Eur. J. Stat. 3 (2023) 9
doi: 10.28924/ada/stat.3.9

Group Acceptance Sampling Plans for Resubmitted Lots Under Odd Generalized Exponential Log-Logistic Distribution

D.C.U. Sivakumar[1], K. Kalyani[2], G. Srinivasa Rao[3,*], K. Rosaiah[4]

[1]Department of Community Medicine, Alluri Sitarama Raju Academy of Medical Sciences, Eluru – 534005, India

dsk2000.siva@gmail.com

[2]Department of Mathematics and Statistics, Vignan University, Vadlamudi, Guntur - 522 213, India

kruthiventikalyani@gmail.com

[3]Department of Statistics, The University of Dodoma, P.O.Box: 259, Tanzania

gaddesrao@gmail.com

[4]Department of Statistics, Acharya Nagarjuna University, Guntur - 522 510, India

rosaiah1959@gmail.com

[*]Correspondence: gaddesrao@gmail.com

Abstract: In this manuscript, we developed resubmitted lots with group acceptance sampling plan for the lifetime of the product follows the odd generalized exponential log logistic distribution introduced by Rosaiah *et al.* (2016c). The values of the design parameters of the proposed plan are obtained which are satisfying the both producer's as well consumer's risk by fixing the experiment termination time. An application of the proposed plan to the industry is presented and the Kolmogorov-Smirnov test was conducted. However, this plan provides reasonable fit for lifetime of items of ball bearings data. Finally, the advantage of the proposed plan reduces the sample size as compared with the ordinary group sampling scheme. An example is given to illustrate the methodology.

1. Introduction

Mostly, all most all sampling schemes, the ordinary single acceptance sampling plan is widely used one due to it is simplicity for practical implementation. The decision on the lot disposition (acceptance or rejection) by the single acceptance sampling is based on the single inspection or life test. Currently, industries

Received: 12 Feb 2023.

Key words and phrases. resubmitted lot; group acceptance sampling plan; consumer's risk and producer's risk.

concentration is on producing of high reliability products. Due to the fact that it could take long experimental time to observe the complete lifetime of a high reliability item, for a product the life test should be ended within a specified schedule and such a process is called a truncated life test. To collect product lifetime information, products must suffer a destructive life test. Many sampling schemes are available in the literature, including single, double, multiple, sequential, group, two-stage group sampling plans, which provide the inspection of the product. Typically the lot sentencing is based upon the decision of either acceptance or rejection of lot.

The final yield of the items totally depends on the attributes which are collected from a random sample which is chosen from a lot or batch was described by Dodge (1943). This technique is known as acceptance sampling plan (ASP) or lot sentencing. In ASP, for a submitted lot the decision is to reject or accept the lot but not to estimate the quality of the lot. This inspection plan is a middle way between no inspection (i.e., 0%) and 100% inspection. Among all ASPs, the single attribute acceptance sampling scheme is the most widely used due to its easy for practical utility. If one is intent to collect lifetimes of all the observations of a random sample, the experimentation time takes very much time, hence the life test must be terminated within a specified lifetime and such a life test is known as truncated life test.

For more evidence about the single ASP under a truncated life test, it is suggested to refer to Epstein (1954), Goode and Kao (1960), Gupta (1962). In several situations the consumer may not accepted the products from the submitted lots based on a single ASP. The manufacturer can contest the first sample evidence, chose the second sample of the same size for testing and create an inference that discards the past results. Therefore, the rejected lot to be resubmitted is known as resubmitted acceptance sampling plan. Performance measurement of the re-sampling technique with a single sampling plan for the examination of the resubmitted lot, different from the usual single sampling plans which was proposed by Govindaraju and Ganesalingam

(1997). Recently, Rao *et al.* (2016a, 2016b) developed the new ASPs based on percentiles for odds exponential log-logistic and exponentiated Fréchet distributions.

Aslam and Jun (2009b) developed the GASP for the inverse Rayleigh and log-logistic distribution using single point on operating characteristics (OCs) curve. The GASP for generalized exponential distribution and Marshall-Olkin extended Lomax distributions are discussed by Rao (2009a, 2009b). Many authors are discussed the GASP based on truncated life tests for various distributions are Rao and Rameshnaidu (2015), Rao *et al.* (2016), Rosaiah *et al.* (2016d). In GASP, the total number of sample products n is to be inspected by dividing n items into equal group sizes according to the number of available experimental tests. The total of g groups with each group consisting of r items, then $n = rg$. In this life test, a test is called a group and the number of items r in each tester is called the group size. For various distribution based on resubmitted lots several authors are discussed, it is advised to refer to Aslam *et al.* (2011), Rao and Rao (2014), Rosaiah *et al.* (2017). Recently, Rao *et al.* (2019) proposed GASP for resubmitted lots under exponentiated Fréchet distribution.

Here an attempt is made to develop a proposed plan for the lifetime of the product follows the odd generalized exponential log logistic distribution introduced by Rosaiah *et al.* (2016c). The design of GASP for percentile lifetimes under a truncated life test is described for the resubmitted lots in Section 2. The applications of the proposed plan to the industry are discussed in Section 3. The proposed sampling plan for the resubmitted lot is compared with the ordinary group sampling plan is given in Section 4. Finally, Conclusions are given in Section 5.

2. Gasp for Resubmitted Lots

An attribute ASP has huge utilization in numerous ways. For specimen, before manufacturing of the products, it is used to test the submitted items to satisfy the prerequisite conditions. The attribute ASP has three design parameters: batch size (*N*), sample size (*n*) and the acceptance number (*c*). The plan is implemented as; select a random sample of size n items from a batch of size N with the acceptance number c. If the number of defective items is more than c, then reject the lot; otherwise accept the lot. The measure of quality p shows that the percentage of fraction defective items. There is no argument to use the attribute acceptance sampling plan, If $p = 0\% \text{ or } 100\%$. On the other side, if p lies between 0% and 100%, an attribute ASP are useful to take a decision whether to accept or reject the item on the basis of the random sample which is taken from the lot.

The presumptions of the sampling inspection for the resubmitted lots are as follows:

i) Abiding by the provisions of a contract or statute, the information of the original inspection resulting in non-acceptance is required to discard.

ii) Consumer has confidence and producer who will not deliberately take the advantage of re-sampling.

The iterative algorithm for the proposed plan under a truncated life test for resubmitted lots is implemented as follows:

Step 1: Select a random sample of size n from the lot and distributing the products into g groups evenly such that each group has r items and $n = r \times g$. Now perform the original group sampling plan and choose about the experiment time period, t_0 and the acceptance number c. Reject the lot if the number of failures from all g groups is larger than c within the t_0; otherwise, the lot is accepted. Truncate the experiment once the number of failures from all the groups exceeds c, or the time of the experiment is terminated, whichever is earlier.

Step 2: On non-acceptance of lot from Step-1, apply the referenced group sampling plan at most w times and reject the lot on the with inspection if the lot could not be accepted before or at the $(w-1)^{th}$ resubmission. Here the referenced group sampling plan is the original group sampling plan since resubmission is allowed.

The present research is proposing a study intended specifically designed for certain situations. Subsequent steps are given below for its uses:

i. Obtain the number of group size g when the number of testers r is pre-specified. Select $n = r \times g$ items form a lot and allocate r items to each group g.

ii. The required sample size in the life test is $n = r \times g$.

iii. compute the acceptance number c for every group and specify the termination time t_0.

iv. Terminate the experiment and reject the lot if more than c or $(c+1)$ failures occur in any group.

v. The manufacturer must respect the consumer's confidence and must not take undue advantage of the re-sampling. Perform the GASP *i.e.*, steps (i) to (iv), on non acceptance of the original GASP, apply the proposed plan m times and reject the submitted lot if it is not accepted on $(w-1)^{th}$ resubmission.

The probability density function (p.d.f.) and cumulative distribution function (c.d.f) of the OGELLD respectively, are given by,

$$f(t;\sigma,\lambda,\theta,\gamma) = \frac{\gamma\theta}{\lambda\sigma}\left(\frac{t}{\sigma}\right)^{\theta-1} e^{-\frac{1}{\lambda}\left(\frac{t}{\sigma}\right)^{\theta}} \left[1 - e^{-\frac{1}{\lambda}\left(\frac{t}{\sigma}\right)^{\theta}}\right]^{\gamma-1} ;\ t>0,\ \sigma,\lambda,\theta>0, \gamma>1 \quad (1)$$

$$F(t;\sigma,\lambda,\theta,\gamma) = \left[1 - e^{-\frac{1}{\lambda}\left(\frac{t}{\sigma}\right)^{\theta}}\right]^{\gamma} ;t>0,\ \sigma,\lambda,\theta>0, \gamma>1 \quad (2)$$

where λ, σ are the scale parameters and θ, γ are shape parameters.

The 100q-th percentile of the OGELLD is given as:

$$t_q = \sigma\eta_q;\ \text{where } \eta_q = \left[-\lambda \ln(1-q^{1/\gamma})\right]^{\frac{1}{\theta}} \quad (3)$$

The parameters of the proposed plan for resubmitted lots are determined by fulfilling the specified producer's and consumer's risks according to the experiment termination time and the number of testers.

The median life is 50th percentile of the OGELLD and is given by

$$t_{0.5} = \sigma\left[-\lambda \ln\left(1-(0.5)^{1/\gamma}\right)\right]^{1/\theta} \quad (4)$$

Let us assume that the parameters λ and θ are known, the 50th percentile given in Eq. (4) is the function of parameters σ and γ. Based on the number of failures form all the groups the probability of accepting lot for the ordinary group sampling plan is given by

$$L(p) = \sum_{i=0}^{c} \binom{rg}{i} p^{i} (1-p)^{rg-i} \quad (5)$$

where c is the acceptance number, r is size of the group, g is the number of testers and p is the probability of getting a failure within the life test schedule t_0.

If the product lifetime follows OGELLD, then $p = F(t_0; \sigma, \lambda, \theta, \gamma)$. Customarily, it would be convenient to obtain the experiment termination time t_0, as $t_0 = \delta_q t_q^0$ for a constant δ_q and the targeted 100q-th percentile lifetime t_q^0. Let t_q be the true 100q-th percentile lifetime, then p can be expressed as:

$$p = \left[1-\exp\left\{-\frac{1}{\lambda}\left(\frac{\delta_q \eta_q}{t_q / t_q^0}\right)^{\theta}\right\}\right]^{\gamma} \quad (6)$$

The operating characteristic (O.C) function of sampling plan for resubmitted lots with *(w-1)* resubmissions is given by (Govindaraju and Ganesalingam (1997)),

$$P_a(p) = 1-\left[1-L(p)\right]^{w} \quad (7)$$

Therefore, the probability of the acceptance of the proposed plan for resubmitted lots is

$$P_a(p) = 1-\left[1-\left\{\sum_{i=0}^{c} \binom{rg}{i} p^{i} (1-p)^{rg-i}\right\}\right]^{w} \quad (8)$$

Here, we have two parameters g and c for the proposed scheme, so for the given group size r and the pre-specified termination time schedule

Here, we have two parameters g and c for the proposed plan, for the given the group size r and the pre-specified truncated life test time schedule, $t_0 = \delta_q t_q^0$, in terms of a various targeted lifetime percentile t_q^0. When the true 100q-th lifetime percentile t_q is lesser than or equal to the target one, the lot is known a bad lot; otherwise, it is known as a good lot. Usually, there are two risks attached with an ASP. The probability of rejecting a good lot is called Type-I error i.e., producer's risk α and the probability of accepting a bad lot is called Type-II error i.e., consumer's risk β. Both producer and consumer require a sampling plan to make the decision satisfy their respective specified risks. Producer requires the lot acceptance probability at least $1-\alpha$ at the acceptable reliability level (ARL) and consumer wants the lot acceptance probability at most β at the lot tolerance reliability level (LTRL). Let p_1 be the probability of a failure corresponding to the producer's risk at ARLs, in terms of $t_q/t_q^0 = 2,\ 4,\ 6,\ 8$ in Eq. (6) and let p_2 be the probability of a failure corresponding to the consumer's risk at LTRL, in terms of $t_q/t_q^0 = 1$, in Eq. (6). Therefore, the plan parameters c and g can be determined by solving the following in-equalities simultaneously,

$$1-\left[1-\left\{\sum_{i=0}^{c}\binom{rg}{i}p_1^i(1-p_1)^{rg-i}\right\}\right]^w \geq 1-\alpha \tag{9}$$

and $$1-\left[1-\left\{\sum_{i=0}^{c}\binom{rg}{i}p_2^i(1-p_2)^{rg-i}\right\}\right]^w \leq \beta \tag{10}$$

where $p_1 = 1-\exp\left\{-\frac{1}{\lambda}\left(\frac{\eta_q\ \delta_q^0}{\left(t_q/t_q^0\right)}\right)^\theta\right\}$ and $p_2 = 1-\exp\left\{-\frac{1}{\lambda}\left(\eta_q\ \delta_q^0\right)^\theta\right\}$

for a given producer's risk $\alpha = 0.05$ and termination time schedule $t_0 = \delta_q t_q^0$ with $\delta_q = 0.5, 1.0$, the three parameters of the proposed GASP under the truncated life test at the pre-specified time t_0, for the resubmitted lot $\hat{\lambda}=39.8486, \hat{\theta}=1.0471, \hat{\gamma}=4.7161$ and $w=2, 3$ are determined according to the consumer's confidence levels $\beta=0.25, 0.10, 0.05 \text{ and } 0.01$. The plan parameters are presented in Tables 1 to 4 for $w=2,3;$ $\lambda=2.0, \theta=\gamma=1.5,$ at 50th, 25th percentiles respectively. Tables 5 to 8 are constructed for estimated parameters $\hat{\lambda}=39.8486, \hat{\theta}=1.0471, \hat{\gamma}=4.7161$ using maximum likelihood for $w=2,3$ at 50th , 25th percentiles which are found from the fitted real data set given in Section 3. We observed from Tables 1 to 8 that percentile ratio increases, the number of groups g decreases and the acceptance number c is increases or same as $\delta_q=0.5$ to 1.0.

3. Application of the Proposed Plan to Industry

In this section, we will give an example to illustrate our proposed plan for industrial uses. The data set is related to tests on endurance of deep groove ball bearings Lawless (1982). The data are the number of million revolutions before failure for each of the 23 ball bearings in life test and they are 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, and 173.40.

We show a rough indication of the goodness of fit for our model by plotting the superimposed for the data shows that the OGELLD is a good fit in Figure 1 and also goodness of fit is emphasized with Q-Q plot, displayed in Figure 1. The maximum likelihood estimates (MLEs) of the OGELLD for these data are $\hat{\lambda}=39.8486, \hat{\theta}=1.0471, \hat{\gamma}=4.7161$ and the K-S test, we found maximum distance between the empirical distribution functions and the fitted distribution functions is 0.1086 and corresponding value of p is 0.922. Therefore, the OGELLD provides an appropriate fit for lifetime of products of ball bearings data.

Let us consider the lifetime of a product is known to follow OGELLD with MLEs $\hat{\lambda}=39.8486, \hat{\theta}=1.0471, \hat{\gamma}=4.7161$. Assume that it is desired to develop GASP for resubmitted lots to decide about acceptance or rejection of a submitted lot of products. Let an experimenter would like to establish the true unknown 50th percentile lifetime for the 20 million revolutions before failure for each of the ball bearings and protect the producer $\alpha = 0.05$, $t_q / t_q^0 = 2$. The experimenter wants to adopt the proposed group sampling plan having $r = 5$ for resubmitted lot with $w = 2$. Let the termination time schedule ratio be $\beta = 0.25$ and $\delta_q = 1.0$ for this experiment. The above data is well fitted to the OGELLD with $\hat{\lambda} = 39.8486, \hat{\theta} = 1.0471, \hat{\gamma} = 4.7161$. The plan parameters from Table 5 are $c = 2$ and $g = 3$.

This plan is implemented as: performing the original inspection by selecting a random sample of size 15 items from the lot and distribute five items to each tester, accept the lots if the number of failures from three groups is not larger than 2 at the end of the experiment time, $t_0 = 20$ million revolutions. If the number of failures from three groups are larger than 2, the product is not accepted and a second experiment must be conducted again. The lot is accepted if the number of failures is less than or equal to two from the second sample; otherwise reject the lot. According to this plan, the deep groove ball bearings life test of the product could have been accepted because before the termination moment there is only one failure, 20 million revolutions.

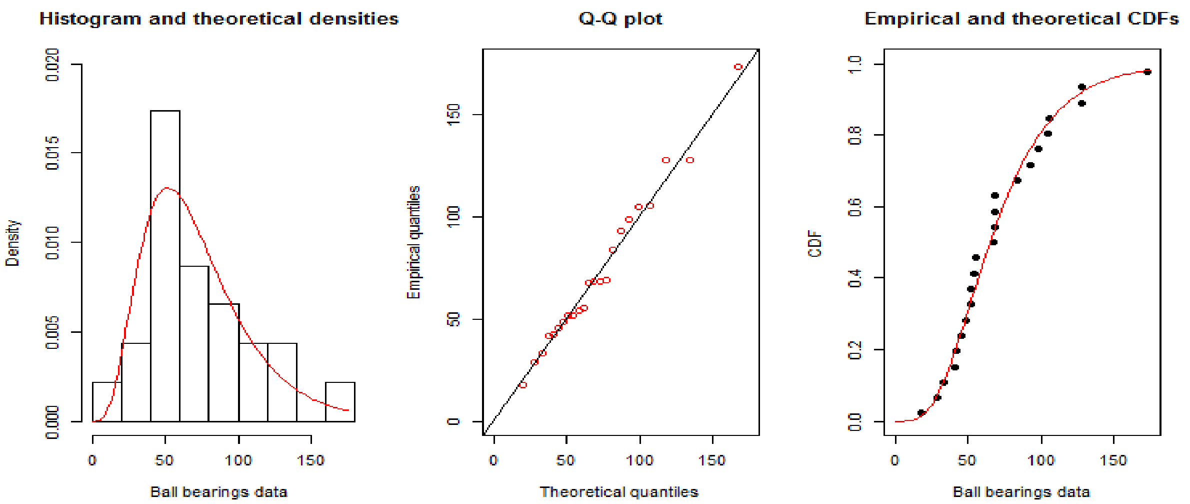

Figure 1: the density, Q-Q and cdf plot of the fitted OGELLD for the ball bearings data

4. Comparison Study

The proposed resubmitted group acceptance sampling plan (RGASP) is a generalization of the ordinary group sampling plan under a truncated life test. In this section, we conducted the comparisons between the proposed group sampling plan and the ordinary GASP. To save the space, we present a comparison of the proposed group sampling plans under w=2 and w=3 with the ordinary GASP for the OGELLD with $\lambda = 2, \theta = \gamma = 1.5$ for a given $q = 0.5, 0.25$ $r = 5, 10$ $\beta = 0.25, 0.10, 0.05, 0.01$ and $\delta_q = 0.5, 1.0$. The design parameters for these group sampling plans are given in Table 9 and Table10 for ready reference. From Table 9, it can be seen that for a given value of t_q / t_q^0 , the proposed group sampling plan parameters are always less than the corresponding plan parameters from the ordinary group sampling plan.

5. Conclusions

In this manuscript, a GASP for the resubmitted lots to ensure the specified product lifetime percentile has been developed for the OGELLD. The plan parameters g and c of the proposed sampling plan are determined such that the lot acceptance probability is larger than $1-\alpha$ at the producer-specified quality level but the lot acceptance probability is smaller than β at the consumer's specified quality level. For industrial use extensive tables have been provided according to various parameters and percentile values. It was detected that the number of groups required increases as the consumer's confidence increases, true quality decreases and as r increases the number of groups reduces for all the parameters. A comparison between the proposed resubmitted group acceptance sampling plan and the ordinary group acceptance sampling plan of sivakumar *et al.* (2019) has also been discussed. It has been noticed that the proposed plan is better than the ordinary group sampling plan with respect to group sizes. The methodology illustrated with real data set.

References

[1] M. Aslam, C. H. Jun, A group acceptance sampling plans for truncated life tests based on the inverse Rayleigh and log-logistic distributions, Pak. J. Stat. 25 (2009b), 107-119.

[2] M. Aslam, C. H. Jun, Y. L. Lio, M. Ahmad, M. Rasool, Group acceptance sampling plans for resubmitted lots under Burr-type XII distributions, J. Chinese Inst. Ind. Eng. 28 (2011), 606615. https://doi.org/10.1080/10170669.2011.651165.

[3] M. Aslam, C.H. Jun, H. Lee, M. Ahmad, M. Rasool, Improved group sampling plans based on time truncated life tests, Chilean J. Stat. 2 (2011), 85–97.

[4] H. F. Dodge, A sampling inspection plan for continuous production, Ann. Math. Stat. 14 (1943), 264-279. https://doi.org/10.1214/aoms/1177731420.

[5] H. F. Dodge, H. G. Romig, Sampling inspection tables single and double sampling, 2nd Edition, John Wiley and Sons, New York, (1959).

[6] B. Epstein,. Truncated life tests in the exponential case, Ann. Math. Stat. 25 (1954), 555-564. https://doi.org/10.1214/aoms/1177728723.

[7] H. P. Goode, J. H. Kao, Sampling plans based on the Weibull distribution (No. TR-1), CORNELL UNIV ITHACA NY. (1960). https://doi.org/10.21236/ad0243881.

[8] K. Govindaraju, S. Ganesalingam, Sampling inspection for resubmitted lots, Comm. Stat.-Simul. Comp. 26 (1997), 1163-1176. https://doi.org/10.1080/03610919708813433.

[9] S. S. Gupta, Life test sampling plans for normal and lognormal distributions, Techno-metrics, 4 (1962), 151-175. https://doi.org/10.1080/00401706.1962.10490002.

[10] J. F. Lawless, Statistical models and methods for lifetime data (No. 04; QA276, L3.). (1982).

[11] G. S. Rao, A group acceptance sampling plans for lifetimes following a generalized exponential distribution, Econ. Qual. Control, 24 (2009a), 75-85. https://doi.org/10.1515/eqc.2009.75.

[12] G. S. Rao, A group acceptance sampling plans based on truncated life tests for Marshall-Olkin extended Lomax distribution, Elec. J. Appl. Stat. Anal. 3 (2009b), 18-27. https://doi.org/10.1285/i20705948v3n1p18.

[13] G. S. Rao, C. Ramesh Naidu, An exponentiated half logistic distribution to develop a group acceptance sampling plans with truncated time, J. Stat. Manage. Syst. 18 (2015), 519-531. https://doi.org/10.1080/09720510.2014.968376.

[14] G. S. Rao, B. S. Rao, Group acceptance sampling plans for resubmitted lots for life tests based on half logistic distribution, J. Data Sci. 12 (2014), 647-659. https://doi.org/10.6339/JDS.201410_12(4).0005.

[15] G. S. Rao, K. Rosaiah, M. S. Babu, Group acceptance sampling plans for resubmitted lots under exponentiated Fréchet distribution, Int. J. Comp. Sci. Math. 10 (2019), 11-21. https://doi.org/10.1504/IJCSM.2019.097633.

[16] G. S. Rao, K. Rosaiah, M. S. Babu, Group Acceptance Sampling Plans for Lifetimes Following an Exponentiated Fréchet Distribution, International Journal of Applied Research and Studies, 5 (2016), 1-13. https://doi.org/10.20908/ijars.v5i3.9466.

[17] G. S. Rao, K. Rosaiah, M. S. Babu, D. C. Sivakumar, New acceptance sampling plans based on percentiles for exponentiated Fréchet distribution, Econ. Qual. Control, 31 (2016b), 37-44. https://doi.org/10.1515/eqc-2015-0011.

[18] G. S. Rao, K. Rosaiah, K. Kalyani, D. C. U. Sivakumar, A new acceptance sampling plans based on percentiles for odds exponential log-logistic distribution, Open Stat. Prob. J. 7 (2016a), 45-52. https://doi.org/10.2174/1876527001607010045.

[19] K. Rosaiah, G. S. Rao, S. V. S. V. S. V. Prasad, A group acceptance sampling plans based on truncated life tests for Type-II generalized log logistic distribution, Prob. Stat. Forum, 9 (2016d), 88-94.

[20] K. Rosaiah, G. S. Rao, K. Kalyani, D. C. U. Sivakumar, Group acceptance sampling plan for resubmitted lots based on life tests for odds exponential log logistic distribution, Int. J. Qual. Reliab. Manage. 34 (2017), 1343-1351. https://doi.org/10.1108/ijqrm-01-2016-0013.

[21] K. Rosaiah, G. S. Rao, D. C. U. Sivakumar, K. Kalyani, The odd generalized exponential log logistic distribution, Int. J. Math. Stat. Invent. 4 (2016c), 21-29.

[22] D. C. U. Sivakumar, K. Rosaiah, G. S. Rao, K. Kalyani, Odd generalized exponential log-logistic distribution group acceptance sampling plan, Stat. Transition new series, 20 (2019), 433-452. https://doi.org/10.21307/stattrans-2019-006.

[23] T. R. Tsai, S. J. Wu, Acceptance sampling based on truncated life tests for generalized Rayleigh distribution, J. Appl. Stat. 33 (2006), 595–600. https://doi.org/10.1080/02664760600679700.

Table 1. Plan parameter values for OGELLD with $\lambda=2, \theta=\gamma=1.5$

and $w=2$ for 50th percentile

β	t_q/t_q^0	r=5						r=10					
		δ_q=0.5			δ_q=1.0			δ_q=0.5			δ_q=1.0		
		c	g	P_a	c	g	P_a	c	g	P_a	c	g	P_a
0.25	2	2	6	0.9864	14	15	0.9512	2	3	0.9864	-	-	-
	4	0	3	0.9842	0	1	0.9660	0	2	0.9731	1	2	0.9642
	6	0	3	0.9972	0	1	0.9935	0	2	0.9951	0	1	0.9759
	8	0	3	0.9992	0	1	0.9981	0	2	0.9986	0	1	0.9927
0.10	2	2	8	0.9542	14	15	0.9512	2	4	0.9542	-	-	-
	4	0	4	0.9731	0	1	0.9660	0	2	0.9731	1	2	0.9642
	6	0	4	0.9951	0	1	0.9935	0	2	0.9951	0	1	0.9759
	8	0	4	0.9986	0	1	0.9981	0	2	0.9986	0	1	0.9927
0.05	2	3	11	0.9687	14	15	0.9512	3	6	0.9525	-	-	-
	4	0	5	0.9598	1	2	0.9966	1	4	0.9975	1	2	0.9642
	6	0	5	0.9924	0	2	0.9759	0	3	0.9893	0	1	0.9759
	8	0	5	0.9978	0	2	0.9927	0	3	0.9969	0	1	0.9927
0.01	2	4	15	0.9674	14	15	0.9512	4	8	0.9535	-	-	-
	4	1	9	0.9963	1	3	0.9859	1	5	0.9946	1	2	0.9642
	6	0	7	0.9857	0	2	0.9759	0	4	0.9816	0	1	0.9759
	8	0	7	0.9958	0	2	0.9927	0	4	0.9946	0	1	0.9927

Table 2. Plan parameter values for OGELLD with $\lambda=2, \theta=\gamma=1.5$

and $w=2$ for 25th percentile

β	t_q/t_q^0	r=5						r=10					
		δ_q=0.5			δ_q=1.0			δ_q=0.5			δ_q=1.0		
		c	g	P_a	c	g	P_a	c	g	P_a	c	g	P_a
0.25	2	1	11	0.9588	2	4	0.9796	2	8	0.9851	5	6	0.9579
	4	0	6	0.9912	0	2	0.9799	0	3	0.9912	0	1	0.9799
	6	0	6	0.9985	0	2	0.9963	0	3	0.9985	0	1	0.9963
	8	0	6	0.9996	0	2	0.9990	0	3	0.9996	0	1	0.9990
0.10	2	2	19	0.9693	3	6	0.9820	2	10	0.9623	5	6	0.9579
	4	0	9	0.9811	0	3	0.9581	0	5	0.9770	1	2	0.9987
	6	0	9	0.9966	0	3	0.9920	0	5	0.9959	0	2	0.9862
	8	0	9	0.9990	0	3	0.9977	0	5	0.9988	0	2	0.9959
0.05	2	3	26	0.9815	3	7	0.9603	3	13	0.9815	5	6	0.9579
	4	0	11	0.9726	0	3	0.9581	0	6	0.9679	1	2	0.9987
	6	0	11	0.9950	0	3	0.9920	0	6	0.9941	0	2	0.9862
	8	0	11	0.9986	0	3	0.9977	0	6	0.9983	0	2	0.9959
0.01	2	4	38	0.9733	5	11	0.9759	4	19	0.9733	5	6	0.9579
	4	1	22	0.9974	1	6	0.9943	1	11	0.9974	1	3	0.9943
	6	0	16	0.9898	0	4	0.9862	0	8	0.9898	0	2	0.9862
	8	0	16	0.9970	0	4	0.9959	0	8	0.9970	0	2	0.9959

Table 3. Plan parameter values for OGELLD with $\lambda = 2, \theta = \gamma = 1.5$ and $w = 3$ for 50th percentile

β	t_q/t_q^0	r=5						r=10					
		δ_q=0.5			δ_q=1.0			δ_q=0.5			δ_q=1.0		
		c	g	P_a	c	g	P_a	c	g	P_a	c	g	P_a
0.25	2	1	5	0.9817	5	6	0.9560	1	3	0.9613	-	-	-
	4	0	3	0.9980	0	1	0.9937	0	2	0.9956	0	1	0.9625
	6	0	3	0.9999	0	1	0.9995	0	2	0.9997	0	1	0.9963
	8	0	3	1.0000	0	1	0.9999	0	2	0.9999	0	1	0.9994
0.10	2	2	8	0.9902	5	6	0.9560	2	4	0.9902	-	-	-
	4	0	4	0.9956	0	1	0.9937	0	2	0.9956	0	1	0.9625
	6	0	4	0.9997	0	1	0.9995	0	2	0.9997	0	1	0.9963
	8	0	4	0.9999	0	1	0.9999	0	2	0.9999	0	1	0.9994
0.05	2	2	9	0.9808	5	6	0.9560	2	5	0.9664	-	-	-
	4	0	5	0.9919	0	2	0.9625	0	3	0.9869	0	1	0.9625
	6	0	5	0.9993	0	2	0.9963	0	3	0.9989	0	1	0.9963
	8	0	5	0.9999	0	2	0.9994	0	3	0.9998	0	1	0.9994
0.01	2	3	14	0.9713	5	6	0.9560	3	7	0.9713	-	-	-
	4	0	7	0.9805	0	2	0.9625	0	4	0.9727	0	1	0.9625
	6	0	7	0.9983	0	2	0.9963	0	4	0.9975	0	1	0.9963
	8	0	7	0.9997	0	2	0.9994	0	4	0.9996	0	1	0.9994

Table 4. Plan parameter values for OGELLD with $\lambda = 2, \theta = \gamma = 1.5$ and $w = 3$ for 25th percentile

β	t_q/t_q^0	r=5						r=10					
		δ_q=0.5			δ_q=1.0			δ_q=0.5			δ_q=1.0		
		c	g	P_a	c	g	P_a	c	g	P_a	c	g	P_a
0.25	2	1	12	0.9877	1	3	0.9821	1	6	0.9877	2	3	0.9674
	4	0	7	0.9987	0	2	0.9972	0	4	0.9981	0	1	0.9972
	6	0	7	0.9999	0	2	0.9998	0	4	0.9999	0	1	0.9998
	8	0	7	1.0000	0	2	1.0000	0	4	1.0000	0	1	1.0000
0.10	2	1	16	0.9598	2	5	0.9882	1	8	0.9598	2	3	0.9674
	4	0	10	0.9965	0	3	0.9914	0	5	0.9965	0	2	0.9818
	6	0	10	0.9997	0	3	0.9993	0	5	0.9997	0	2	0.9984
	8	0	10	1.0000	0	3	0.9999	0	5	1.0000	0	2	0.9997
0.05	2	2	23	0.9837	2	6	0.9674	2	12	0.9794	2	3	0.9674
	4	0	12	0.9943	0	3	0.9914	0	6	0.9943	0	2	0.9818
	6	0	12	0.9995	0	3	0.9993	0	6	0.9995	0	2	0.9984
	8	0	12	0.9999	0	3	0.9999	0	6	0.9999	0	2	0.9997
0.01	2	3	34	0.9828	3	9	0.9578	3	17	0.9828	4	5	0.9870
	4	0	17	0.9855	0	4	0.9818	0	9	0.9832	0	2	0.9818
	6	0	17	0.9988	0	4	0.9984	0	9	0.9986	0	2	0.9984
	8	0	17	0.9988	0	4	0.9997	0	9	0.9998	0	2	0.9997

Table 5. Plan parameter values for OGELLD with $\hat{\lambda}=39.8486, \hat{\theta}=1.0471, \hat{\gamma}=4.7161$ and $w=2$ for 50th percentile

β	t_q/t_q^0	r=5						r=10					
		$\delta_q=0.5$			$\delta_q=1.0$			$\delta_q=0.5$			$\delta_q=1.0$		
		c	g	P_a	c	g	P_a	c	g	P_a	c	g	P_a
0.25	2	0	4	0.9698	2	3	0.9608	0	2	0.9698	-	-	-
	4	0	4	0.9999	0	1	0.9978	0	2	0.9999	0	1	0.9917
	6	0	4	1.0000	0	1	0.9999	0	2	1.0000	0	1	0.9997
	8	0	4	1.0000	0	1	1.0000	0	2	1.0000	0	1	1.0000
0.10	2	1	9	0.9953	2	3	0.9608	1	5	0.9933	-	-	-
	4	0	6	0.9998	0	1	0.9978	0	3	0.9998	0	1	0.9917
	6	0	6	1.0000	0	1	0.9999	0	3	1.0000	0	1	0.9997
	8	0	6	1.0000	0	1	1.0000	0	3	1.0000	0	1	1.0000
0.05	2	1	11	0.9907	2	3	0.9608	1	6	0.9876	-	-	-
	4	0	7	0.9997	0	2	0.9917	0	4	0.9996	0	1	0.9917
	6	0	7	1.0000	0	2	0.9997	0	4	1.0000	0	1	0.9997
	8	0	7	1.0000	0	2	1.0000	0	4	1.0000	0	1	1.0000
0.01	2	1	14	0.9795	2	3	0.9608	1	7	0.9795	-	-	-
	4	0	10	0.9993	0	2	0.9917	0	5	0.9993	0	1	0.9917
	6	0	10	1.0000	0	2	0.9997	0	5	1.0000	0	1	0.9997
	8	0	10	1.0000	0	2	1.0000	0	5	1.0000	0	1	1.0000

Table 6. Plan parameter values for OGELLD with $\hat{\lambda}=39.8486, \hat{\theta}=1.0471, \hat{\gamma}=4.7161$ and $w=2$ for 25th percentile

β	t_q/t_q^0	r=5						r=10					
		$\delta_q=0.5$			$\delta_q=1.0$			$\delta_q=0.5$			$\delta_q=1.0$		
		c	g	P_a	c	g	P_a	c	g	P_a	c	g	P_a
0.25	2	0	13	0.9817	1	3	0.9928	0	7	0.9790	1	2	0.9810
	4	0	13	1.0000	0	2	0.9995	0	7	0.9999	0	1	0.9995
	6	0	13	1.0000	0	2	1.0000	0	7	1.0000	0	1	1.0000
	8	0	13	1.0000	0	2	1.0000	0	7	1.0000	0	1	1.0000
0.10	2	0	18	0.9668	1	4	0.9810	0	9	0.9668	1	2	0.9810
	4	0	18	0.9999	0	3	0.9989	0	9	0.9999	0	2	0.9981
	6	0	18	1.0000	0	3	1.0000	0	9	1.0000	0	2	0.9999
	8	0	18	1.0000	0	3	1.0000	0	9	1.0000	0	2	1.0000
0.05	2	-	-	-	1	4	0.9810	1	17	0.9969	1	2	0.9810
	4	0	23	0.9999	0	3	0.9989	0	12	0.9998	0	2	0.9981
	6	0	23	1.0000	0	3	1.0000	0	12	1.0000	0	2	0.9999
	8	0	23	1.0000	0	3	1.0000	0	12	1.0000	0	2	1.0000
0.01	2	-	-	-	2	7	0.9889	1	23	0.9911	2	4	0.9799
	4	0	32	0.9997	0	4	0.9981	0	16	0.9997	0	2	0.9981
	6	0	32	1.0000	0	4	0.9999	0	16	1.0000	0	2	0.9999
	8	0	32	1.0000	0	4	1.0000	0	16	1.0000	0	2	1.0000

Table 7. Plan parameter values for OGELLD with $\hat{\lambda}=39.8486, \hat{\theta}=1.0471, \hat{\gamma}=4.7161$ and w =3 for 50th percentile

β	t_q/t_q^0	r=5						r=10					
		δ_q=0.5			δ_q=1.0			δ_q=0.5			δ_q=1.0		
		c	g	P_a	c	g	P_a	C	g	P_a	c	g	P_a
0.25	2	0	5	0.9904	1	2	0.9786	0	3	0.9846	-	-	-
	4	0	5	1.0000	0	1	0.9999	0	3	1.0000	0	1	0.9992
	6	0	5	1.0000	0	1	1.0000	0	3	1.0000	0	1	1.0000
	8	0	5	1.0000	0	1	1.0000	0	3	1.0000	0	1	1.0000
0.10	2	0	7	0.9771	1	2	0.9786	0	4	0.9681	-	-	-
	4	0	7	1.0000	0	1	0.9999	0	4	1.0000	0	1	0.9992
	6	0	7	1.0000	0	1	1.0000	0	4	1.0000	0	1	1.0000
	8	0	7	1.0000	0	1	1.0000	0	4	1.0000	0	1	1.0000
0.05	2	0	8	0.9681	1	2	0.9786	0	4	0.9681	-	-	-
	4	0	8	1.0000	0	2	0.9992	0	4	1.0000	0	1	0.9992
	6	0	8	1.0000	0	2	1.0000	0	4	1.0000	0	1	1.0000
	8	0	8	1.0000	0	2	1.0000	0	4	1.0000	0	1	1.0000
0.01	2	1	15	0.9959	2	4	0.9595	1	8	0.9945	-	-	-
	4	0	11	1.0000	0	2	0.9992	1	6	1.0000	0	1	0.9992
	6	0	11	1.0000	0	2	1.0000	1	6	1.0000	0	1	1.0000
	8	0	11	1.0000	0	2	1.0000	1	6	1.0000	0	1	1.0000

Table 8. Plan parameter values for OGELLD with $\hat{\lambda}=39.8486, \hat{\theta}=1.0471, \hat{\gamma}=4.7161$ and w =3 for 25th percentile

β	t_q/t_q^0	r=5						r=10					
		δ_q=0.5			δ_q=1.0			δ_q=0.5			δ_q=1.0		
		c	g	P_a	c	g	P_a	c	g	P_a	c	g	P_a
0.25	2	0	15	0.9963	0	2	0.9774	0	8	0.9956	0	1	0.9774
	4	0	15	1.0000	0	2	1.0000	0	8	1.0000	0	1	1.0000
	6	0	15	1.0000	0	2	1.0000	0	8	1.0000	0	1	1.0000
	8	0	15	1.0000	0	2	1.0000	0	8	1.0000	0	1	1.0000
0.10	2	0	21	0.9908	1	4	0.9974	0	11	0.9896	1	2	0.9774
	4	0	21	1.0000	0	3	1.0000	0	11	1.0000	0	2	0.9999
	6	0	21	1.0000	0	3	1.0000	0	11	1.0000	0	2	1.0000
	8	0	21	1.0000	0	3	1.0000	0	11	1.0000	0	2	1.0000
0.05	2	0	25	0.9855	1	5	0.9925	0	13	0.9840	1	3	0.9831
	4	0	25	1.0000	0	3	1.0000	0	13	1.0000	0	2	0.9999
	6	0	25	1.0000	0	3	1.0000	0	13	1.0000	0	2	1.0000
	8	0	25	1.0000	0	3	1.0000	0	13	1.0000	0	2	1.0000
0.01	2	1	35	0.9660	1	6	0.9831	0	18	0.9636	1	3	0.9831
	4	1	35	1.0000	0	4	0.9999	0	18	1.0000	0	2	0.9999
	6	1	35	1.0000	0	4	1.0000	0	18	1.0000	0	2	1.0000
	8	1	35	1.0000	0	4	1.0000	0	18	1.0000	0	2	1.0000

Table 9. Comparison of RGASP with GASP with $\lambda = 2,\ \theta = \gamma = 1.5,\ w = 2,3\ \ r = 5,10$ and $\delta_q = 0.5, 1.0$ for 50^{th} percentile.

β	t_q/t_q^0	r=5						r=10					
		$\delta_q = 0.5$			$\delta_q = 1.0$			$\delta_q = 0.5$			$\delta_q = 1.0$		
		RGASP		GASP	RGASP		GASP	RGASP		GASP	RGASP		GASP
		w=2	w=3		w=2	w=3		w=2	w=3		w=2	w=3	
		g, c	g, c	g, c	g, c	g, c	g, c	g, c	g, c	g, c	g, c	g, c	g, c
0.25	2	6, 2	5, 1	7, 3	15,14	6, 5	--, --	3, 2	3, 1	5, 4	--, --	--, --	--, --
	4	3, 0	3, 0	4, 1	1, 0	1, 0	3, 2	2, 0	2, 0	2, 1	2, 1	1, 0	5, 4
	6	3, 0	3, 0	2, 0	1, 0	1, 0	2, 1	2, 0	2, 0	2, 1	1, 0	1, 0	2, 1
	8	3, 0	3, 0	2, 0	1, 0	1, 0	2, 1	2, 0	2, 0	2, 1	1, 0	1, 0	2, 1
0.10	2	8, 2	8, 2	10, 4	15, 14	6, 5	--, --	4, 2	4, 2	5, 4	--, --	--, --	--, --
	4	4, 0	4, 0	5, 1	1, 0	1, 0	3, 2	2, 0	2, 0	3, 1	2, 1	1, 0	5, 4
	6	4, 0	4, 0	5, 1	1, 0	1, 0	2, 1	2, 0	2, 0	3, 1	1, 0	1, 0	2, 1
	8	4, 0	4, 0	5, 1	1, 0	1, 0	2, 1	2, 0	2, 0	3, 1	1, 0	1, 0	2, 1
0.05	2	11, 3	9, 2	13, 5	15, 14	6, 5	--, --	6, 3	5, 2	8, 6	--, --	--, --	--, --
	4	5, 0	5, 0	6, 1	2, 1	2, 0	3, 2	4, 1	3, 0	3, 1	2, 1	1, 0	5, 4
	6	5, 0	5, 0	6, 1	2, 0	2, 0	2, 1	3, 0	3, 0	3, 1	1, 0	1, 0	2, 1
	8	5, 0	5, 0	6, 1	2, 0	2, 0	2, 1	3, 0	3, 0	3, 1	1, 0	1, 0	2, 1
0.01	2	15, 4	14, 3	19, 7	15, 14	6, 5	--, --	8, 4	7, 3	10, 7	--, --	--, --	--, --
	4	9, 1	7, 0	8, 1	9, 1	2, 0	3, 2	5, 1	4, 0	4, 1	2, 1	1, 0	5, 4
	6	7, 0	7, 0	8, 1	7, 0	2, 0	2, 1	4, 0	4, 0	4, 1	1, 0	1, 0	2, 1
	8	7, 0	7, 0	8, 1	7, 0	2, 0	2, 1	4, 0	4, 0	4, 1	1, 0	1, 0	2, 1

Table 10. Comparison of RGASP with GASP with $\lambda = 2,\ \theta = \gamma = 1.5,\ w = 2,3\ \ r = 5,10$ and $\delta_q = 0.5, 1.0$ for 25^{th} percentile.

β	t_q/t_q^0	r=5						r=10					
		$\delta_q = 0.5$			$\delta_q = 1.0$			$\delta_q = 0.5$			$\delta_q = 1.0$		
		RGASP		GASP	RGASP		GASP	RGASP		GASP	RGASP		GASP
		w=2	w=3		w=2	w=3		w=2	w=3		w=2	w=3	
		g, c	g, c	g, c	g, c	g, c	g, c	g, c	g, c	g, c	g, c	g, c	g, c
0.25	2	11, 1	12, 1	16, 3	4, 2	3, 1	4, 3	8, 2	6, 1	8, 3	6, 5	3, 2	--, --
	4	6, 0	7, 0	8, 1	2, 0	2, 0	2, 1	3, 0	4, 0	4, 1	1, 0	1, 0	2, 1
	6	6, 0	7, 0	5, 0	2, 0	2, 0	2, 1	3, 0	4, 0	4, 1	1, 0	1, 0	2, 1
	8	6, 0	7, 0	5, 0	2, 0	2, 0	2, 1	2, 0	4, 0	4, 1	1, 0	1, 0	2, 1
0.10	2	19, 2	16, 1	24, 4	6, 3	5, 2	6, 4	10, 2	8, 1	12, 4	6, 5	3, 2	--, --
	4	9, 0	10, 0	12, 1	3, 0	3, 0	3, 1	5, 0	5, 0	6, 1	2, 1	2, 0	2, 1
	6	9, 0	10, 0	7, 0	3, 0	3, 0	3, 1	5, 0	5, 0	6, 1	2, 0	2, 0	2, 1
	8	9, 0	10, 0	7, 0	3, 0	3, 0	3, 1	5, 0	5, 0	6, 1	2, 0	2, 0	2, 1
0.05	2	26, 3	23, 2	32, 5	7, 3	6, 2	8, 5	13, 3	12, 2	16, 5	6, 5	3, 2	--, --
	4	11, 0	12, 0	14, 1	3, 0	3, 0	4, 1	6, 0	6, 0	7, 1	2, 1	2, 0	2, 1
	6	11, 0	12, 0	14, 1	3, 0	3, 0	4, 1	6, 0	6, 0	7, 1	2, 0	2, 0	2, 1
	8	11, 0	12, 0	14, 1	3, 0	3, 0	4, 1	6, 0	6, 0	7, 1	2, 0	2, 0	2, 1
0.01	2	38, 4	34, 3	48, 7	11, 5	9, 3	12, 7	19, 4	17, 3	24, 7	6, 5	5, 4	--, --
	4	22, 1	17, 0	20, 1	6, 1	4, 0	7, 2	11, 1	9, 0	10, 1	3, 1	2, 0	5, 4
	6	16, 0	17, 0	20, 1	4, 0	4, 0	5, 1	8, 0	9, 0	10, 1	2, 0	2, 0	2, 1
	8	16, 0	17, 0	20, 1	4, 0	4, 0	5, 1	8, 0	9, 0	10, 1	2, 0	2, 0	2, 1