

Time Series Regression Modeling with AR(1) Errors

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ABSTRACT. When ordinary regression analysis is performed using time-series variables, it is common for the errors (residuals) to have a time-series structure. This violates the usual assumption of independent errors in ordinary least squares (OLS) regressions. Consequently, the estimates of the coefficients and their standard errors are incorrect if the time-series structure of the errors is ignored. In this study, an investigation of a regression model with time-series variables, particularly a simple case, was conducted using the conventional method. The 'AirPassengers Dataset' was downloaded from the **R** repository used for the analysis. Ordinary least squares and Cochrane-Orcutt procedures were used as methodologies. The results show that the adjusted regression model with autoregressive errors outperformed the ordinary regression model.

1. INTRODUCTION

Time-series regression is a technique for modeling time-series data or variables using a regression model. This technique is an extension of the existing classical ordinary regression model when the variable structure is a time-series. Similar to the regression model, this method is used to predict a future response based on autoregressive dynamics or response history [1]. The common use of time-series regression includes the modeling and forecasting of economic, financial, biological, and engineering systems [2].

A regression model represented using a functional relationship between the dependent and independent variables is given by

$$y_i = \beta_0 + \sum_{i=1}^p \beta_i x_i + \epsilon_i, \text{ for } i = 1, 2, \dots, p \text{ and } \epsilon_i \sim i.i.d \mathcal{N}(0, \sigma^2), \quad (1)$$

where y is the dependent variable, x the independent variable, β_0 the intercept, and β_i the gradient or slope. The model presented in Equation (1) is a general linear regression of dependent variable

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y on independent variable x . Similarly, a general linear regression model with time-series variables is represented as

$$y_t = \beta_0 + \sum_{i=1}^q \beta_i x_t + \epsilon_t, \text{ for } i = 1, 2, \dots, q \text{ and } \omega_t \sim i.i.d \mathcal{N}(0, \sigma^2). \quad (2)$$

where time-dependent and independent variables are y_t and x_t , respectively; β_0 is the intercept, and β_i is the gradient or slope. The model presented in Equation (2) is a general linear regression with a time-series dependent variable y_t on the independent variable x_t . Time series regression can help understand and predict the behavior of dynamic systems from experimental or observational data [3]. Several studies (e.g., [4–7]) utilized ordinary regression and built a predictive model from time-series variables. Running a linear regression model with time series data results in an incorrect estimate of the parameters of interest, together with an inflated standard error term [8–10]. Furthermore, the method of least squares eliminates the error term, which violates the assumption that the error (i.e., $\epsilon_t \sim i.i.d \mathcal{N}(0, \sigma^2)$) in the time-series model is structured as either the autoregressive (AR) or moving average (MA) model. Consequently, the estimates of coefficients and their standard errors will be incorrect if the time-series structure of the errors is ignored. In this study, an investigation into a regression model with time-series variables, and particularly a simple case, will be used against the ordinary least squares method. This study helps explain how to handle time-series variables applied to a linear regression model in statistical analysis. Furthermore, this study also guides researchers in choosing the appropriate technique when both the dependent and independent variables appear to be time series. The scope of this study is to apply time-series variables to investigate a linear regression model. Hence, the analysis is limited to the simple case of linear regression and will be investigated on the "AirPassengers" dataset, which can be found in the **R** software repository. To build up the literature, materials, and methods of this study, we reviewed some related works.

Several studies have been conducted to investigate the linear regression model applied to time-series variables. For instance, a study by [11] presented data analytics of the influence of climate factors on the impact of malaria incidence using a regression model with autoregressive error structure AR(1). They found that the relative humidity was the most influential climatic predictor of malaria incidence in the study area. Similarly, a novel regression method for harmonic analysis of time series and the results show that Harmonic Adaptive Penalty Operator (HAPO) exhibits a highly accurate model result, and HAPO has consistently smaller bias than ridge was investigated by [12]. Furthermore, a passion autoregressive model to understand COVID-19 contagion dynamics using a statistical model that can be employed to understand the contagion dynamics of COVID-19 was presented by [13]. The results show that the model can be applied to any country, region, or period. A spatial regression model analyzed by [14] to investigate Pulmonary TB cases In North Sumatra Province using ArcGIS in the processing data and Geo-Data was used in the regression analysis,

which showed a positive spatial autocorrelation. Linear Regression Analysis To Predict the Number of Death In India Due To SARS-COV-2 At 6 Weeks From Day (100 cases march 14th 2020) using a validated database, multiple regression, and linear regression analysis, the results showed that the current measured for containment of COVID-19 must be strengthened or supplemented [15]. Mental health related investigation was carried out by [16] to study conversations on social media and crisis episodes: a time series regression analysis using time analysis of retroactively collected data from Twitter and two London mental health providers, and the result showed that SLAM crisis episodes were 15% higher (p -value <0.001) on higher volume schizophrenia tweet days 9% higher (p -value <0.001) on higher volume supportive depression tweet. However, [17] modeled non-stationary emotion dynamics in dyads, and the results showed that the time-varying and standard value model indicated that all the parameters pertaining to the males were statistically significant. In addition, [18] studied time series regression with a unit root, and the results showed that the method outlined in the section for the refinement of first-order asymptotic theory may be applied in general time series models with unit roots. Furthermore, [19] investigated a time-series regression analysis to evaluate the economic impact of COVID-19 cases in Indonesia using the transfer function model and vector autoregressive moving average with exogenous regressors (VARMAX) model. The results show that an increase in the number of COVID-19 cases in Indonesia significantly affected the USD/IDR exchange rate. However, discretize-optimize vs. optimize-discrete for time series regression and continuous normalizing flows using ordinary differential equations (ODEs), and the results showed the improved convergence of Disc-Opt over the Opt-Disc approach on image classification tasks investigated by [20].

The remainder of this paper is organized as follows. In Section 2, we describe the proposed method. Section 3 presents the results of this study. The conclusions and future work are summarized in Section 4.

2. MATERIALS AND METHODS

This section presents the methodology for solving regression analysis with time-series variables or data, focusing only on the simple case of the regression model.

2.1. Source of Data. Monthly time-series data on international airline passengers recorded between 1949 and 1960 were retrieved from the **R** repository [21]. The data were named the classic Box and Jenkins airline data [22]. The **R** code `data()` is used to open all datasets in the repository and then selected to open the "*AirPassengers*" for this study.

2.2. Ordinary Regression Model. Consider a simple linear regression (SLR) model given by

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \text{ for } i = 1, 2, \dots, n. \quad (3)$$

where y_i is the dependent variable, x_i is the time-series independent variable, and ϵ_i is the error or residual term. To estimate the parameters β_0 and β_1 (i.e., the intercept and slope, respectively), the ordinary least squares (OLS) method is used to obtain the following estimates:

$$\hat{\beta}_0 = \bar{y} - \beta_1 \bar{x} \text{ and } \hat{\beta}_1 = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n \sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2}. \quad (4)$$

The detail proving of the estimates (β_0 and β_1) could be found in many statistical textbooks [23]. As stated earlier, when ordinary regression analysis is performed using time-series variables, it is common for the errors (residuals) to have a time-series structure [24–27]. This violates the usual assumption of independent errors made in ordinary least squares regression. The consequence is that the estimates of coefficients and their standard errors will be wrong if the time series structure of the errors is ignored. However, it is possible to adjust the estimated regression coefficients and standard errors when the errors have an *AR* structure. More generally, we can make adjustments when errors have a general ARIMA structure.

2.3. Time-Series Regression Model. Consider a simple linear regression model with time series variables given by

$$y_t = \beta_0 + \beta_1 x_t + \epsilon_t. \quad (5)$$

where y_t is the time-series of the dependent variable, x_t is the time- of the independent variable, and ϵ_t is the time-series structured error term. Suppose that ϵ_t has *AR* structure given by

$$\epsilon_t = \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \phi_3 \epsilon_{t-3} + \dots + \omega_t, \text{ where } \omega_t \sim i.i.d \mathcal{N}(0, \sigma^2). \quad (6)$$

Using the backshift operator \mathcal{B} to evaluate Equation (6) we obtain

$$\epsilon_t = \phi_1 \mathcal{B} \epsilon_t + \phi_2 \mathcal{B}^2 \epsilon_t + \phi_3 \mathcal{B}^3 \epsilon_t + \dots + \omega_t. \quad (7)$$

Taking the L.H.S of Equation (7) except ω_t to the R.H.S, and obtaining

$$\epsilon_t - \phi_1 \mathcal{B} \epsilon_t - \phi_2 \mathcal{B}^2 \epsilon_t - \phi_3 \mathcal{B}^3 \epsilon_t - \dots = \omega_t. \quad (8)$$

By factoring ϵ_t in the L.H.S of Equation (8), we obtain

$$(1 - \phi_1 \mathcal{B} - \phi_2 \mathcal{B}^2 - \phi_3 \mathcal{B}^3 - \dots) \epsilon_t = \omega_t. \quad (9)$$

Let $\Phi(\mathcal{B}) = 1 - \phi_1 \mathcal{B} - \phi_2 \mathcal{B}^2 - \phi_3 \mathcal{B}^3 - \dots$ is a polynomial. By substituting $\Phi(\mathcal{B})$ into the polynomial in Equation (9) we obtain

$$\Phi(\mathcal{B}) \epsilon_t = \omega_t. \quad (10)$$

Suppose the inverse $\Phi(\mathcal{B})^{-1}$ exists, which means that the $\det|\Phi(\mathcal{B})| \neq 0$; then, Equation (10) reduces to

$$\epsilon_t = \Phi(\mathcal{B})^{-1} \omega_t, \text{ where } \epsilon_t \sim i.i.d \mathcal{N}(0, \sigma^2). \quad (11)$$

Now, substituting Equation 11 into Equation (5), becomes

$$y_t = \beta_0 + \beta_1 x_t + \Phi(\mathcal{B})^{-1} \omega_t. \quad (12)$$

2.4. Cochrane-Orcutt. The Cochrane-Orcutt procedure is a method for estimating the parameters of a linear regression model that has a first-order serial correlation in the errors. First-order serial correlation occurs when the error terms in a regression model correlate with the error terms in the immediately preceding observations. This can occur when there is some type of temporal dependence in the data [30], such as time-series data.

The Cochrane-Orcutt procedure can be implemented in statistical software packages, such as Stata, SAS, and R [28]. It is important to note that the Cochrane-Orcutt procedure assumes that the error terms in the model follow an AR(1) process, which may not be appropriate for all data types. Additionally, the procedure can be sensitive to the choice of starting values and number of iterations [29]; therefore, it is important to conduct sensitivity analyses to assess the robustness of the results.

2.4.1. Cochrane-Orcutt Procedure. The Cochrane-Orcutt procedure involves the following steps:

- **Step 1:** Estimate the model using ordinary least squares (OLS) regression.
- **Step 2:** Estimate the autocorrelation coefficient (ρ) using the residuals from the OLS regression.
- **Step 3:** Use the estimated ρ to transform the data by differencing each variable and taking the lagged values of the transformed variables.
- **Step 4:** Estimate the transformed model using OLS regression.
- **Step 5:** Iterate steps 2 to 4 until the estimated value of ρ converges to a stable value.

2.4.2. Parameter Estimation. The estimation of parameters in the simple regression time-series model presented in Equation (12) is solved using the Cochrane-Orcutt theory [31], and the solution is explained accordingly. We start by multiplying Equation (12) with $\Phi(\mathcal{B})$ to obtain

$$\Phi(\mathcal{B})y_t = \Phi(\mathcal{B})\beta_0 + \Phi(\mathcal{B})\beta_1 x_t + \omega_t. \quad (13)$$

Then, let

$$\begin{aligned} y_t^* &= \Phi(\mathcal{B})y_t &= y_t - \Phi_1 y_{t-1} - \Phi_2 y_{t-2} - \cdots - \Phi_p y_{t-p}. \\ x_t^* &= \Phi(\mathcal{B})x_t &= x_t - \Phi_1 x_{t-1} - \Phi_2 x_{t-2} - \cdots - \Phi_p x_{t-p}. \\ \beta_0^* &= \Phi(\mathcal{B})\beta_0 &= (1 - \Phi_1 - \Phi_2 - \cdots - \Phi_p)\beta_0. \end{aligned} \quad (14)$$

where β_0 is an unknown constant that does not move over time or is independent of time (t). By substituting Equation (15) into Equation (14), we obtain the reduced form as:

$$y_t^* = \beta_0^* + \beta_1 x_t^* + \omega_t, \text{ where } \omega_t \sim i.i.d \mathcal{N}(0, \sigma^2). \quad (15)$$

Therefore, Equation (15) is a simple linear regression model based on a transformation. By making β_0 the subject of the formula in Equation (14), we obtain

$$\hat{\beta}_0 = \frac{\hat{\beta}_0^*}{(1 - \hat{\Phi}_1 - \hat{\Phi}_2 - \dots - \hat{\Phi}_p)}. \quad (16)$$

Similarly, the standard error for $\hat{\beta}_0$ is given by

$$\text{s.e}(\hat{\beta}_0) = \frac{\text{s.e}(\hat{\beta}_0^*)}{(1 - \hat{\Phi}_1 - \hat{\Phi}_2 - \dots - \hat{\Phi}_p)}. \quad (17)$$

3. RESULTS AND DISCUSSION

The time-series plot presented in Figure 1 was produced using the *AirPassengers* dataset displayed in [21].

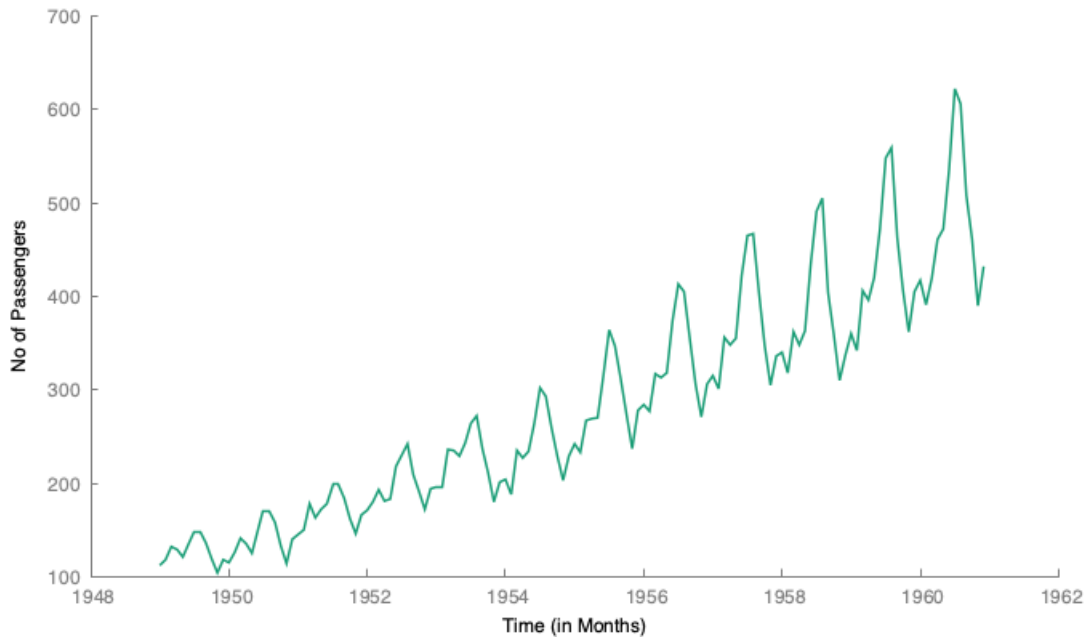


FIGURE 1. Time-series plot

The pattern of airline passengers, as depicted in the plot, shows an upward increase in the trend with seasonality over time.

3.1. Ordinary Least Squares. The results presented in Table 1 show the analysis of variance (ANOVA) for the simple linear regression model built from the air passenger dataset. The constant $\hat{\beta}_0 = 87.6528$ (p -value = 0.0000) contributed significantly to the model. Similarly, the slope $\hat{\beta}_1 = 2.6572$ (p -value = 0.0000) also contributes significantly to the model of air passengers. The model is mathematically represented by

$$\hat{y} = 87.6528 + 2.6572\hat{x}. \quad (18)$$

Since the p -value = 0.0000 for both the parameters, thus indicating a strong evidence for rejecting H_0 at 5% level of significance. Following this decision rule, the estimated parameters ($\hat{\beta}_0$ and $\hat{\beta}_1$) have a greater impact to development of the model and prediction of the passengers. Subsequently, the model is adequately yields a strong coefficient of determination, $R^2 = 85\%$. However, the

TABLE 1. Test of Significance on Parameters Estimated using OLS.

Variable	Coefficient	Std. Error	t -statistic	P -value	95% CI
Const	87.6528	5.6203	15.6000	0.0000***	76.5426, 98.7630
Months	2.6572	0.1041	25.5200	0.0000***	2.4513, 2.8630

*** is indicating significant at $\alpha = 5\%$, $F(1, 142) = 651.0966$, CI = confidence interval and $R^2 = 85\%$

overall results of the analysis of variance of the estimated parameters in Table 1 ($F_{cal}(1, 142) = 651.0966 \gg F_{tab}(1, 142) = 3.908$) show that the ordinary regression model is also significant for p -value = 0.0000 when compared with $\alpha = 0.05$. The value of $F_{tab}(1, 142)$ is not readily available in F table therefore, $F_{tab}(1, 142)$ can be found between $F_{tab}(1, 140)$ and $F_{tab}(1, 180)$. An interpolation technique is used to obtain the value of $F_{tab}(1, 142)$. As a result, $F_{tab}(1, 142) = 3.908$ and the t -statistic corresponding to $\beta_0 = 87.6528$ (15.6000) and $\beta_1 = 2.6572$ (25.5200) are significant compared with the tabulated value. The regression coefficients are robust estimates as they take no zero value in the spectrum of the 95% confidence interval.

3.2. **Cochrane-Orcutt Results.** The estimated relationship between y_t and x_t is given by

$$y_t = 86.8847 + 2.65021x_t + 0.731812\epsilon_{t-1} + \omega_t, \quad (19)$$

where the errors are $\epsilon_t = 0.731812\epsilon_{t-1} + \omega_t$ and $\omega_t \sim i.i.d \mathcal{N}(0, \sigma^2)$. The predicted y_t is a linear function of x_t at this time and the residual at the previous time. The accuracy of the time-series regression model with an autoregressive error is 93%, which is better than that of the model presented in Equation (18) with $R^2 = 85\%$. Cochrane-Orcutt procedure does not minimize the residual sum of squares, as it does in OLS [32]. The overall results of the analysis of variance

TABLE 2. Test of Significance on Parameters Estimated using Cochrane-Orcutt.

Variable	Coefficient	Std. Error	t -statistic	P -value	95% CI
Const	86.8847	20.5453	4.2290	0.0000***	46.2679, 127.5010
Months	2.6502	0.2382	11.1300	0.0000***	2.1793, 3.1211

*** is indicating significant at $\alpha = 5\%$, $F(1, 142) = 123.7768$, CI = confidence interval and $R^2 = 93\%$

of the estimated parameters in Table 2 show that the time series regression model developed using the Cochrane-Orcutt technique is significant, p -value = 0.0000 ($F_{cal}(1, 142) = 123.7768 \gg F_{tab}(1, 142) = 3.908$). A diagnostic test is performed to test the adequacy of the model presented in Equation (19). The residuals of the model (Equation (19)) are assumed to be normal, and the

test statistic is $\chi^2(2) = 3.8983$ with p -value = 0.1424, where H_0 is rejected as the p -value is greater than $\alpha = 0.05$. Table 3 presents the iteration procedure for ρ convergence in estimating the Cochrane-Orcutt parameters of the model presented in Equation (19).

TABLE 3. Performing iterative calculation of ρ .

Iteration	ρ	ESS
1	0.73175	140225
2	0.73181	140225
3	0.73181	140225

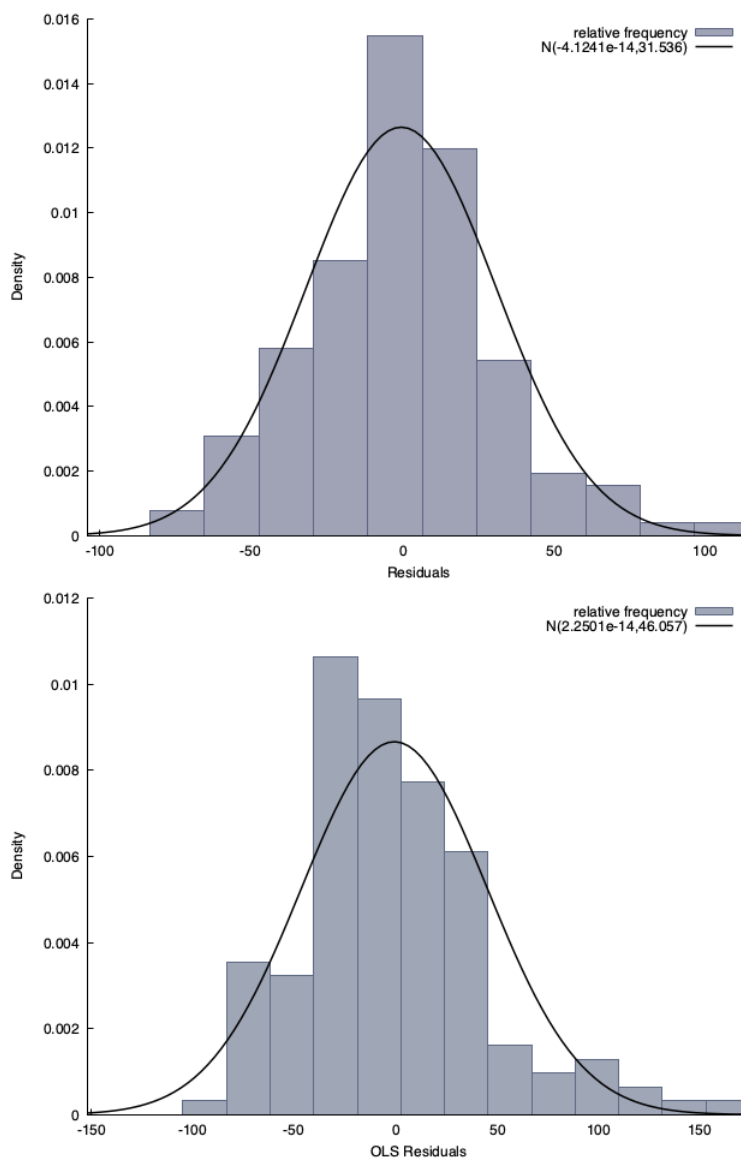


FIGURE 2. Residual plots of the OLS and Cochrane-Orcutt method.

3.3. **Performance Accuracy.** Table 4 presents the model performance for various measures; however, the most commonly used measure is the coefficient of determination (R^2). Using the ordinary least squares method, $R^2 = 85\%$, and Cochrane–Orcutt produced $R^2 = 93\%$, which accurately explained the variation in the air passenger model.

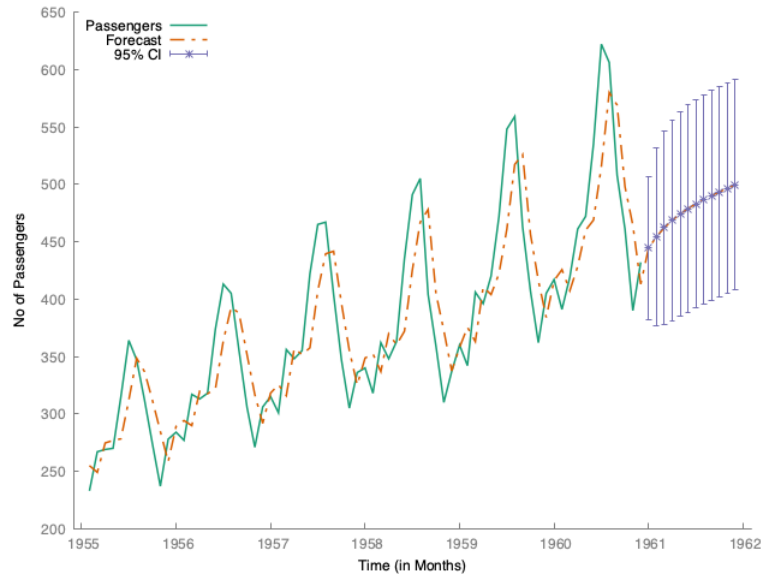


FIGURE 3. AirPassengers prediction plot.

TABLE 4. Model Evaluation.

Performance Measures	Ordinary Least Squares (OLS)	Cochrane-Orcutt
Mean dependent var	280.2986	281.4755
Sum squared residual	140225.3	301219.1
R^2	0.853638	0.930907
Adjusted R^2	0.852607	0.930417
$F(1, 142)$	651.0966 (p -value = 0.0000)	123.7768 (p -value = 0.0000)

4. CONCLUSION

Regression modeling with time-series variables results in an incorrect estimate of the parameters and inflated standard errors. In this study, a simple linear regression model with time-series variables was investigated. Cochrane–Orcutt procedure was used to estimate the precise coefficients of the adjusted time-series regression model through iteration, and the results were compared with OLS. The results showed that the errors exhibit an AR(1) pattern, and the Cochrane–Orcutt procedure outperformed OLS, affirmed by R^2 and other accuracy measures. However, the higher-order AR model fit the residuals well. This study provides an approach for adjusting estimated regression coefficients and their corresponding standard errors. This study will be extended to investigate multiple regression cases with ARIMA errors.

COMPETING INTERESTS

The authors declare that they have no competing interests.

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