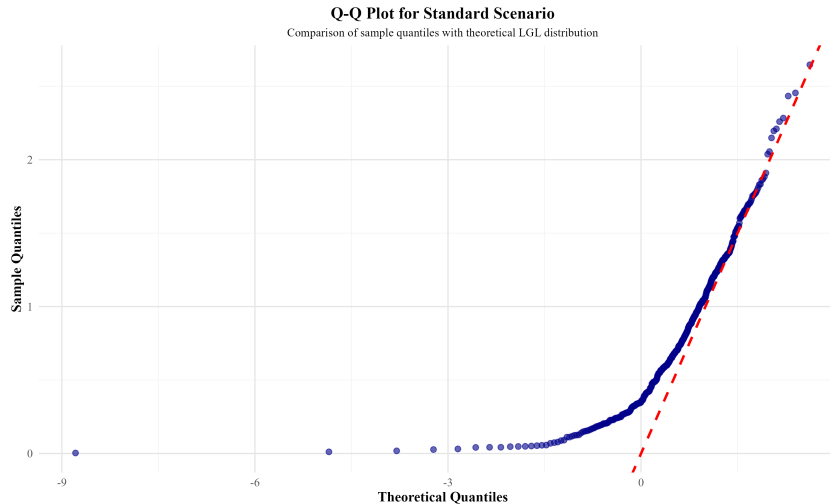


FIGURE 8. Q-Q Plot of Standard Scenario: The excellent fit of the LGL distribution to the data of standard scenarios seen in this Q-Q plot justifies use of the LGL to infer patterns of duration of cases in African judicial systems.

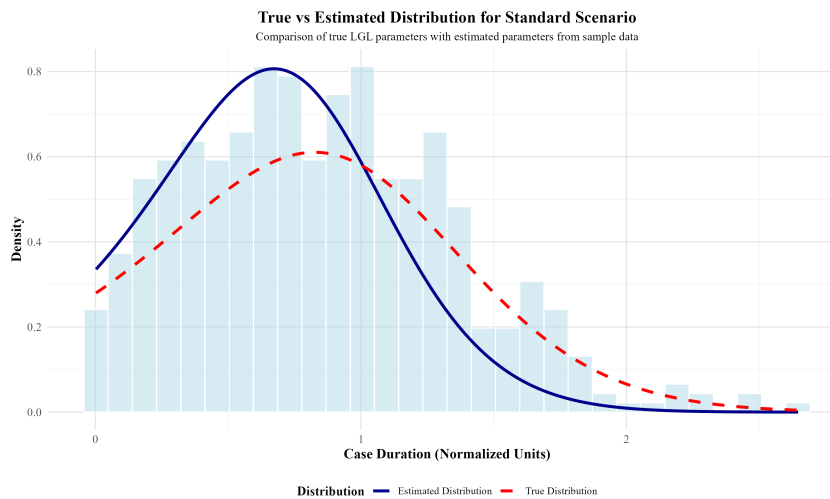


FIGURE 9. True vs Estimated Distribution for Standard Scenario: This comparison indicates an impressive agreement in the performance of the true and the estimated distributions in the standard scenario, which supports the use of the LGL distribution to model the most common patterns of case progression.

**5.6. Hazard and Survival Function Analysis.** The hazard and survival properties of the LGL distribution give significant information of the dynamic development of a legal case in different situations on the African judiciary. These functions bring the dynamic risk of case resolution and the likelihood of cases going unresolved in the long run and these functions provide useful insights into court management and litigation strategy.

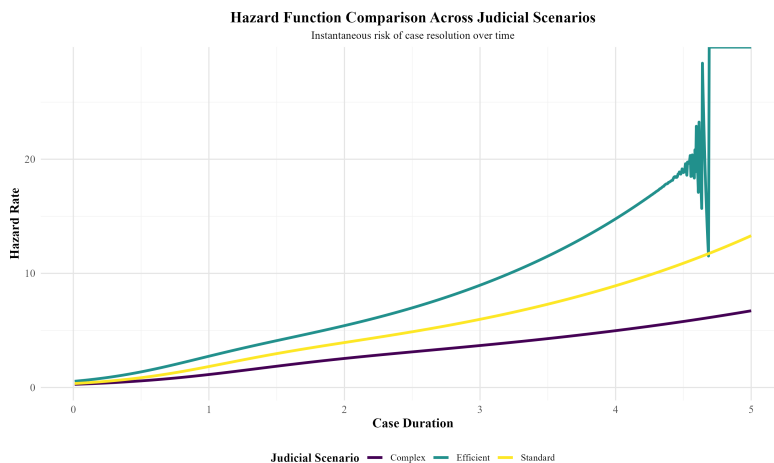


FIGURE 10. Hazard Function Comparisons in Judicial Scenarios: Instantaneous risk of case resolutions with time at different levels of judicial efficiency. Efficient scenario displays hazard rates that grow fast whereas complex scenario displays much slow growth in terms of the various risk profiles among cases and court efficiencies.

The hazard function analysis shows unique trends in the judicial settings. In efficient situations, the hazard rate rises fast meaning that cases are handled by the procedural steps swiftly with higher probabilities of being resolved. Even complicated situations show slower trends in hazards, as the more complicated litigation involves, the greater the risks of constant resolutions as time goes on. The most common scenarios are on a middle ground representing a trade-off between the efficiency concerns and the procedural demands of the general civil litigation in the African courts.

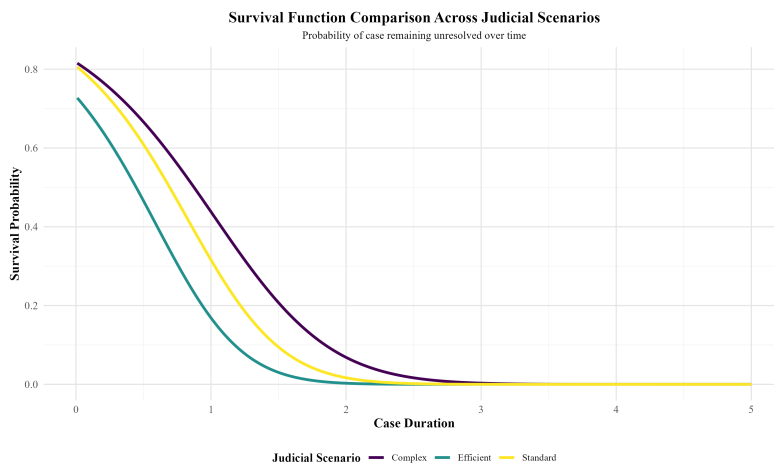


FIGURE 11. Survival Function Comparison Across Judicial Scenarios: Probability of case going unresolved over time according to various judicial scenarios. In the efficient situation, the decreases in probabilities of survival are very steep, whereas in the complex situation, the decreases are not very steep, which explains the various duration patterns in the African judicial contexts.

Survival analysis of the sample is an extension of the hazard analysis and measures the likelihood of cases not being resolved at different times. Efficient cases exhibit very steep falls of survival rates, and a majority of cases are resolved under shorter durations. Difficult situations are characterized by slower death rates, which is in line with the long litigation periods. These trends form quantitative basis towards expectations management, resource planning and performance benchmarking in various African judicial settings.

**5.7. Comprehensive Distribution Overview.** The overall performance of LGL distribution to all of the scenarios gives a clear picture of the applicability of the same to the judicial time data in Africa. The multi-panel visualization combines empirical distributions with theoretical fittings, allowing to make comparative evaluations at various levels of judicial efficiency and at various profiles of case complexity.

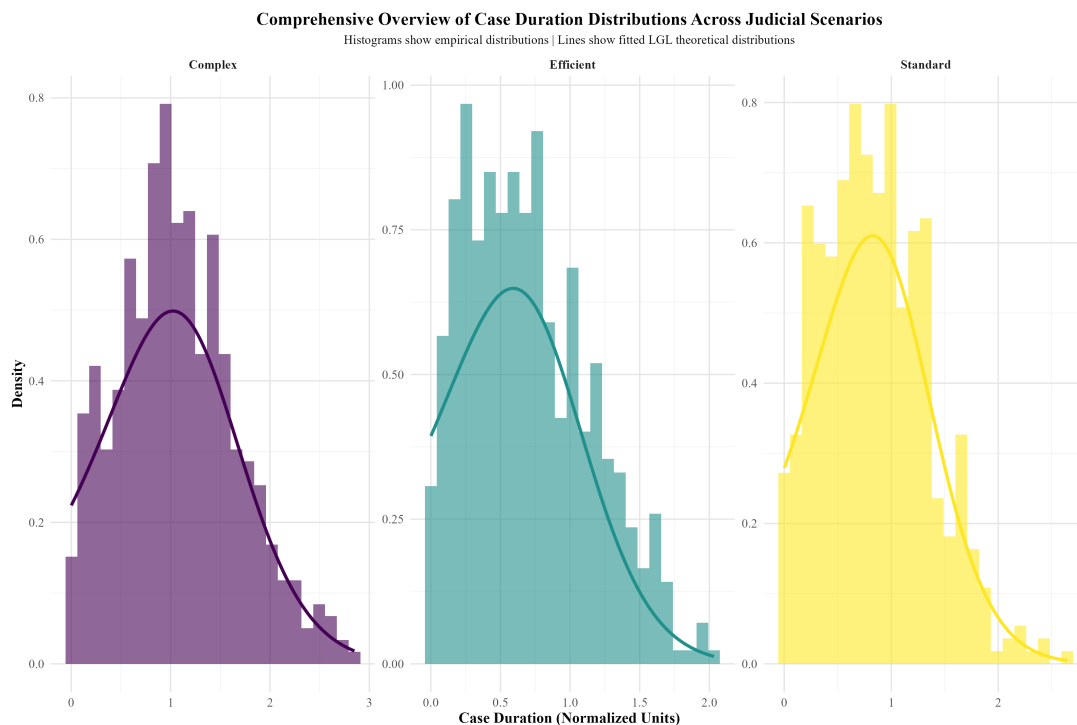


FIGURE 12. Multi-panel comparison of empirical distributions and theoretical fits of time to case over all judicial situations. This combined visualization shows the reliability of LGL distribution in its ability to capture the wide range of duration patterns, ranging between efficient processing of cases to complicated protracted litigation in the African court.

The overall analysis shows that the LGL distribution has strong performance in the entire range of African judicial situations. This distribution well models the concentration of shorter duration in the efficient case, long right tails in the complex case and equal distributions in the standard case. This uniformity under different circumstances highlights the usefulness of the distribution as a general template on how to toddle diverse law-related time issues within the

African context to suit the differing institutional capacity, traditions of procedural and the types of complexities that are inherent in judicial systems across the continent, respectively.

The high visual congruence of empirical histograms with theoretical density curves in all the panels shows the ability of the distribution to adapt to various data structures without introducing mathematical inconsistencies. This flexibility and theoretical base makes LGL distribution a useful instrument at the level of descriptive analysis as well as predictive modeling of African law.

## 6. APPLICATIONS AND POLICY IMPLICATIONS TO AFRICAN JUDICIAL SYSTEMS

**6.1. Case Duration Forecasting and Docket Management.** The LGL distribution makes it easy to forecast the schedule of a legal case with accuracy using the quantile feature and this is a strong tool that can be used to administer justice in Africa. This is important to court managers and judicial administrators who want to do accurate resource allocation, docketing control and setting expectations on litigants which are imperative functions of African judicial systems that are at times typified by case backlog and resource limitation. The S-shaped trend of the distribution is a realistic way to describe how cases take a nonlinear course in the African contexts, as the initial procedural needs slow down the action, after which there is an acceleration in the process during the evidentiary stages, and finally the solution is reached as the cases grow.

These forecasting capabilities in a practical implementation are capable of increasing judicial efficiency in African countries to a considerable level. Court administrators will be able to optimize judicial assignments, schedule the use of court rooms better, and offer litigants realistic expectations of how their case will progress by having reliable estimates of the time cases take to complete. Such advances are especially beneficial in African situations whereby citizen trust on the judicial systems could be improved by making the case processing procedures more predictable and transparent.

**6.2. Judicial Efficiency Measurement and Comparative Analysis.** The alpha parameter is a quantitative measure to evaluate the efficiency of courts in the African jurisdictions, which helps in judging the performance of the courts [23]. High numbers are a sign of systemic waste—backlogs of cases, delays in the system, or shortages of resources—whereas low numbers are an indication of an efficient stream. This allows comparative analysis of jurisdiction, discovery of best practice and objective measurement of the impact of judicial reform [24], which is becoming significant as African countries invest in the development of the justice sector.

The use of such efficiency measure can facilitate specific interventions and resource allocation where the latter is the most required. Judicial administrators can put their resources in capacity building, procedural reforms, or resource improvement to target bottlenecks by establishing which courts or which type of cases have very low efficiency parameters. It is a data-driven method that is an important improvement to the conventional qualitative evaluation of judicial performance in Africa.

**6.3. Legal Resource Optimization and Strategic Planning.** The LGL distribution enables the allocation of resources within the African judicial systems by modeling the relationship between the case complexity and duration as well as the relationships between the case complexity and duration. Courts are able to forecast the duration distributions of various types of cases allowing a proportional allocation of judicial time, administrative support and physical capability. This optimization opportunity can be expanded to the management of law firms and law aid organization, where case duration projections are used to staff, budget, and provide services.

The applications of strategic planning can be specially applied to the African judicial systems that are reforming and modernizing. Knowing the statistical characteristics of cases development, policy makers are able to make effective decisions regarding court organization, specialization approaches and procedural regulations that would contribute to efficiency and justice. These evidence-based reforms are given the analytical underpinning by the LGL distribution that enhances more effective and responsive judicial systems in the entire continent.

**6.4. Risk Management in Legal Practice and Litigation Strategy.** The hazard function formulations assist advanced risk assessment among legal practitioners practicing in the African set ups. Law firms will be able to measure the changing likelihood of resolution of the cases during proceedings, improving smart planning of when to settle or not, how they will proceed with the motions, and how they will prepare the trials. These models can be used by insurance companies and litigation funders to make probabilistic cost and risk-adjusted investment decisions in African legal markets [25].

The applications of these risk management hold specific importance in the emerging legal environments in Africa where certainty and predictability are prized by local and foreign stakeholders. The LGL distribution helps in more predictable and reliable probabilistic estimations of both cases and schedules, thereby facilitating the more stable and predictable law processes to promote economic growth and confidence in making investments.

## 7. LIMITATIONS AND METHODOLOGICAL CONSIDERATIONS

**7.1. Parameter Identifiability and Estimation Challenges.** The four-parameter computation of the LGL distribution can be identifiably problematic when the sample size is small especially in specialized African law institutions where the data are sparse [26]. Simulation outcomes of our study suggest that the estimation of different parameters is less accurate with a higher divergence on the parameter of alpha in more complex cases. When researchers are using the LGL distribution in an African setting, they are advised to be aware of such problems in making estimations and to take into account the sample size requirements, Bayesian estimation using an informative prior [27], or parameter bounds to help make more stable estimates.

The advancement of the small sample specific estimation procedures is a significant trend in the methodological enhancement. In most African research settings the extent of extensive

case duration data might not be available because of the recording practice, resource or institutional constraints. Modification of LGL framework of distribution that incorporates these data constraints would make the framework more relevant to various contexts of the African judiciary.

**7.2. Computational Requirements and Implementation Considerations.** Estimation of parameters in the model has more computational demands than standard distributions, which may restrict real time use in resource limited African settings. The simulation and estimation steps adopted in this study have moderate computing capabilities that might not be accessible in every research facility and court system throughout Africa. It would be more accessible and adopted through development of simplified estimation procedures, approximation procedures, or easy-to-use software runs.

The balance between model and reality in terms of practicability must be handled with caution in Africa. Although the LGL distribution has great benefits of capturing the legal time dynamics, its usefulness lies in its practical application in the face of the available institutional capacity. The computational infrastructure and technical capacity building strategic investments can be required to reach maximum analytical potential of the distribution.

**7.3. Contextual Adaptation.** This flexibility of distribution requires cautious model selection methods to prevent overfitting especially when compared to simple alternatives in African applications where parsimony might be appreciated. Researchers and practitioners are supposed to use intensive model comparison structures, cross-validation methods and the incorporation of domain knowledge to enable proper model selection in particular African judicial settings.

Another aspect of significance is the contextual adaptation of the LGL distribution. Judicial systems in Africa are characterized by a great variation in legal tradition, rules of procedure and institutional design. A change of the distribution framework to reflect these contextual differences; by interpreting its parameters, choice of estimating methods or methods of application would render it more relevant and useful among the various jurisdictions in Africa.

## 8. CONCLUSION AND FUTURE RESEARCH DIRECTIONS

**8.1. Theoretical Contributions and Analytical Advancements.** The Logistic-Gompertz-Law distribution presented in the paper offers a strong statistical framework of legal temporal data modeling in the African context. We have also shown its usefulness in a variety of judicial situations that occur throughout the continent through a well-developed theoretical base, the simulative model of validation, and discussion of its practical application. The capacity of the distribution to embody typical S-shaped pattern of progression, elasticity of hazard formulations, and the interpretations of its parameter as intuitive, make it a useful tool to the African judicial systems in the legal analytics.

The theoretical implications of the study are not limited to the given formulation of the distribution, but also to the methodology of the legal temporal analysis as applied to African conditions. This piece of work proves that high-level statistical tools can be used to improve the

administration of justice and study of law throughout the continent by combining mathematical complexity with situational awareness. The strict research and validation system predetermines the norm of new methodological innovations in African law analytics in the future.

**8.2. Practical Implications and Implementation Strategies.** The LGL distribution has a practical implication in a number of spheres of the African court administration and legal practice. Its forecasting abilities can be taken advantage of by court managers as they can be exploited to optimize resources and manage dockets. Its efficiency metrics can be used by policy-makers in evidence-based reforms in the judiciary. Its risk assessment capabilities can be applied by legal practitioners to manage cases on a strategic basis. These multi purposes underscore the distribution capabilities of improving more effective, transparent as well as accessible justice systems in Africa.

The implementation will have to be done through strategic approaches with the African contextual factors to be taken into consideration to achieve success. Adoption in various jurisdictions can be achieved through capacity building programs, development of easy to use software, and through exchange of knowledge. The cooperation among researchers, judicial administrators, and legal practitioners will be necessary in order to put advances in methodology into effective changes in judicial operations and access to justice.

**8.3. Future Research Directions and Development of the Methodology.** There are a number of avenues that the future research directions can take this work to the African contexts. Multivariate extensions have the potential to add case features, partisan attributes, and court variables to duration modelling, to reflect the rich interactions that determine the course of cases. Regression equations would allow the covariate impacts on all the aspects of distribution providing finer analysis of the influence of factors on legal timelines in various African jurisdictions.

Proportional data that depends on time, e.g. court clearance rates or filing patterns which have seasonal or trend factors, could be modeled using time series applications. Combination with machine learning systems would improve predictive accuracy, but still have probabilistic interpretability, which could take advantage of the increasing accessibility of legal information in digital forms in the court systems of Africa.

This advancement of specialized computational algorithms and software implementations will enable widespread adoption in the African legal practice and legal scholarship. Theoretical extensions such as characterization theorems, limit distributions and links with other distribution families would contribute to solidifying the mathematical base but remain relevant to the African judicial applications.

**8.4. Concluding Reflections.** With more and more pressure on African judicial systems to prove their efficiency, transparency and accessibility, advanced statistical techniques such as the LGL distribution will become ever more important in evidence-based judicial administration

and legal reform. This effort to reconcile the higher order of statistical practice with adaptation to the local context of law and legal problems will bring the revolution of legal practice and research to this day and further as the legal community increasingly turns towards quantitative rigor and empirical confirmation.

The ongoing evolution and use of specific analytical frameworks of African jurisdictional systems is an encouraging direction of increased justice delivery and institutional fortification. The LGL distribution suggests a specialized approach to the legal temporal analysis as well as a methodology that could be used in the future to prompt more innovations at the crossroads between statistical science and legal informatics in the African continent.

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**Data Availability.** The R code that followed the LGL distribution along with a simulation, parameter estimation and visualization features can be obtained by the respective author on reasonable request on academic and research purposes. The artificial data that were created as part of this research can be verified and developed to enhance the methods and are open to the standard academic use policies. The authors are open to share their data and assist in the implementation of collaborative applications of the LGL distribution in judicial settings in Africa and are open to contact by scholars who wish to share information about the application.

**Conflict of Interest.** The authors do not express any possible conflict of interests in their research in this paper. This study was not supported financially in any way and remains independent in terms of developing the methods of work and drawing analytical conclusions. This study was done with the interest of the authors in their academic duties and the need to promote statistical methodology to be used in the African judiciary.

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