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Different Types of Topological Structures by Graphs

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Abstract. In this paper, we will represent relation of graph which bring different type of topological structure to the graph [2], then, consider certain properties of the graph. We will discuss mainly blood circulation in lungs and some different diseases of it [4] and relate them with graph and make topologies [8]. Moreover, certain applications in medical field will be represent. We can also use results in real life [11].

1. Introduction and Preliminaries

Initially in eighteenth century swiss mathematician Leonhard Euler gave the basic idea about graph [2]. He resolved famous problems. He drew any tenth spectral graph theory introduced in decade of 1950, while in 1980 introduced monograph spectra by Cvetkovics, Doob and Sachs. Recently graph theory has become very large field not only for mathematicians but also for other fields of life [13]. In real life graph theory playing its vital role of life, Very common example of it is all roads and motorways form a large network which is used by cruising services e.g. goggle maps when working on different routs between two points. Graph theory is the study of graph, which mathematically used to develop pairwise relationship between objects [13]. This is also a collection of points and lines. Points are known as vertices and lines are edges. The collection of vertices of any graph G is vertex set and collection of edges is known as edge set denominated as V(G) and E(G) respectively [4]. The number of vertices and edges in G is known as order and size of G respectively. If an edge has same end is loop. Whenever more than one edges having same final point than it will consider parallel edges [12]. Mapping [14] play a specific role in graph theory also.

Notions on closure operations are helpful for algebra, topology, basic graph theory and also for many other fields [4]. Topology is very advance field of mathematics. It deals with thins independently. It allow to increase or decrease things without cutting. Consider [11] X might be

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nonempty set furthermore ϕ is collection for X, than (1) ϕ, X belongs to τ (2) Absolute union for number of τ belongs to τ (3) Limited intersection for τ belongs to τ. Than τ will be consider topology over X so, (X, τ) is called topological space. Topology also helpful in different properties like Convergence, Existence, Convexity and many other. All elements within topology known as open set and complement might be close [13].

Consider G is any graph, than two adjacent vertices are called nbhd of each other

$$N(V) = \{u \in v(G) \mid u \text{ be nbhd of } V\}$$

is open nbhd for V and $N[V] = N(V) \cup \{V\}$ is closed nbhd for V. [10]

Loops and parallel edge free graph is simple graph. If any two distinct vertices joined by an edge is named as complete graph. [12]

If vertices of two sets A and B joined by each edge between A and B is called bipartite graph. If each vertex from A connected with every vertices of B with only a single edge is complete bipartite graph [8].

If we delete any edge from a subgraph G is called spanning subgraph while deleting any vertex is induced subgraph [3].

Consider that if any subgraph do not contain their final point that channel P will be nominated by topological open subgraph while having its initial and final point is topological closed graph [7].

Consider $G = (V, E)$ be any connected graph. Moreover, $(V(G), \tau)$ be topology [7] generate with

$$\beta_j = \{V(G), \phi, \{V_j\}, \{N(V_j)\}\}$$

is basis moreover consider S_1 and S_2 be two open paths than

$$(I)\, V(S_1) \subseteq Cl\, V(S_1)$$

$$(II)\, S_1 \subseteq S_2$$

and

$$Cl(V(S_1)) \subseteq Cl(V(S_2))$$

2. Relation Over Graph

Suppose that U is vertex in any graph G having l^* loop and m multiple edges than

$$(\deg_G(u))_u = (2l_u + m_u)_u$$

while simple graph is $(\deg_G(u))_u$. [2]

[4] Here is relation R for any graph G is deformed by

$$R = \{((2l_u + m_u)_u, (2l_w + m_w)_w), u, w \in V\}$$

While l_u and l_w are number for loops for vertices u ,w from each furthermore m_u , m_w are multiple edges for vertex u and w respectively. Consider that G be simple graph,

$$R = \{((\deg_G(u)_u, \deg_G(w)_w)); u, w \in V\}$$

if $l = 0$ than

$$R = \{(l_u)_u, (m_w)_w, u, w \in W\}$$

if $m = 1$ and $l = 0$ than

$$R = \{(l_u, l_w) u, w \in V$$

Consider G is directed along with simple than

$$R = \{(l_u, l_w) = (U, W) \ \ u, w \in V\}$$

while if G is undirected than

$$R = \{(l_u, l_w) = (u, w)$$

or

$$(w, u) \ \ u, w \in V \ \}$$

Example 1. *[4]*

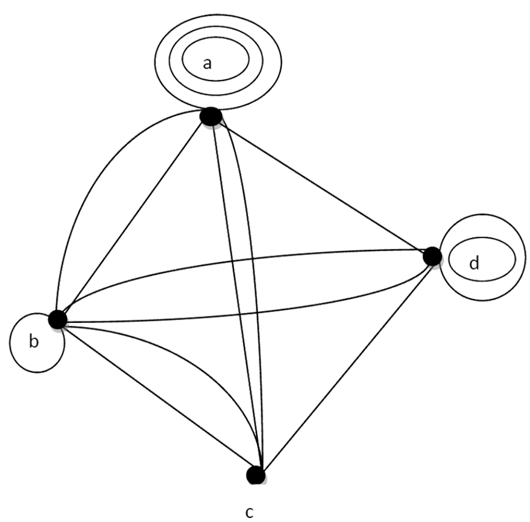


Fig 1. A pesudograph G

Suppose that G is undirected graph given above Fig.1.

$$\begin{aligned} R &= \{(11_a, 8_b), (11_a, 5_c), (11_a, 8_d), (8_b, 5_c), (8_b, 8_d), \\ &(5_c, 8_d), (11_a, 11_a), (8_b, 8_b), (8_d, 8_d) \end{aligned}$$

Example 2. *[1]*

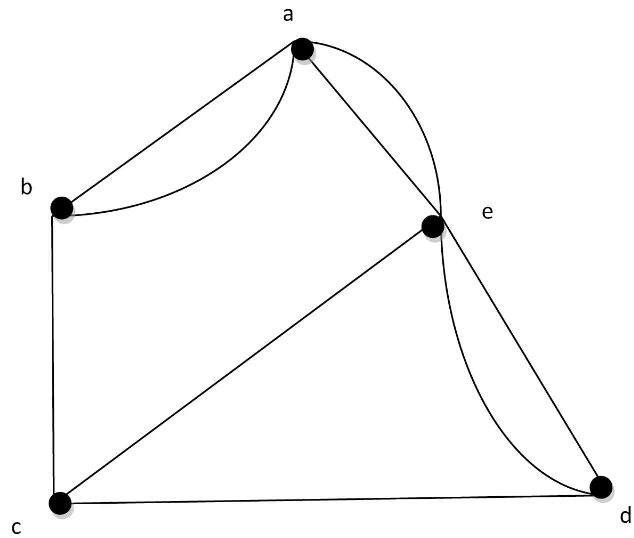


Fig 2. A Multiple graph

Let G be a graph given in figure 02

$$R = \{(3_c, 3_b), (4_a, 5_e), (3_b, 3_c), (3_c, 3_d), (3_c, 5_e), (5_e, 3_d)\}$$

Example 3. *[2]*

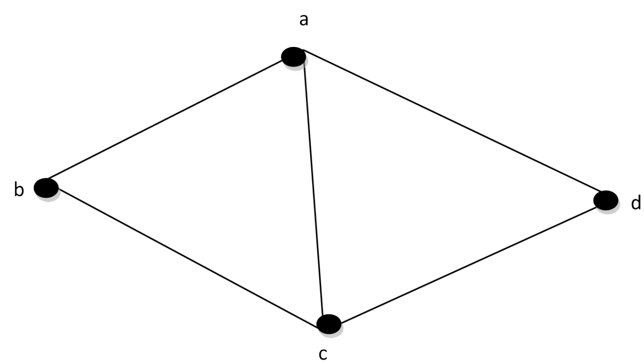


Fig 3. A Simple graph

Let G be a graph in figure 3

$$R = \{(3_a, 2_b), (3_a, 2_d), (3_a, 3_c), (2_b, 3_c), (3_c, 2_d)$$

3. Topological Structure on Graph

By previous illustration (1) created a topology. According to this example the vertices are given as

$(11_a)\,R = \{8_b, 8_d, 5_c\}, (8_b)\,R = \{11_a, 5_c, 8_d\}, (5_c)\,R = \{8_b, 8_d, 11_a\}, (8_d)\,R = \{11_a, 8_b, 5_c\}$

Subbase

$$\begin{aligned} & S_G \\ = \; & \{\{8_b, 8_d, 5_c\}, \{11_a, 5_c, 8_d\}, \{8_b, 8_d, 11_a\}, \{11_a, 8_b, 5_c\}\} \end{aligned}$$

Topology

$$\begin{aligned} & \tau_G \\ = \; & \{X, \phi, \{8_b, 8_d, 5_c\}, \{11_a, 5_c, 8_d\}, \{8_b, 8_d, 11_a\}, \\ & \{11_a, 8_b, 5_c\}, \{8_d, 5_c\}, \{8_b, 8_d\}, , \{8_b, 5_c\}, \\ & \{11_a, 8_d\}, \{11_a, 5_c\}, \{11_a, 8_b\}, \{8_b, 5_c, 8_d\}, \\ & \{11_a, 5_c, 8_d\}, \{11_a, 8_b, 8_d\}, \{11_a, 8_b, 5_c\} \end{aligned}$$

By previous illustration(2) created a topology. According to this example the vertices are given as

$$\begin{aligned} & (4_a)\,R \\ = \; & \{3_b, 5_e\}, (3_b)\,R = \{4_a, 3_c\}, (5_e)\,R \\ = \; & \{4_a, 3_c, 3_d\}, (3_c)\,R = \{3_b, 3_d, 5_e\}, (3_d)\,R = \{3_c, 5_e\} \end{aligned}$$

Subbase

$$\begin{aligned} & S_G \\ = \; & \{\{3_b, 5_e\}, \{4_a, 3_c\}, \{4_a, 3_c, 3_d\}, \{4_a, 3_d, 5_e\}, \{3_c, 5_e\}\} \end{aligned}$$

Base

$$\begin{aligned} & \beta_G \\ = \; & \{X, \phi, \{3_b, 5_e\}, \{4_a, 3_c\}, \{4_a, 3_c, 3_d\}, \{3_b, 3_d, 5_e\}, \{3_c, 5_e\}, \{5_e\}, \{3_c\}, \{3_d\} \end{aligned}$$

Topology

$$\begin{aligned} & \tau_G \\ = \; & \{X, \phi, \{3_b, 5_e\}, \{4_a, 3_c\}, \{4_a, 3_c, 3_d\}, \{3_b, 3_d, 5_e\}, \{3_c, 5_e\}, \{3_c\}, \\ & \{3_d\}, \{5_e\}, \{4_a, 3_b, 3_c, 5_e\}, \{4_a, 3_c, 3_d, 5_e\}, \{3_b, 3_c, 5_e\}, \{4_a, 3_c, 5_e\}, \{4_a, 3_d\}, \\ & \{3_b, 3_c, 3_d, 5_e\}, \{3_c, 3_d, 5_e\}, \{3_c, 3_d\}, \{3_c, 5_e\}, \{3_d, 5_e\} \end{aligned}$$

By previous example (3) it is given as.

$$\begin{aligned} & (3_a)\, R \\ = \; & \{2_b, 3_c, 2_d\}, (2_b)\, R = \{3_a, 3_c\}, (3_c)\, R \\ = \; & \{2_b, 3_a, 2_d\}, (2_d)\, R = \{3_a, 3_c\} \end{aligned}$$

Subbase

$$S_G = \{\{2_b, 3_c, 2_d\}, \{3_a, 3_c\}, \{2_b, 3_a, 2_d\}, \{3_a, 3_c\}$$

Base

$$\begin{aligned} & \beta_G \\ = \; & \{X, \phi, \{2_b, 3_c, 2_d\}, \{3_a, 3_c\}, \{3_a, 2_b, 2_d\}, \\ & \{3_a, 3_c\}, \{3_c\}, \{2_b, 2_d\}, \{3_a\} \end{aligned}$$

Topology

$$\begin{aligned} & \tau_G \\ = \; & \{X, \phi, \{2_b, 3_c, 2_d\}, \{3_a, 3_c\}, \{3_a, 2_b, 2_d\}, \\ & \{3_a, 3_c\}, \{3_c\}, \{2_b, 2_d\}, \{3_a\}, \{2_b, 3_c, 2_d\} \end{aligned}$$

Consider that $G = (V^*, E^*)$ is graph moreover H is induced subgraph for G. So,

$$cl\,(V^*\,(H)) = V\,(H)\, U\{x \in v^*\,(G)\,; xR \cap V\,(H) \neq \phi$$

furthermore

$$xR = \{\left(\deg_G\,(a_r)_{a_r}\right)\}$$

$\forall\ r \in I$ and a_r is set of every adjacent vertices V_i.

Suppose that $G = (V^*, E^*)$ is graph. Moreover H IS induced subgraph for G and

$$Int\,(V^*\,(H)) = \{x \in V^*\,(G)\,; xR \subseteq V\,(H)\,, xR = \{\left(\deg_G\,(a_r)_{a_r}\right)$$

$\forall\ r \in I$ and a_r adjacent for X.

4. Some Applications

In this part we will give an example of blood circulation in lungs. We will also draw topological structure of this circulation. We will relate mathematics with medical field. We will made graph of it. Moreover, we will discuss few reasons of disability in lungs and cause of dangerous diseases. We will explain these diseases mathematically.

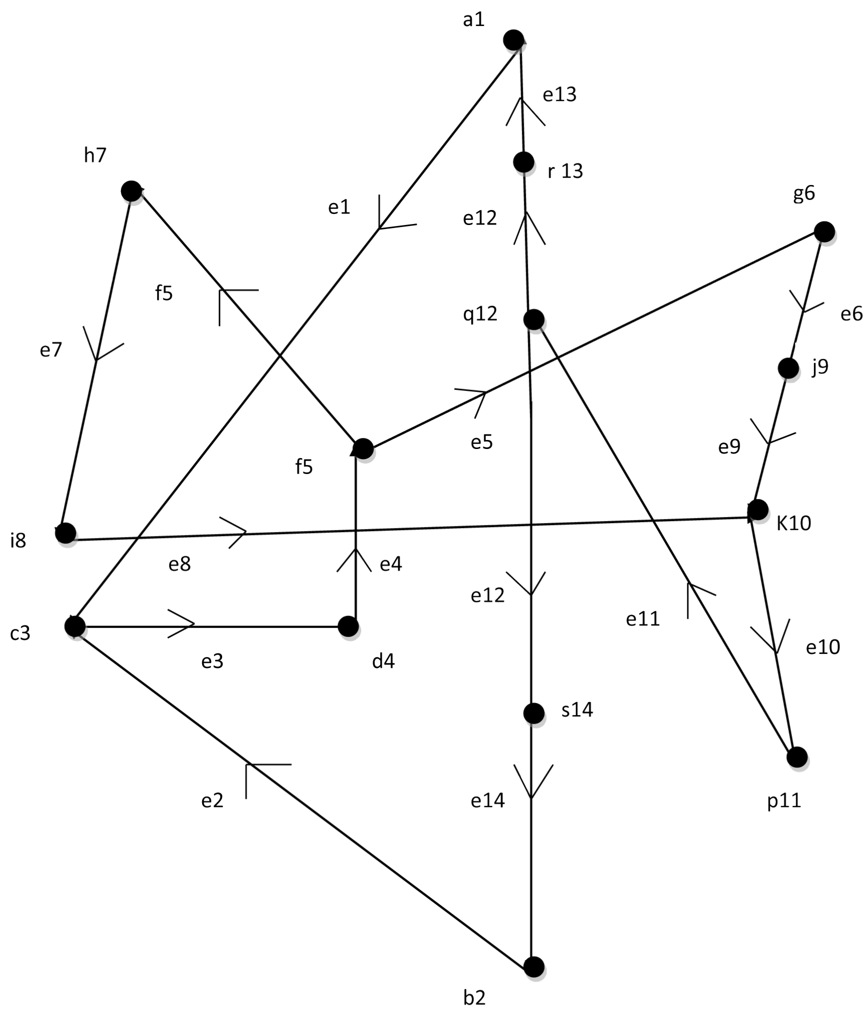
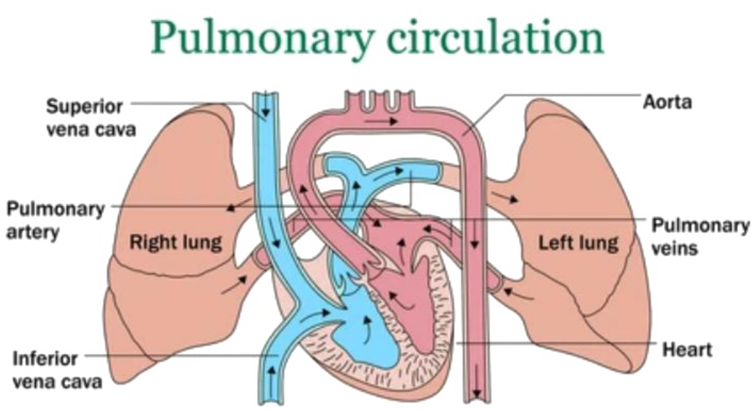


Fig 4



Here we will utilize our work discussed above in medical field. We will introduced the technique in which connected graph is modifying condition in the medical field. Diagram represent to graph..

We can notice the blood circulation in lungs is representation of set of vertices and edges. Than, we can define a topological structure τ_G on that. Post classes for vertices in graph are the following given below.

$(a_1)\,R = \{c_3\}, (b_2)\,R = \{c_3\}, (c_3)\,R = \{d_4\}, (d_4)\,R = \{f_5\}, (f_5)\,R = \{g_6, h_7\}, (g_6)\,R = \{j_9\}, (h_7)\,R = \{i_8\}, (i_8)\,R = \{k_{10}\}, (j_9)\,R = \{k_{10}\}, (k_{10})\,R = \{p_{11}\}, (p_{11})\,R = \{q_{12}\}, (q_{12})\,R = \{r_{13}, s_{14}\}$,

$(r_{13})\,R = \{a_1\}, (s_{14})\,R = \{b_2\}$

The subbase has a form

$$\begin{aligned} & S_G \\ = \ & \{\{c_3\}, \{d_4\}, \{f_5\}, \{g_6, h_7\}, \{j_9\}, \{i_8\}, \{k_{10}\}, \{p_{11}\}, \\ & \{q_{12}\}, \{r_{13,} s_{14}\}, \{a_1\}, \{b_2\}\} \end{aligned}$$

Base has a form

$$\begin{aligned} & \beta_G \\ = \ & \{X, \phi, \{c_3\}, \{d_4\}, \{f_5\}, \{g_6, h_7\}, \{j_9\}, \{i_8\}, \\ & \{k_{10}\}, \{p_{11}\}, \{q_{12}\}, \{r_{13}, s_{14}\}, \{a_1\}, \{b_2\}\} \end{aligned}$$

Topology on a graph G have

$$\begin{aligned} & \tau_G \\ = \ & \{X, \phi, \{c_3\}, \{d_4\}, \{f_5\}, \{g_6, h_7\}, \{j_9\}, \{i_8\}, \{k_{10}\}, \{p_{11}\}, \\ & \{q_{12}\}, \{r_{13}, s_{14}\}, \{a_1\}, \{b_2\}, \\ & \{c_3, d_4\}, \{c_3, f_5\}, \{c_3, g_6, h_7\}, \{c_3, j_9\}, \{c_3, i_8\}, \\ & \{c_3, k_{10}\}, \{c_3, p_{11}\}, \{c_3, q_{12}\}, \\ & \{c_3, r_{13}, s_{14}\}, \{c_3, a_1\}, \{c_3, b_2\}, \{d_4, f_5\}, \\ & \{d_4, g_6, h_7\}, \{d_4, j_9\}, \{d_4, i_8\}, \{d_4, k_{10}\}, \\ & \{d_4, p_{11}\}, \{d_4, q_{12}\}, \{d_4, r_{13}, s_{14}\}, \{d_4, a_1\}, \\ & \{d_4, b_2\}, \{f_5, g_6, h_7\}, \{f_5, j_9\}, \{i_8\}, \{f_5, k_{10}\}, \\ & \{f_5, p_{11}\}, \{f_5, q_{12}\}, \{f_5, r_{13}, s_{14}\}, \{f_5, a_1\}, \\ & \{f_5, b_2\}, \{g_6, h_7, j_9\}, \{g_6, h_7, i_8\}, \{g_6, h_7, k_{10}\}, \\ & \{g_6, h_7, p_{11}\}, \{g_6, h_7, q_{12}\}, \{g_6, h_7, r_{13}, s_{14}\}, \\ & \{g_6, h_7, a_1\}, \{g_6, h_7, b_2\}, \{j_9, i_8\}, \{j_9, k_{10}\}, \\ & \{j_9, p_{11}\}, \{j_9, q_{12}\}, \{j_9, r_{13}, s_{14}\}, \{j_9, a_1\}, \end{aligned}$$

$$\{j_9, b_2\}, \{k_{10}, p_{11}\}, \{k_{10}, q_{12}\}, \{k_{10}, r_{13}, s_{14}\},$$
$$\{k_{10}, a_1, b_2\}, \{p_{11}, q_{12}\}, \{p_{11}, r_{13}, s_{14}\},$$
$$\{p_{11}, a_1\}, \{p_{11}, b_2\}, \{q_{12}, r_{13}, s_{14}\}, \{q_{12}, a_1, \}$$
$$\{q_{12}, b_2\}, \{r_{13}, s_{14}, a_1\}, \{r_{13}, s_{14}, b_2\}, \{a_1, b_2\}$$

Initially we get closure of graph. If H is any subgraph

$$H = \{b_2, c_3, e_2, e_3, e_4\}$$

that is

$$V(H) = \{b_2, c_3\}$$

by definition of closure for subgraph H be

$$cl(V(H)) = \{b_2, c_3, d_4\}$$

Medically, here we will use that illustration for circulation of blood in lungs will be true. Blood flow in lungs by directed path to complete its cycle. But due to any fault flow of blood distribute and stop. It create serious diseases. Moreover, we can find interior for graph over subgraph

$$H = \{f_5, e_5, g_6, e_7, h_7\}$$

but from definition we can assume

$$intV(H) = \{f_5, g_6\}$$

In this example we note that end point does not include. This contradiction in heart but suitable for lungs medically because due to some disorder people can also survive with only one lungs this is gift of God.

5. Some Serious Diseases in Lungs

There are some diseases in lungs due to some disorder. These diseases are divided in to some categories. We will discuss reasons of these diseases and express them graphically. Moreover, we will also show topological structure. [3]

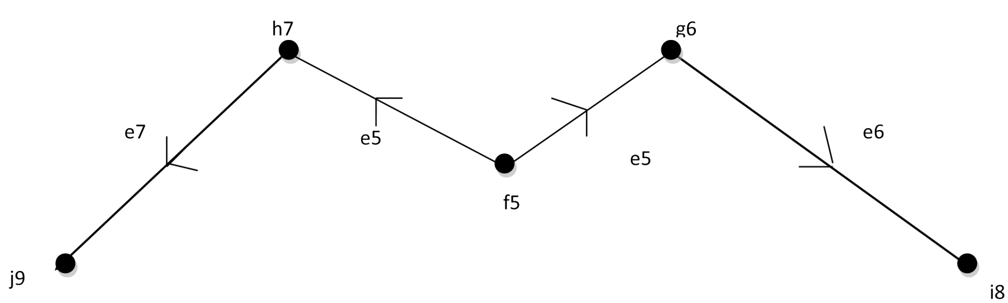


Fig 5

5.1. **Pulmonary Arterial Hypertension.** Heart problem autoimmune system can cause high blood pressure in pulmonary arteries.

$$(f_5)\,R = \{g_6, h_7\}, (g_6)\,R = \{i_8\}, (h_7)\,R = \{j_9\}$$

Subbase

$$S_G = \{\{g_6, h_7\}, \{i_8\}, \{j_9\}\}$$

Base

$$\beta_G = \{X, \phi, \{g_6, h_7\}, \{i_8\}, \{j_9\}\}$$

Topology

$$\begin{aligned} &\tau_G \\ = \;& \{X, \phi, \{g_6, h_7\}, \{i_8\}, \{j_9\}, \{g_6, h_7, i_8\}, \{g_6, h_7, j_9\}, \{h_8, j_9\}\} \end{aligned}$$

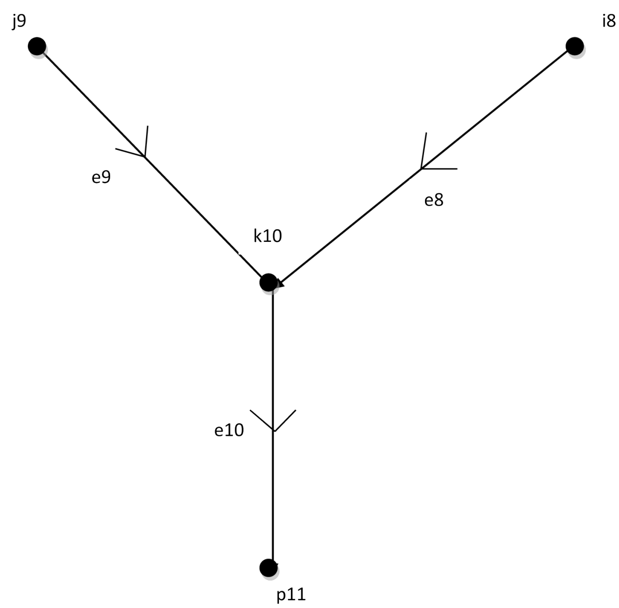


Fig. 6

5.2. **Pulmonary Venous Hypertension.** Any damage or self eating of mitral valve can cause higher blood pressure in pulmonary veins.

$$(i_8)\,R = \{k_{10}\}, (j_9)\,R = \{k_{10}\}, (k_{10})\,R = \{p_{11}\}$$

Subbase

$$S_G = \{\{k_{10}\}, \{p_{11}\}\}$$

Base

$$\beta_G = \{X, \phi, \{k_{10}\}, \{p_{11}\}\}$$

Topology

$$\tau_G = \{X, \phi, \{k_{10}\}, \{p_{11}\}, \{k_{10}, p_{11}\}\}$$

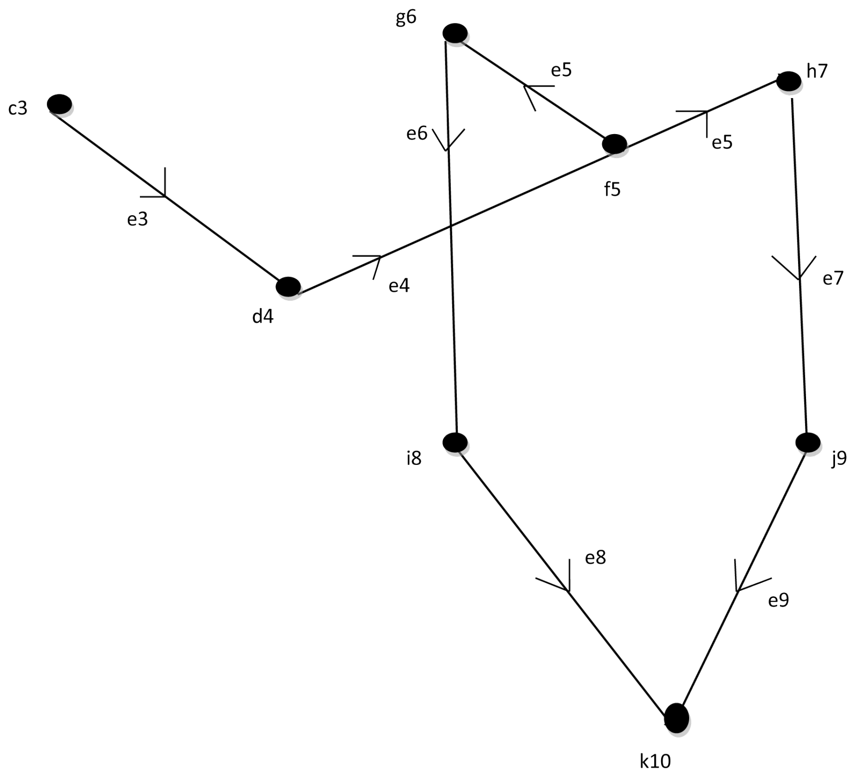


Fig. 7

5.3. **Pulmonary Embolism.** Any coagulation of blood or fat droplets can travel to lungs from heart and cause blockage of lungs blood vessels.

$$\begin{aligned} & (c_3)\, R \\ = \; & \{d_4\}, (d_4)\, R = \{f_5\}, (f_5)\, R = \{g_6, h_7\}, \\ & (g_6)\, R \\ = \; & \{i_8\}, (h_7)\, R = \{j_9\}, (i_8)\, R = \{k_{10}\}, (j_9)\, R = \{k_{10}\} \end{aligned}$$

Subbase

$$S_G = \{\{d_4\}, \{f_5\}, \{g_6, h_7\}, \{i_8\}, \{j_9\}, \{k_{10}\}\}$$

Base

$$\beta_G = \{X, \phi, \{d_4\}, \{f_5\}, \{g_6, h_7\}, \{i_8\}, \{j_9\}, \{k_{10}\}, \}$$

Topology

$$\begin{aligned} & \tau_G \\ = & \{X, \phi, \{d_4\}, \{f_5\}, \{g_6, h_7\}, \{i_8\}, \{j_9\}, \\ & \{k_{10}\}, \{d_4, f_5\}, \{d_4, g_6, h_7\}, \{d_4, i_8\}, \\ & \{d_4, j_9\}, \{d_4, k_{10}\}, \{f_5, g_6, h_7\}, \{f_5, i_8\}, \\ & \{f_5, j_9\}, \{f_5, k_{10}\}, \{g_6, h_7, i_8\}, \{g_6, h_7, j_9\}, \\ & \{g_6, h_7, k_{10}\}, \{i_8, j_9\}, \{i_8, k_{10}\}, \{j_9, k_{10}\} \end{aligned}$$

6. Conclusion

We derive topological structure by using different relations defined above. We also use different type of graphs. We also mentioned the method of general topology its graph and relationship between both of them. We also represent medical field, blood circulation in lungs, its diseases made topologies by using graph.

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