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A Novel Discrete Distribution: Properties and Application Using Nipah Virus Infection Data Set

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Abstract. In this study, the Poisson (PD) distribution was compounded with a continuous distribution to produce the Poisson XLindley distribution (PXLD). Its raw moments and central moments are acquired as a result of a general expression for its r^{th} factorial moment concerning origin being derived. Additionally, the expressions for its coefficient of variation, skewness, kurtosis and index of dispersion have been provided. For the estimate of its parameters, in particular, the methods of maximum likelihood and moments have been addressed. The applicability of the proposed distribution in modeling real data sets on Nipah virus infection, number of Hemocytometer yeast cell count data, and epileptic seizure counts data is examined by analyzing two real-world data sets.

1. Introduction

Adding a distribution attributed to Lindley to the Poisson distribution, a compound Poisson distribution is created. With probability mass function, the Poisson-Lindley distribution derived as

$$f(x;\theta)=\frac{\theta^2(x+\theta+2)}{(1+\theta)^{x+3}},\ \ x=0,1,\ldots;\ \ \theta>0$$

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was introduced by Sankaran (1970) to model count data. The distribution derives from the Poisson distribution when its parameter λ follows a Lindley (1958) distribution with the probability density function:

$$g(\lambda;\theta)=\frac{\theta^2(1+\lambda)}{(1+\theta)}e^{-\theta\lambda},\ \lambda>0;\ \theta>0.$$

Compounding the Poisson distribution with the generalized Lindley distribution yielded the extended form of the compound Poisson distribution that Mahmoudi and Zakerzadeh (2010) suggested. Zakerzadeh and Dolati (2009) then further examined this distribution. Shanker (2016a, 2016b) proposed compound Poisson distributions, named the Poisson–Amarendra distribution (PAD) and Poisson–Sujatha distribution (PSD), by compounding Poisson and Amarendra (Sujatha) distributions. In 2017, a Poisson mixture of Garima distribution named, Poisson-Garima distribution (PGD) has been proposed by Shanker. Grine and Zeghdoudi (2017) introduced the Poisson Quasi-Lindley distributions. Zeghdoudi and Nedjar (2017, 2020) proposed compound Poisson distributions, named the Poisson Pseudo-Lindley distribution and Poisson Gamma Lindley (PGaL), by compounding Poisson and pseudo-Lindley (gamma Lindley) distributions.

Chouia and Zeghdoudi (2021) introduced a new distribution, named XLindley distribution (XLD) having the probability density function (p.d.f.)

$$f(x;\theta)=\frac{\theta^2(2+\theta+x)}{(1+\theta)^2}e^{-\theta x},\ x>0;\ \theta>0. \quad (1.1)$$

The first four moments about origin and the variance of XLD obtained by Chouia and Zeghdoudi (2021) are as follows:

$$\mu_1'=\frac{(1+\theta)^2+1}{\theta(1+\theta)^2},\ \mu_2'=\frac{2(\theta^2+2\theta+3)}{\theta^2(1+\theta)^2},$$

$$\mu_2=\frac{(1+\theta)^4+4\theta^2+6\theta+1}{\theta^2(1+\theta)^4}.$$

In order to describe lifetime data and biological sciences, this study will create a new lifetime distribution by compounding Poisson and XLindley distributions.

A Poisson XLindley distribution (PXLD) has been presented in the present study. Authors have determined and analyzed its raw and central moments as well as central moments-based features such as coefficient of variation, skewness, kurtosis, and index of dispersion. Its model parameters have also been covered. For estimating the PXLD parameter, the moment approach and the method of maximum likelihood estimation have been studied.

2. Poisson XLindley distribution

Consider $dF(\lambda)=e^{\lambda\Phi}h(\lambda)B(\Phi)d\lambda$, where $h(\lambda)=2-\Phi+\lambda$ and $B(\Phi)=\frac{\Phi^2}{(1-\Phi)^2}$, then the compound Poisson distribution is (Sankaran, 1970).

$$P_X=\int_0^\infty \frac{e^{-\lambda}\lambda^x}{x!}\cdot\frac{\Phi^2}{(1-\Phi)^2}(2-\Phi+\lambda)e^{\lambda\Phi}d\lambda$$

$$=\frac{\Phi^2}{(1-\Phi)^2x!}\int_0^\infty e^{\lambda(\Phi-1)}\cdot\left(2\lambda^x-\Phi\lambda^x+\lambda^{x+1}\right)d\lambda$$

Then, replace Φ with $-\theta$:

$$P_X=\frac{\theta^2}{(1+\theta)^2x!}\int_0^\infty e^{-\lambda(1+\theta)}\cdot\left(2\lambda^x+\theta\lambda^x+\lambda^{x+1}\right)d\lambda$$

$$=\frac{\theta^2}{(1+\theta)^2x!}\left[(2+\theta)\int_0^\infty \lambda^x e^{-\lambda(1+\theta)}d\lambda+\int_0^\infty \lambda^{x+1}e^{-\lambda(1+\theta)}d\lambda\right]$$

$$=\frac{\theta^2}{(1+\theta)^2x!}\left[(2+\theta)\frac{\Gamma(x+1)}{(1+\theta)^{x+1}}+\frac{\Gamma(x+2)}{(1+\theta)^{x+2}}\right]$$

$$P_X=\frac{\theta^2}{(1+\theta)^2}\left[(2+\theta)\frac{\Gamma(x+1)}{(1+\theta)^{x+1}}+\frac{\Gamma(x+2)}{(1+\theta)^{x+2}}\right]$$

$$=\frac{\theta^2}{(1+\theta)^2}\left[\frac{\theta^2+3\theta+3+x}{(1+\theta)^{x+2}}\right]$$

The p.m.f. of the Poisson XLindley distribution (PXLD) with parameter θ can be obtained as

$$P_{PXL}(x;\theta)=\frac{\theta^2\left(\theta^2+3\theta+3+x\right)}{(1+\theta)^{x+4}},\quad x=0,1,2,\ldots;\theta>0. \quad (2.1)$$

To study the nature and behavior of PXLD for varying values of parameter θ, a number of graphs of the pmf of PXLD have been drawn and presented in the figure 1.

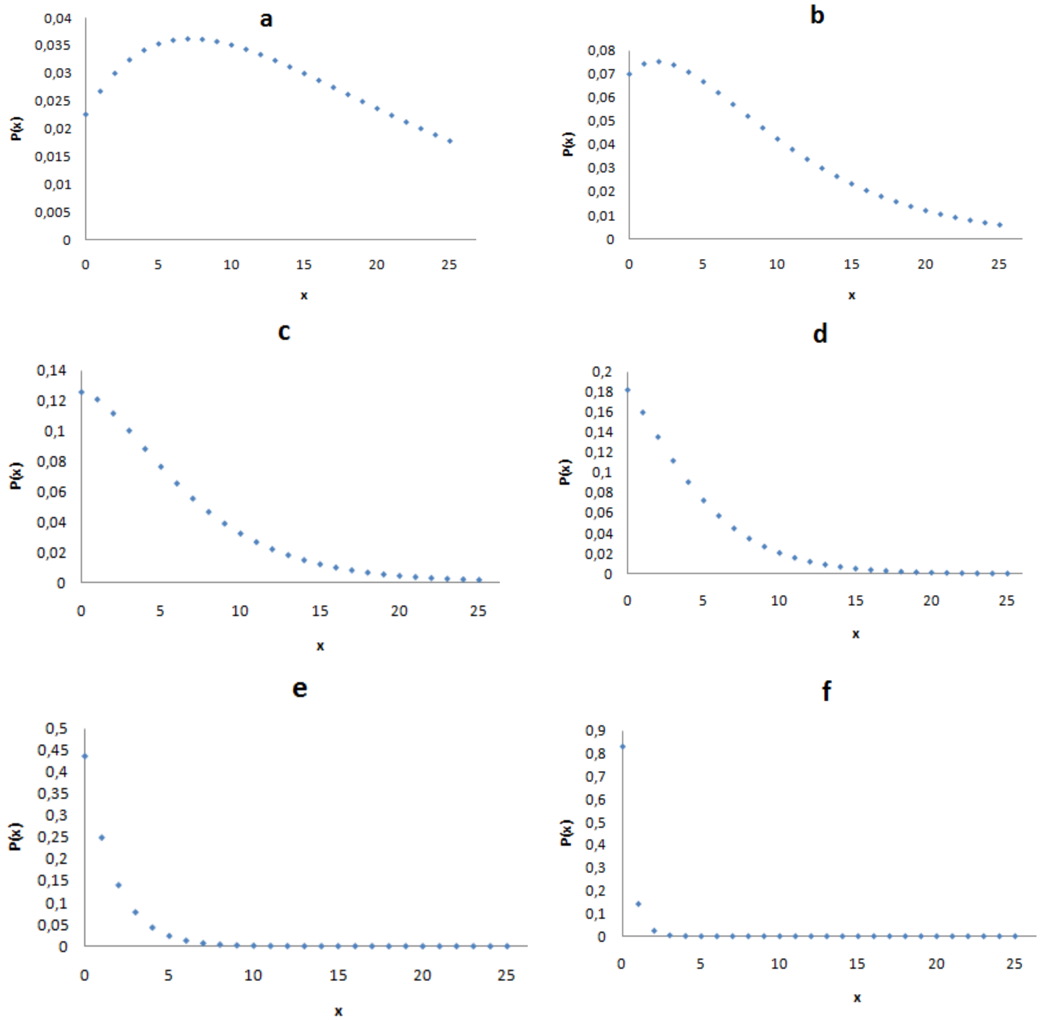

Figure 1: Plots of the p.m.f. of the Poisson XLindley distribution: (a) $\theta = 0.1$ (top left); (b) $\theta = 0.2$ (top right); (c) $\theta = 0.3$ (middle left); (d) $\theta = 0.4$ (middle right); (e) $\theta = 1$ (bottom left); and, (f) $\theta = 5$ (bottom right).

3. Moments and Moments Based Measures

The r^{th} factorial moment about origin of the PXLD (2.1) can be obtained as

$$\mu'_{(r)} = E\left(E\left(X^{(r)} \middle| \lambda\right)\right)$$
$$= \int_0^\infty \left[\sum_{x=0}^\infty x^{(r)} e^{-\lambda} \frac{\lambda^x}{x!}\right] \cdot \frac{\theta^2}{(1+\theta)^2}(2+\theta+\lambda)e^{-\theta\lambda} d\lambda$$
$$= \frac{\theta^2}{(1+\theta)^2} \int_0^\infty \lambda^r \left[\sum_{x=r}^\infty e^{-\lambda} \frac{\lambda^{x-r}}{(x-r)!}\right] \cdot (2+\theta+\lambda)e^{-\theta\lambda} d\lambda$$

Taking $x+r$, in place of x we get:

$$\mu'_{(r)} = \frac{\theta^2}{(1+\theta)^2} \int_0^\infty \lambda^r \left[\sum_{x=0}^\infty e^{-\lambda} \frac{\lambda^x}{x!}\right] \cdot (2+\theta+\lambda)e^{-\theta\lambda} d\lambda$$
$$= \frac{\theta^2}{(1+\theta)^2} \int_0^\infty \lambda^r \cdot (2+\theta+\lambda)e^{-\theta\lambda} d\lambda$$
$$= \frac{\theta^2}{(1+\theta)^2}\left[(2+\theta)\int_0^\infty \lambda^r \cdot e^{-\theta\lambda} d\lambda + \int_0^\infty \lambda^{r+1} \cdot e^{-\theta\lambda} d\lambda\right]$$

Using gamma integral and a little algebraic simplification, we get finally a general expression for the r^{th} factorial moment of PXLD as

$$\mu'_{(r)} = \frac{r!\left(\theta^2 + 2\theta + r + 1\right)}{\theta^r (1+\theta)^2}; \qquad r = 1, 2, 3, \ldots$$

The first four moments about origin of PXLD can be obtained as

$$\mu'_{(1)} = \frac{\theta^2 + 2\theta + 2}{\theta(1+\theta)^2}$$
$$\mu'_{(2)} = \frac{2\left(\theta^2 + 2\theta + 3\right)}{\theta^2 (1+\theta)^2}$$
$$\mu'_{(3)} = \frac{6\left(\theta^2 + 2\theta + 4\right)}{\theta^3 (1+\theta)^2}$$
$$\mu'_{(3)} = \frac{24\left(\theta^2 + 2\theta + 5\right)}{\theta^4 (1+\theta)^2}$$

Now using the relationship between factorial moments about origin and the moments about origin, the first four moments about origin of PXLD (2.1) can be obtained as

$$\mu_1' = \frac{\theta^2+2\theta+2}{\theta(1+\theta)^2}, \quad \mu_2' = \frac{\theta^3+4\theta^2+6\theta+6}{\theta^2(1+\theta)^2}$$

$$\mu_3' = \frac{\theta^4+8\theta^3+20\theta^2+30\theta+24}{\theta^3(1+\theta)^2}$$

$$\mu_4' = \frac{\theta^5+16\theta^4+66\theta^3+138\theta^2+192\theta+120}{\theta^4(1+\theta)^2}$$

Now, using the relationship between central moments and the moments about origin, the central moments of the PXLD (2.1) are thus obtained as

$$\mu_2 = \frac{\theta^5+5\theta^4+11\theta^3+14\theta^2+10\theta+2}{\theta^2(1+\theta)^4}$$

$$\mu_3 = \frac{\theta^8+9\theta^7+36\theta^6+87\theta^5+141\theta^4+152\theta^3+98\theta^2+30\theta+4}{\theta^3(1+\theta)^6}$$

$$\mu_4 = \frac{\begin{bmatrix}\theta^{11}+18\theta^{10}+127\theta^9+515\theta^8+1395\theta^7+2692\theta^6+\\ 3747\theta^5+3678\theta^4+2430\theta^3+1010\theta^2+240\theta+24\end{bmatrix}}{\theta^4(1+\theta)^8}$$

The coefficient of variation $(C.V)$, coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, coefficient of Kurtosis (β_2) and index of dispersion (γ) of the PXLD (2.1) are thus obtained as

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\theta^5+5\theta^4+11\theta^3+14\theta^2+10\theta+2}}{\theta^2+2\theta+2}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{(\mu_2)^{3/2}} = \frac{\theta^8+9\theta^7+36\theta^6+87\theta^5+141\theta^4+152\theta^3+98\theta^2+30\theta+4}{\left(\theta^5+5\theta^4+11\theta^3+14\theta^2+10\theta+2\right)^{\frac{3}{2}}}$$

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{\begin{bmatrix}\theta^{11}+18\theta^{10}+127\theta^9+515\theta^8+1395\theta^7+2692\theta^6+\\ 3747\theta^5+3678\theta^4+2430\theta^3+1010\theta^2+240\theta+24\end{bmatrix}}{\left(\theta^5+5\theta^4+11\theta^3+14\theta^2+10\theta+2\right)^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\theta^5+5\theta^4+11\theta^3+14\theta^2+10\theta+2}{(1+\theta)^2\left(\theta^3+2\theta^2+2\theta\right)}$$

To study the nature and behavior of $\mu_1', \mu_2, CV, \sqrt{\beta_1}, \beta_2$ and γ of PXLD, numerical values of these characteristics for varying values of the parameter θ have been presented in table 1.

It is clear that μ_1', μ_2 and γ are decreasing whereas $CV, \sqrt{\beta_1}, \beta_2$ are increasing for increasing values of the parameter θ.

Table 1: Values of θ for Poisson XLindley Distribution

θ	1	2	3	4	5	6
μ_1'	1.2500000	0.5555556	0.3541667	0.2600000	0.2055556	0.1700680
μ_2	2.6875000	0.8580247	0.4787326	0.3274000	0.2477469	0.1989680
CV	1.311488	1.667333	1.953614	2.421447	2.421447	2.622823
$\sqrt{\beta_1}$	1.883180	2.237235	2.458191	2.652708	2.833442	3.003631
β_2	8.586804	9.975260	11.027291	12.029514	13.024644	14.019731
γ	2.150000	1.544444	1.351716	1.259231	1.205255	1.169932

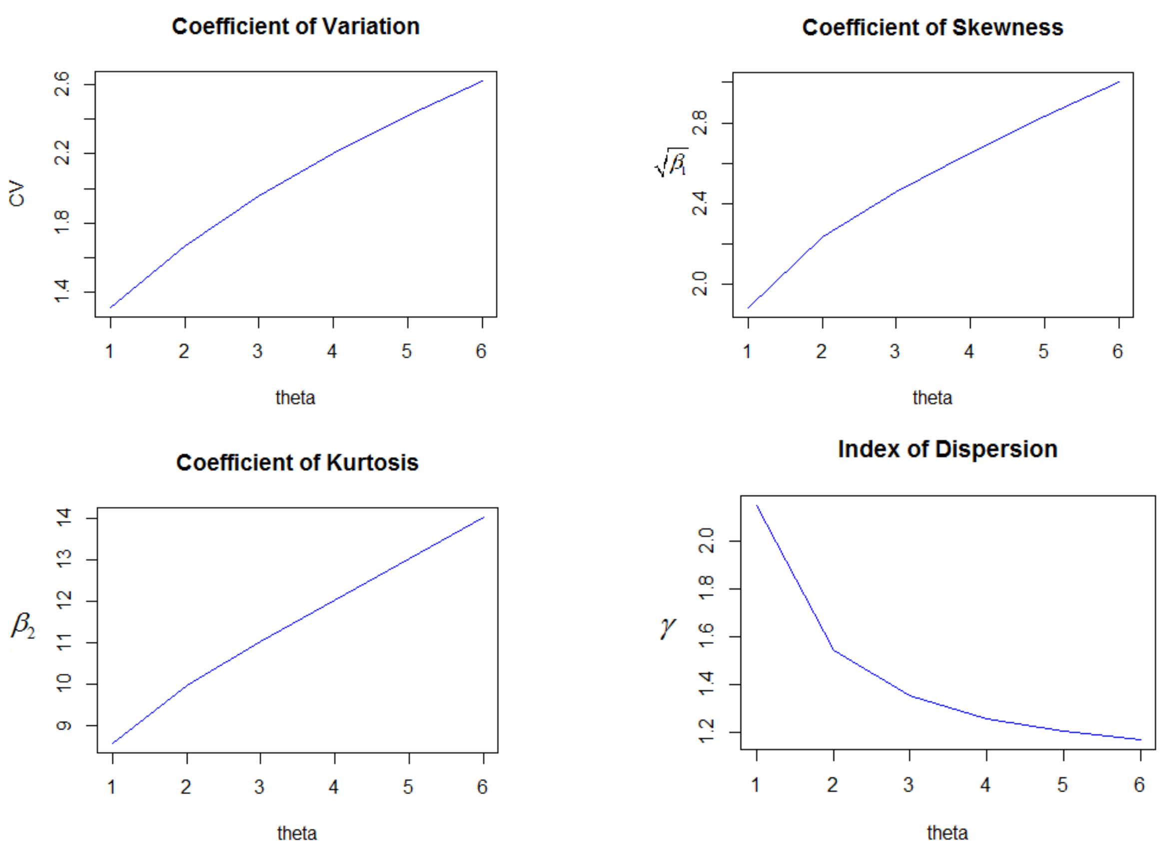

Figure 2: Graphs of coefficient of variation, coefficient of skewness, coefficient of kurtosis and index of dispersion of PXLD for varying values of the parameter θ.

Table 1 and figure 2 depicts graphs to examine the structure and performance of the PXLD (2.1) parameter's coefficient of variation, skewness, kurtosis, and index of dispersion for a range of parameter θ values. It is clear from the graphs that when the parameter θ value increases, PXLD's coefficients of variation, skewness, and kurtosis all increase as well.

4. Statistical Properties of PXLD

4.1. Over-Dispersion

The PXLD (2.1) is always over-dispersed $\left(\sigma^2 > \mu\right)$. We have

$$\begin{aligned}
\sigma^2 &= \frac{\theta^5 + 5\theta^4 + 11\theta^3 + 14\theta^2 + 10\theta + 2}{\theta^2 (1+\theta)^4} \\
&= \frac{\theta^2 + 2\theta + 2}{\theta(1+\theta)^2}\left[\frac{\theta^5 + 5\theta^4 + 11\theta^3 + 14\theta^2 + 10\theta + 2}{\theta(1+\theta)^2\left(\theta^2 + 2\theta + 2\right)}\right] \\
&= \frac{\theta^2 + 2\theta + 2}{\theta(1+\theta)^2}\left[1 + \frac{\theta^4 + 4\theta^3 + 8\theta^2 + 8\theta + 2}{\theta(1+\theta)^2\left(\theta^2 + 2\theta + 2\right)}\right] \\
&= \mu\left[1 + \frac{\theta^4 + 4\theta^3 + 8\theta^2 + 8\theta + 2}{\theta(1+\theta)^2\left(\theta^2 + 2\theta + 2\right)}\right] > \mu
\end{aligned}$$

This shows that PXLD (2.1) is always over-dispersed.

4.2. Reliability Propertie

We now give some basic propertie of the PXLD model. Since

$$\frac{P(x+1;\theta)}{P(x;\theta)} = \left(\frac{1}{1+\theta}\right)\left[1 + \frac{1}{\theta^2 + 3\theta + 3 + x}\right]$$

is a deceasing function of x. Therefore, PXLD is unimodal.

4.3. Generating Function

Probability Generating Function: The probability generating function of the PXLD (2.1) can be obtained as

$$P_X(t)=E\left(t^X\right)=\sum_{x=1}^{\infty}t^xP(x;\theta)$$
$$=\frac{\theta^2}{(1+\theta)^4}\left[\sum_{x=1}^{\infty}x\left(\frac{t}{1+\theta}\right)^x+\left(\theta^2+3\theta+3\right)\sum_{x=1}^{\infty}\left(\frac{t}{1+\theta}\right)^x\right]$$
$$=\frac{\theta^2 t}{(1+\theta)^4}\left[\frac{(1+\theta)}{(\theta+1-t)^2}+\frac{\left(\theta^2+3\theta+3\right)}{(\theta+1-t)}\right]$$

Moment Generating Function: The moment generating function of the PXLD (2.1) is given by

$$M_X(t)=E\left(e^{tX}\right)=\frac{\theta^2e^t}{(1+\theta)^4}\left[\frac{(1+\theta)}{\left(\theta+1-e^t\right)^2}+\frac{\left(\theta^2+3\theta+3\right)}{\left(\theta+1-e^t\right)}\right]$$

5. Estimation of Parameters

5.1. Maximum Likelihood Estimate (MLE)

Given a random sample $x_1, x_2, ..., x_n$ of size n from the PXLD distribution with p.m.f. (2.1) is,

$$P_{PXL}(x;\theta)=\frac{\theta^2\left(\theta^2+3\theta+3+x\right)}{(1+\theta)^{x+4}},\ \ x=0,1,2,...;\theta>0.$$

The likelihood function will be

$$L(x_i;\theta)=\left(\frac{\theta^2}{(1+\theta)^4}\right)^n\prod_{i=1}^{n}\frac{1}{(1+\theta)^{\sum_{i=1}^{n}x_i}}\cdot\left(\theta^2+3\theta+3+x_i\right)$$

Taking log both sides

$$\log L=2n\log(\theta)-4n\log(1+\theta)+\sum_{i=1}^{n}\log\left(\theta^2+3\theta+3+x_i\right)-\log(1+\theta)\sum_{i=1}^{n}x_i$$

The maximum likelihood estimate $\hat{\theta}$ of θ of PXLD (2.1) is the solutions of the following log likelihood equations

$$\frac{\partial\log L}{\partial\theta}=\frac{2n}{\theta}-\frac{4n}{1+\theta}-\frac{n\overline{X}}{(1+\theta)}+\sum_{i=1}^{n}\frac{2\theta+3}{\theta^2+3\theta+3+x_i} \qquad (4.1)$$

where $\overline{X}$ is the sample mean.

The maximum likelihood estimate (MLE), $\hat{\theta}$ of θ is the solution of the equation $\frac{\partial\log L}{\partial\theta}$.

Any numerical iteration approach, including the Regula-Falsi method, the Bisection method, and the Newton-Raphson method, can be used to solve this nonlinear problem. The Newton-Raphson method has been applied in this study to estimate the parameter.

5.2. Estimates from Moments

Using the first moment about PXLD, we have

$$\overline{X} = \mu_1^{'} = \frac{\theta^2 + 2\theta + 2}{\theta(1+\theta)^2}$$

The MoM estimate $\hat{\theta}$ of θ is the solution of the equation:

$$\theta^3 \overline{X} + \theta^2 \left(2\overline{X} - 1\right) + \theta\left(\overline{X} - 2\right) - 2 = 0 \, (4.2)$$

Where $\overline{X}$ is the sample mean.

$$\hat{\theta}_{MoM} = -\frac{1}{3\overline{X}}\left(2\overline{X} - 1\right) + \frac{\frac{2}{9\overline{X}} + \frac{1}{9\overline{X}^2} + \frac{1}{9}}{\sqrt[3]{\sqrt{\frac{1}{27\overline{X}} + \frac{13}{36\overline{X}^2} + \frac{1}{9\overline{X}^3} + \frac{1}{27\overline{X}^4}} + \frac{11}{18\overline{X}} + \frac{1}{9\overline{X}^2} + \frac{1}{27\overline{X}^3} + \frac{1}{27}}}$$
$$+ \sqrt[3]{\sqrt{\frac{1}{27\overline{X}} + \frac{13}{36\overline{X}^2} + \frac{1}{9\overline{X}^3} + \frac{1}{27\overline{X}^4}} + \frac{11}{18\overline{X}} + \frac{1}{9\overline{X}^2} + \frac{1}{27\overline{X}^3} + \frac{1}{27}}$$

6. Applications of Poisson XLindley distribution

Illustration 1

We present the PXLD, PD, PLD, and PPsLD goodness of fit for four count data sets in this section. By taking into account an actual data set on Nipah virus infection, we demonstrate the usefulness of PXL distribution in this section. *(world health organization source).*

The information in Table 2 shows the survival times (in months) of (56), at Kerala, Indian state, resident who were infected with the Nipah virus in 2017. We compared these survival times using Poisson distribution, Poisson Lindley distribution (PLD), Poisson Pseudo Lindley distribution (PPsLD), and PXLD distributions.

Table 2 shows that the PXLD offers the least χ^2 values when compared to other distributions and, as a result, better matches the data of all the models taken into consideration.

Table 2: Comparison between Poisson, Poisson-Lindley, Poisson Pseudo Lindley and Poisson XLindley distributions.

Number of attached particles	Obs freq.	PD $\theta=0.75$	PLD $\theta=1.8081$	PPsLD $\theta=6.9277$ $\beta=0.2383$	PXLD $\theta=1.54$
0	33	26. 45	31.49	23.03	31.9
1	12	19. 84	14.17	25.54	13.89
2	6	7. 44	6.09	6.08	5.93
3	3	1. 86	2.54	1.13	2.6
4	1	0.35	1.04	0.19	1.07
5	1	0.05	0.42	0.03	0.453
Total	56	56	56	56	56
χ^2		24. 1	1.289	49.405	0.9740
-LL		71.58235	66.9821	77.03363	66.9288
AIC		145.1647	135.9642	158.0673	135.8575
AICc		145.2388	136.0383	158.2937	133.929
BIC		147.1901	137.9896	162.118	137.8829

The negative log-likelihood (-LL), the Akaike information criterion (AIC), the corrected Akaike information criterion (AICc) and the Bayesian information criterion (BIC) are also given in Table 2.

In Table 2, PXLD has the lowest negative log-likelihood values when compared to PPsLD, PLD, and PD. The PXLD is the most logical forecasting and data production model based on the values of AIC, AICc, and BIC. Our distribution can serve as a crucial model for data in biological sciences.

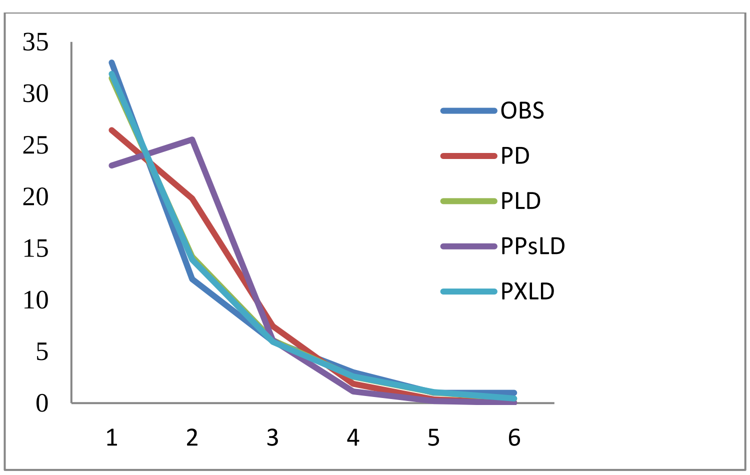

Figure 3: OBS, PD, PLD, PPsLD and PXLD graphs

Illustration 2

In this section, we present the goodness of fit of PXLD, PD, PLD, and PQLD for four count data sets.

Table 3 shows some distributions of observed and expected number of Hemocytometer yeast cell count per square observed by Student (1907) obtained by fitting Poisson, Poisson-Lindley Poisson-Quasi Lindley and PXLD.

Table 3: Comparison between Poisson, Poisson-Lindley, Poisson Quasi-Lindley and PXLD

No of errors per group	Obs freq.	PD $\theta = 1.08$	PLD $\theta = 1.547$	PQLD $\theta = 1.398$ $\beta = 0.786$	PXLD $\theta = 1.3179$
0	35	25.207	31.856	32.152	32.42
1	11	22.686	16.031	15.320	15.6
2	8	10.209	7.677	7.602	7.4231
3	4	3.062	3.557	3.542	3.5021
4	3	0.689	1.609	1.613	1.7
5	1	0.124	0.715	0.749	0.8
Total	62	62	62	62	62
χ^2		24.524	3.271	3.216	2.6836

Table 3 shows that PXLD offers the smallest χ^2 values when compared to other distributions and, as a result, better matches the data of all the models taken into consideration.

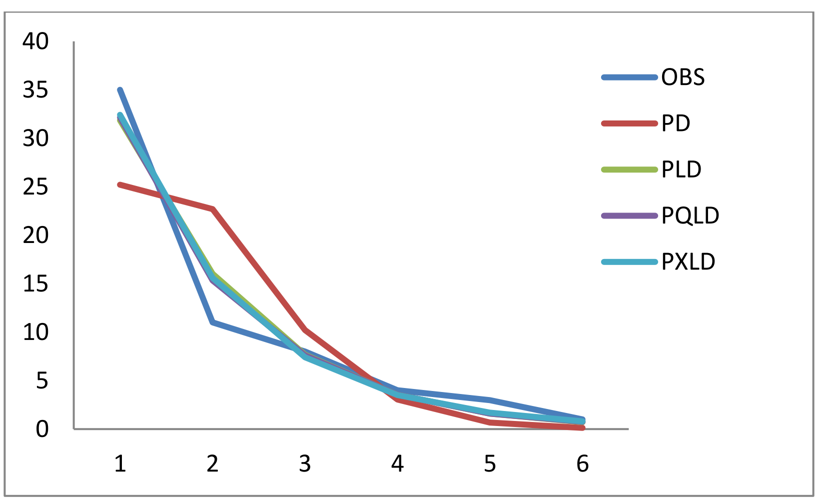

Figure 4: OBS, PD, PLD, PQLD and PXLD graphs

Illustration 3

In this section, we demonstrate that our proposed model fits well when compared to other competing models by fitting it to two real datasets, the first of which represents the number of epileptic seizures (see Chakraborty (2010)). The data set used to indicate the number of epileptic seizures has a lengthy right tail and steadily decreases toward zero. Table 4 contains the data set.

Table 4: Dataset representing epileptic seizure counts (Chakraborty, 2010)

Epileptic seizure (X)	0	1	2	3	4	5	6	7	8
Observed count	126	80	59	42	24	8	5	4	3

The parameters of each of these distributions are determined using the maximum likelihood approach. Authors used R program to examine the data. Table 5 provides parameter estimates and the model function for the fitted distributions.

Table 5: Estimated Parameters by ML method for fitted distributions for dataset representing epileptic seizure counts

Distribution	Parameter estimates	Model function
Poisson XLindley distribution	$\theta = 0.839$	$P(x) = \frac{\theta^2\left(\theta^2 + 3\theta + 3 + x\right)}{(1+\theta)^{x+4}};\ x = 0,1,2,\ldots;\ \theta > 0.$
Poisson Xgamma distribution	$\theta = 1.23$	$P(x) = \frac{\theta^2\left(2(1+\theta)^2 + \theta(1+x)(2+x)\right)}{2(1+\theta)^{x+4}};\ x = 0,1,2,\ldots;\theta > 0.$
Poisson distribution	$\theta = 1.544$	$P(x) = \frac{e^{-\theta}\theta^x}{x!},\ \theta > 0;\ x = 0,1,2,\ldots$
Zero inflated Poisson	$\theta = 2.11,\ \beta = 0.27$	$P(x) = \begin{cases} \beta + (1-\beta)\frac{e^{-\theta}\theta^x}{x!},\ \theta > 0;\ x = 0 \\ (1-\beta)\frac{e^{-\theta}\theta^x}{x!},\ \theta > 0;\ x = 0,1,2,\ldots \end{cases} \quad 0 \prec \beta \prec 1$
Geometric distribution	$p = 0.393$	$P(x) = p(1-p)^x;\ 0 \prec p \prec 1;\ x = 0,1,2,\ldots$

Negative binomial distribution	$\hat{r}=1.55,\ p=0.501$	$P(x)=\binom{x+r-1}{x}p^r q^x;\ 0\prec p\prec 1;\ r\succ 0;\ x=0,1,2,\ldots$
Discrete Weibull	$q=0.66,\ \beta=1.16$	$P(x)=q^{x^\beta}-q^{(x+1)^\beta};\ 0\prec q\prec 1;\ x=0,1,2,\ldots$
Discrete Lindley	$p=0.472,\ \theta=0.615$	$P(x)=\frac{p^x}{(1+\theta)}\left(\theta(1-2p)+(1-p)(1-\theta x)\right);$ $0\prec p\prec 1;\ \theta\succ 0;\ x=0,1,2,\ldots$
Poisson quasi-Lindley	$\theta=1.11,\ \beta=0.383$	$P(x)=\frac{\theta\left(\beta(1+\theta)+\theta(1+x)\right)}{(1+\beta)(1+\theta)^{x+2}};\ x=0,1,2,\ldots;\theta>0;\ \beta>-1.$

We compute the expected frequencies for fitting Poisson XLindley, Poisson Xgamma, Poisson, Zero Inflated Poisson, Geometric, Negative Binomial, discrete Weibull (Nakagawa & Osaki (1975)), discrete Lindley (Bakouch et al (2012)) and Poisson Quasi Lindley (Altun (2019)) distributions with the help of Pearson's chi-square test is applied to check the goodness of fit of the models.

Table 6: Fitted proposed distribution and other competing models to a dataset representing epileptic seizure counts

Epileptic seizure	Observed counts	Poisson XLindley distribution	Poisson Xgamma distribution	Poisson distribution	Zero inflated Poisson	Geometric distribution	Negative binomial distribution	Discrete Weibull	Discrete Lindley	Discrete Poisson quasi Lindley
0	126	135.40	132.981	74.935	126.000	137.963	120.201	120.120	121.867	121.868
1	80	85.00	83.371	115.712	65.080	83.736	93.009	92.875	90.942	90.941
2	59	53.00	53.367	89.339	68.974	50.823	59.184	59.036	58.745	58.744
3	42	32.02	33.497	45.985	48.733	30.847	34.949	35.133	35.216	35.216
4	24	19.30	20.391	17.752	25.824	18.722	19.837	20.071	20.167	20.167
5	8	11.52	12.037	5.482	10.948	11.363	10.987	11.131	11.197	11.197
6	5	6.82	6.914	1.411	3.868	6.897	5.984	6.029	6.079	6.079
7	4	4.02	3.878	0.311	1.171	4.186	3.221	3.202	3.245	3.246
8	3	2.35	2.132	2.132	0.072	0.402	6.464	3.627	1.673	1.710
Total	351	351	351	351	351	351	351	351	351	351
df		8	6	4	4	6	5	5	5	5
p-value		0.547	0.431	<0.001	0.012	0.144	0.259	0.261	0.331	0.332

Table 6 lists the expected counts and chi square p-value for each fitted model. In comparison to other distributions, the PXLD satisfactorily better fits the data set reflecting epileptic seizure counts, as shown by the chi-square p-value.

Additionally, we take into account the AIC (Akaike information criterion), AICC (corrected Akaike information criterion), and BIC (Bayesian information criterion) criteria in order to evaluate the proposed distribution to the other competing models mentioned. The lower AIC, AICC, and BIC values are correlated with the better distribution.

$$AIC = 2k - 2\log L,\ \ AICC = AIC + \frac{2k(k+1)}{n-k-1},\ \ BIC = k\log n - 2\log L$$

where k is the number of parameters in the statistical model, n is the sample size and $-2\log L$ is the maximized value of the log-likelihood function under the considered model.

Table 7: Model comparison criterion for fitted models to a dataset representing epileptic seizure counts

Criterion	Poisson XLindley distribution	Poisson Xgamma distribution	Poisson distribution	Zero inflated poisson	Geometric distribution	Negative binomial distribution	Discrete weibull	Discrete lindley	Discrete poisson quasi lindley
$-2\log L$	328.1955	595.343	636.045	599.637	598.396	594.942	594.749	594.482	594.482
AIC	658.3909	1192.687	1274.091	1203.274	1198.791	1193.884	1193.499	1192.964	1192.964
BIC	662.2402	1196.547	1277.952	1210.996	1202.652	1201.605	1201.220	1200.685	1200.685

Table 7 shows that, when compared to other competing models, the PXLD has lowered AIC and BIC values. Therefore, we draw the conclusion that, when studying the data set shown in Table 4, the PXLD provides a better fit than the other competing models.

7. Conclusion

The PXLD has just been unveiled. Both the method of maximum likelihood and the method of moments have been discussed for the estimation of its parameters. a model's application to actual data Additionally, the comparison fit study was conducted using other well-known one and two parameters. Density plots and term χ^2 value plots were used to evaluate the suitability of fits. Authors can demonstrate how the PXLD works well for assessing actuarial science and real lifetime data.

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