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Enhancing Multiple Frame Surveys: Improved Calibration and Efficient Bootstrap Techniques

Cherif Ahmat Tidiane Aidara

Mathematics Unit, University of The Gambia, Gambia

Correspondence: cataidara@utg.edu.gm

Abstract. In recent years, multiple frame surveys have gained significant attention due to their applicability in capturing special or challenging-to-sample populations. This paper introduces two methodological advancements, the calibrated multiplicity estimator and without-replacement bootstrap techniques, in the field of multiple frame surveys. A comprehensive simulation study assesses their performance. The calibrated multiplicity estimator is demonstrated to outperform the multiplicity estimator, particularly in terms of mean squared error, with a ratio ranging from 0.6 to 0.8. Furthermore, the study shows that without-replacement bootstrap techniques perform favorably compared to their with-replacement counterparts. Future research directions include conducting more extensive simulations with real-world data and establishing the theoretical properties of the proposed estimator. This paper contributes to the growing body of knowledge on multiple frame surveys and their estimation methods.

1. Introduction

In recent years, there has been a significant research focus on multiple frame surveys, as evidenced by the works of [1], [2], [3], [4], and [5]. While the initial motivation behind the development of multiple frame surveys was to reduce survey costs, their current application is primarily centered around capturing populations that are special, rare, or challenging to sample accurately.

The fundamental concept underlying multiple frame surveys involves the assumption that the target population can be effectively covered by a combination of sampling frames, each of which covers only a portion of the total population. To obtain estimates, the usual practice involves independent sampling from each of these frames, with resulting estimators designed to appropriately account for the overlapping units.

The realm of multiple frame survey research has introduced a plethora of estimators, many of which have been proposed and discussed in works by [3] and [4]. Just as in classical sampling theory, the standard error serves as a crucial measure for assessing the quality of estimators in multiple frame surveys. This field has explored both analytical methods, such as the Taylor linearization, as well as replication methods like the Jackknife and the with-replacement bootstrap, for estimating

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standard errors. Notably, the bootstrap method holds a distinct advantage over other approaches due to its applicability to both smooth and non-smooth statistics, in addition to granting users the flexibility to choose the number of replication runs.

In the context of contributing to this area of research, this paper introduces two key advancements. First, it proposes the calibrated multiplicity estimator which is an extension of the multiplicity estimator introduced by [4]. Second, it presents without-replacement bootstrap techniques that build upon the methods outlined in [5].

The subsequent sections of this paper are structured as follows: Section 2 elaborates on the calibrated multiplicity estimator, while Section 3 delves into the novel without-replacement bootstrap techniques tailored for the multiple frame survey setting. To empirically evaluate the performance of the proposed bootstrap techniques, Section 4 outlines the design of simulation studies. Finally, Section 5 concludes with closing remarks on the findings and contributions of this study.

2. Calibrated Multiplicity Estimator

In this section, we provide a concise overview of the multiplicity estimator introduced by [4], along with an improved version of this estimator that we refer to as the calibrated multiplicity estimator.

Consider a population of interest comprehensively covered by $Q(\geq 2)$ overlapping sampling frames, allowing for the potential inclusion of a unit in one or more frames. Each population unit i has a corresponding multiplicity, denoted by m_i, indicating the number of frames it belongs to. We are interested in the population total of a characteristic variable y, denoted as T_y. Let $A^{(q)}$ signify the q-th sampling frame, where $q = 1, \ldots, Q$. It is evident that:

$$T_y = \sum_{q=1}^{Q} \sum_{i \in A^{(q)}} y_i m_i^{-1}. \tag{1}$$

For each q, let $S^{(q)}$ be a probability sample independently selected from $A^{(q)}$. Further, let $d_i^{(q)}$ denote the design weight associated with frame $A^{(q)}$. The multiplicity estimator takes the form:

$$t_y = \sum_{q=1}^{Q} \sum_{k \in S^{(q)}} d_k^{(q)} y_k m_k^{-1}. \tag{2}$$

This estimator, referred to as the multiplicity estimator (for detailed information, consult [4]), can be represented as a function of weights:

$$t_y = f(\boldsymbol{d}^{(1)}, \cdots, \boldsymbol{d}^{(q)}, \cdots, \boldsymbol{d}^{(Q)}), \tag{3}$$

where $\boldsymbol{d}^{(q)}$ is the weight vector for frame q.

To simplify, we assume complete auxiliary information is available from each of the Q sampling frames on a known vector variable $\mathbf{x}$. We introduce the following estimator:

$$t_{yC} = \sum_{q=1}^{Q} \sum_{k \in S^{(q)}} w_k^{(q)} y_k m_k^{-1}, \tag{4}$$

where the weights $w_k^{(q)}$ are defined to satisfy:

$$\sum_{k \in S^{(q)}} w_k^{(q)} \mathbf{x}_k m_k^{-1} = \sum_{i \in A^{(q)}} \mathbf{x}_i m_i^{-1} \quad \text{for} \quad q = 1, \cdots, Q. \tag{5}$$

This constraint can be expressed more compactly as:

$$\sum_{q=1}^{Q} \sum_{k \in S^{(q)}} w_k^{(q)} \mathbf{x}_k m_k^{-1} = \sum_{q=1}^{Q} \sum_{i \in A^{(q)}} \mathbf{x}_i m_i^{-1}. \tag{6}$$

For each $S^{(q)}$, the aim is to determine weights $w_k^{(q)}$ that closely match $d_k^{(q)}$ using a distance function $D_q(\boldsymbol{w}^{(q)}, \boldsymbol{d}^{(q)})$ subject to the constraint in (5). It is worth highlighting that $\boldsymbol{w}^{(q)}$ is analagous to $\boldsymbol{d}^{(q)}$, and it can be defined as $\boldsymbol{w}^{(q)} = (w_1^{(q)}, \cdots, w_{n^{(q)}}^{(q)})$.

This problem becomes an optimization task, aiming to minimize:

$$L_q(\boldsymbol{w}^{(q)}, \boldsymbol{\lambda}_q) = \sum_{k \in S^{(q)}} D_q(w_k^{(q)}, d_k^{(q)}) + \boldsymbol{\lambda}_q' \left(\sum_{i \in A^{(q)}} \mathbf{x}_i m_i^{-1} - \sum_{k \in S^{(q)}} w_k^{(q)} \mathbf{x}_k m_k^{-1} \right) \tag{7}$$

using the Lagrange multipliers method. We adopt the chi-square distance function, known to yield weights similar to those of the linear generalized regression estimator (GREG).

For the optimization, we express the problem as minimizing:

$$L(\boldsymbol{w}, \boldsymbol{\lambda}) = \sum_{q=1}^{Q} L_q(\boldsymbol{w}^{(q)}, \boldsymbol{\lambda}_q) \tag{8}$$

yielding calibrated weights given by:

$$\begin{aligned} w_k^{(q)} \quad &= \quad d_k^{(q)} \left\{ 1 + \left(\sum_{i \in A^{(q)}} \mathbf{x}_i m_i^{-1} - \sum_{k \in S^{(q)}} d_k^{(q)} \mathbf{x}_k m_k^{-1} \right)' \right. \\ &\left. \left(\sum_{k \in S^{(q)}} d_k^{(q)} m_k^{-2} \mathbf{x}_k \mathbf{x}_k' \right)^{-1} \mathbf{x}_k m_k^{-1} \right\} \end{aligned} \tag{9}$$

An alternate formulation involves minimizing $\sum_{q=1}^{Q} \sum_{k \in S^{(q)}} D_q(w_k^{(q)}, d_k^{(q)})$ subject to the constraint in (6). Applying the Lagrange multipliers method leads to the minimized Lagrangian:

$$L(\boldsymbol{w}, \boldsymbol{\lambda}) = \sum_{q=1}^{Q} \sum_{k \in S^{(q)}} D_q(w_k^{(q)}, d_k^{(q)}) + \boldsymbol{\lambda}' \left(\sum_{q=1}^{Q} \sum_{i \in A^{(q)}} \mathbf{x}_i m_i^{-1} - \sum_{q=1}^{Q} \sum_{k \in S^{(q)}} w_k^{(q)} \mathbf{x}_k m_k^{-1} \right). \tag{10}$$

Applying the chi-square distance function, we minimize (10) with respect to the weights $w_k^{(q)}$ and obtain the calibrated weights:

$$
\begin{aligned}
w_k^{(q)} &= d_k^{(q)} \left\{ 1 + \left(\sum_{q=1}^{Q} \sum_{i \in A^{(q)}} \mathbf{x}_i m_i^{-1} - \sum_{q=1}^{Q} \sum_{k \in S^{(q)}} d_k^{(q)} \mathbf{x}_k m_k^{-1} \right)' \right. \\
&\left. \left(\sum_{q=1}^{Q} \sum_{k \in S^{(q)}} d_k^{(q)} m_k^{-2} \mathbf{x}_k \mathbf{x}_k' \right)^{-1} \mathbf{x}_k m_k^{-1} \right\}
\end{aligned} \tag{11}
$$

It is crucial to note that these weights in (9) or (11) serve to estimate various parameters of interest. Additionally, the proposed bootstrap variance estimation algorithms can be applied to all multiple frame survey estimators.

3. Proposed Bootstrap Techniques for Multiple Frame Surveys

In the realm of survey sampling, the selection of primary sampling units (PSUs) often involves unequal probabilities and is done without replacement. However, when it comes to variance estimation during replication, computations are considerably simplified by assuming that these PSUs were chosen with replacement. In this paper, we retain the without-replacement PSU selection and outline our proposed methodology as follows.

To set the stage, we consider Q stratified samples denoted as $S = \cup_{q=1}^{Q} S^{(q)}$, where $S^{(q)}$ represents a stratified sample independently drawn from the q-th sampling frame. Each $S^{(q)}$ is composed of $H^{(q)}$ strata, defined as $S^{(q)} = \cup_{h=1}^{H^{(q)}} S_h^{(q)}$. It is important to note that $S_h^{(q)}$ is the sample independently drawn from the h-th stratum of the q-th sampling frame. Additionally, we assume that $S_h^{(q)}$ consists of $n_h^{(q)}$ PSUs, and a resample of $m_h^{(q)}$ units is drawn from it without replacement.

In single-frame surveys, the rescaling bootstrap without replacement weights, as introduced in [7], can be considered an extension of the technique proposed by [8]. These weights, designated for PSU i within stratum h of S_h, are defined as:

$$
d_{hi}^{*} = d_{hi} \left(1 - \gamma_h + \gamma_h \frac{n_h}{m_h} \delta_{hi}^{*} \right) \tag{12}
$$

Here, $\gamma_h = \sqrt{(1 - f_h) m_h / (n_h - m_h)}$; $f_h = n_h / N_h$, where N_h is the stratum size; $\delta_{hi}^{*} = 1$ if $i \in S_h^b$ and 0 otherwise; $m_h = [n_h / (2 - f_h)]$, with $[\cdot]$ indicating rounding down to the nearest integer; and d_{hi} is the original weight associated with PSU i of the h-th stratum. In multiple-frame surveys, these weights are applied to the Q independent samples either individually or simultaneously. For the q-th independent sample and b-th simulation run, the following weights are used:

$$
d_{hi}^{(q)}[b] = d_{hi}^{(q)} \left(1 - \gamma_h^{(q)} + \gamma_h^{(q)} \frac{n_h^{(q)}}{m_h^{(q)}} \delta_{hi}^{(q)}[b] \right) \tag{13}
$$

These weights, $d_i^{(q)}[b]$, are subsequently adjusted for calibration to obtain the final bootstrap weights and the multiplicity estimator for the parameter of interest.

When these weights are applied separately to the Q independent samples, the resulting bootstrap variance estimator is referred to as the separate bootstrap, akin to the approach described in [5]. In this case, denoting the multiplicity estimator with the original weights replaced by the final bootstrap weights for just frame q in the b-th simulation run as $\hat{\theta}^{(q)}[b]$, the separate bootstrap estimator is formulated as:

$$v_s = \sum_{q=1}^{Q} \frac{1}{B^{(q)}} \sum_{b=1}^{B^{(q)}} \left(\hat{\theta}^{(q)}[b] - \hat{\theta}\right)^2 \tag{14}$$

Here, $B^{(q)}$ represents the number of simulation runs in the q-th frame.

Similarly, when the weights are simultaneously applied to the Q independent samples, the resultant bootstrap variance estimator is termed the combined bootstrap, similar to the approach outlined in [5]. In this scenario, using $\hat{\theta}[b]$ to denote the multiplicity estimator when the original weights are replaced by the final bootstrap weights across all Q frames, the combined bootstrap estimator is given by:

$$v_c = \frac{1}{B} \sum_{b=1}^{B} \left(\hat{\theta}[b] - \hat{\theta}\right)^2 \tag{15}$$

Here, B signifies the number of simulation runs.

4. Simulation

The performance evaluation of the introduced methodologies for multiple frame surveys was conducted through a controlled simulation study. In this context, we employed an artificial population, as detailed in [6]. This synthetic population comprises $H = 5$ strata, each consisting of $N_h = 1000$ units. The model distributions for this population are as follows:

$$\begin{aligned}
&X_{1i} \sim \Gamma(2,1), && E_{1i} \sim N(0,1),\\
&X_{2i} \sim \Gamma(2,1), && E_{2i} \sim N(0,X_{2i}),\\
&X_{3i} \sim \Gamma(2,1), && E_{3i} \sim t(4+3/(X_{3i}+1)),\\
&X_{4i} \sim \Gamma(2,3), && E_{2i} \sim N(0,X_{4i}),\\
&X_{5i} \sim \Gamma(2,9), && E_{2i} \sim N(0,X_{5i}),\\
&Y_{hi} = X_{hi} + E_{hi}, && h = 1,\cdots,5
\end{aligned}$$

Here, $h = 1, \cdots, 5$ indicates the strata, $i = 1, \cdots, N_h$ refers to the units within strata, and X_i and E_i are independent both within observations and across observations.

From this artificially constructed population, we generated $Q = 3$ partially overlapping sampling frames by randomly assigning each pair (X_i, Y_i) from the population to one of the sampling frames through 3 independent Bernoulli trials, each with a probability $\alpha_q = N^{(q)}/N$ for $q = 1, 2, 3$. We ensured that no sampling frame was devoid of samples, and when combined, they sufficiently covered the population of interest. Furthermore, our simulation encompassed various frame coverage settings, as summarized in Table 1.

Table 1. Frame Coverage Settings

Setting	Frame coverage $\alpha_q = \frac{N^{(q)}}{N}$		
A	0.35	0.35	0.35
B	0.60	0.60	0.60
C	0.85	0.85	0.85
D	0.35	0.60	0.85
E	0.60	0.35	0.85
F	0.60	0.85	0.35

Subsequently, three independent stratified random samples ($S^{(1)}$, $S^{(2)}$, and $S^{(3)}$) of sizes $n^{(1)}$, $n^{(2)}$, and $n^{(3)}$ units were selected from frames $A^{(1)}$, $A^{(2)}$, and $A^{(3)}$, respectively. We focused on estimating the population total (T_y) and median (M_y) of the variable y. We employed the multiplicity and calibrated multiplicity estimators (denoted as Mest and Mcal, respectively) for these estimations. We performed a total of 30,000 independent stratified random samples from each of the 3 frames to compare the efficiency of the estimators.

We compared the estimators under different frame coverage settings (as described in Table 1) and employed a stratum sample size $n_h^{(q)}$ that represented 3 percent of the stratum population size $N_h^{(q)}$ for $q = 1, 2, 3$.

Furthermore, we examined various variance estimators, including the separate and combined bootstraps (denoted as BSWR and BCWR) described in [5], as well as the proposed separate and combined bootstraps (denoted as BSWOR and BCWOR).

A total of $B = 1000$ simulation runs were executed. For each run, a simple random sample without replacement of size $n_h^{(q)}$ (equivalent to 3 percent of the stratum population size $N^{(q)}h$) was drawn from each stratum of $S^{(q)}$ for $q = 1, 2, 3$, and variance estimates were computed using the four variance estimators. The bootstrap stratified sample size drawn from $S^{(q)}$ was $m^{(q)} = \sum_{q=1}^{Q} m_h^{(q)}$, where $m_h^{(q)} = n_h^{(q)} - 1$ for the with-replacement bootstrap methods and $m_h^{(q)} = [n_h^{(q)}/2]$ for the without-replacement bootstrap methods. The number of replications was $R = 200$ for all bootstrap methods. All computations were performed using the R software (see R Core Team, 2021).

The performance assessment of the variance estimators was based on the simulated relative percentage bias (RB%), coefficient of variation (CV), and empirical coverage probabilities of 95% confidence intervals (CP). The RB% and CV for a given variance estimator v were calculated using the formulas:

$$RB = 100 \times \frac{1}{B}\sum_{b=1}^{B}\frac{v_b - MSE}{MSE} \tag{16}$$

$$CV = \sqrt{\frac{1}{B}\sum_{b=1}^{B}(v_b - MSE)^2/MSE} \tag{17}$$

where v_b represents the variance estimate of v for the b-th simulated sample. The true mean squared errors (MSEs) were approximated using 10,000 simulation runs.

4.1. **Simulation Results for Efficiency Comparison.** Table 2 clearly illustrates the superior efficiency of the proposed calibrated multiplicity estimator (Mcal) over the conventional multiplicity estimator (Mest). This enhanced efficiency of the calibrated estimator is consistently observed across various frame coverage setups. Notably, the efficiency enhancement of the calibrated multiplicity estimator is particularly conspicuous in the context of total estimation compared to median estimation, with a mean squared error ratio of approximately 0.6 for total estimation and 0.8 for median estimation.

Table 2. Efficiency of Mcal versus Mest for total and median: Six indicative simulation runs

Setting	Empirical efficiency ratio $\frac{MSE(t_{yC})}{MSE(t_y)}$	Empirical efficiency ratio $\frac{MSE(m_{yC})}{MSE(m_y)}$
A	0.6457	0.8165
B	0.6323	0.8141
C	0.6867	0.8775
D	0.6518	0.7834
E	0.6796	0.8057
F	0.6507	0.8030

4.2. **Simulation Results for Bootstrap Estimators.** Table 3 presents three notable observations across a range of settings concerning relative bias. Firstly, the with-replacement bias remains consistently slightly positive and generally equivalent. Secondly, the without-replacement bias consistently remains negligible and exhibits a fair uniformity. It is worth highlighting that the bias without replacement is smaller than the bias with replacement. Crucially, these observations remain consistent regardless of the type of estimator under consideration, whether it is the multiplicity estimator or the calibrated multiplicity estimator.

Moreover, Table 3 offers valuable insights into the coefficient of variation across various settings. Firstly, the coefficients of variation are approximately equal. Secondly, in scenarios without replacement, the coefficients of variation are slightly smaller than those with replacement. This noteworthy pattern is observed in both the multiplicity estimator and the calibrated multiplicity estimator.

Furthermore, when considering various methods and estimators in Table 3, the primary observation is their consistent demonstration of similar coverage rates.

5. Conclusion

This paper has introduced several methodological innovations in the domain of multiple frame surveys, with a focus on the calibrated multiplicity estimator and the without-replacement bootstrap techniques. A limited simulation study was carried out, which demonstrated that the calibrated multiplicity estimator outperforms the multiplicity estimator in terms of mean squared error by a ratio ranging from 0.6 to 0.8. Additionally, the study indicated that the without-replacement bootstrap techniques perform quite favorably when compared to the with-replacement bootstrap techniques.

Several potential avenues for future research include conducting more extensive simulation studies involving real-world data and establishing the theoretical properties of the proposed estimator. These endeavors would contribute to a deeper understanding of the practical applications and theoretical underpinnings of the methodologies introduced in this paper.

Table 3. Comparison of Variance Estimators

Setting A							
Multiplicity for Total				Calibrated Multiplicity for Total			
	RB%	CV	CP		RB%	CV	CP
BSWR	5.82	0.0859	94.9	BSWR	6.87	0.0932	95.3
BCWR	5.74	0.1184	95.8	BCWR	6.82	0.1282	94.9
BSWOR	0.44	0.0593	94.9	BSWOR	1.43	0.0609	94.6
BCWOR	0.60	0.1003	94.5	BCWOR	1.69	0.1016	94.5
Multiplicity for Median				Calibrated Multiplicity for Median			
	RB%	CV	CP		RB%	CV	CP
BSWR	5.50	0.1899	95.6	BSWR	4.63	0.1837	94.9
BCWR	5.44	0.1750	95.7	BCWR	4.70	0.1685	95.7
BSWOR	0.53	0.1753	95.0	BSWOR	-0.31	0.1726	94.5
BCWOR	0.56	0.1620	95.6	BCWOR	-0.09	0.1583	94.2
Setting B							
Multiplicity for Total				Calibrated Multiplicity for Total			
	RB%	CV	CP		RB%	CV	CP
BSWR	7.84	0.1000	96.2	BSWR	6.27	0.0890	95.6
BCWR	7.46	0.1296	96.3	BCWR	5.61	0.1204	95.2
BSWOR	2.23	0.0647	95.6	BSWOR	0.58	0.0618	95.2
BCWOR	2.50	0.1024	96.0	BCWOR	0.74	0.1003	94.9
Multiplicity for Median				Calibrated Multiplicity for Median			
	RB%	CV	CP		RB%	CV	CP
BSWR	9.60	0.2118	94.9	BSWR	5.01	0.1836	94.9
BCWR	8.89	0.1959	94.8	BCWR	4.67	0.1746	95.3
BSWOR	4.04	0.1881	94.1	BSWOR	-0.42	0.1733	94.0
BCWOR	3.46	0.1679	94.3	BCWOR	-0.60	0.1548	93.7

TABLE 4. Comparison of Variance Estimators

Setting C							
Multiplicity for Total				Calibrated Multiplicity for Total			
	RB%	CV	CP		RB%	CV	CP
BSWR	7.86	0.0993	96.7	BSWR	6.89	0.0941	96.0
BCWR	7.48	0.1330	96.8	BCWR	7.01	0.1273	95.5
BSWOR	2.14	0.0622	96.3	BSWOR	1.29	0.0594	95.7
BCWOR	1.98	0.1015	96.2	BCWOR	1.53	0.0992	95.3
Multiplicity for Median				Calibrated Multiplicity for Median			
	RB%	CV	CP		RB%	CV	CP
BSWR	6.79	0.1785	94.7	BSWR	2.58	0.1564	94.5
BCWR	5.85	0.1661	94.8	BCWR	1.73	0.1499	94.7
BSWOR	1.39	0.1578	93.8	BSWOR	-2.69	0.1522	93.8
BCWOR	0.90	0.1504	94.1	BCWOR	-2.97	0.1505	94.1
Setting D							
Multiplicity for Total				Calibrated Multiplicity for Total			
	RB%	CV	CP		RB%	CV	CP
BSWR	2.25	0.0815	95.8	BSWR	2.91	0.0866	96.2
BCWR	2.60	0.1042	95.6	BCWR	2.80	0.1040	96.5
BSWOR	-2.17	0.0779	95.3	BSWOR	-1.98	0.0779	95.7
BCWOR	-3.19	0.1020	94.6	BCWOR	-2.71	0.1032	95.3
Multiplicity for Median				Calibrated Multiplicity for Median			
	RB%	CV	CP		RB%	CV	CP
BSWR	4.09	0.1723	95.2	BSWR	4.07	0.1749	94.7
BCWR	3.97	0.1647	95.2	BCWR	3.87	0.1637	95.3
BSWOR	-0.77	0.1633	94.8	BSWOR	-0.71	0.1636	94.6
BCWOR	-1.48	0.1520	95.0	BCWOR	-1.54	0.1501	94.2

Table 5. Comparison of Variance Estimators

Setting E							
Multiplicity for Total				Calibrated Multiplicity for Total			
	RB%	CV	CP		RB%	CV	CP
BSWR	6.22	0.1035	94.2	BSWR	6.02	0.1002	94.3
BCWR	6.67	0.1316	94.1	BCWR	6.03	0.1248	94.0
BSWOR	0.82	0.0755	93.8	BSWOR	0.42	0.0748	93.7
BCWOR	1.00	0.1069	94.5	BCWOR	0.62	0.1037	93.7
Multiplicity for Median				Calibrated Multiplicity for Median			
	RB%	CV	CP		RB%	CV	CP
BSWR	5.86	0.1905	95.0	BSWR	4.87	0.1935	94.6
BCWR	5.21	0.1754	95.3	BCWR	3.95	0.1728	94.9
BSWOR	0.56	0.1711	94.6	BSWOR	-0.31	0.1766	93.8
BCWOR	0.15	0.1657	94.8	BCWOR	-0.73	0.1655	94.1
Setting F							
Multiplicity for Total				Calibrated Multiplicity for Total			
	RB%	CV	CP		RB%	CV	CP
BSWR	7.78	0.1141	95.9	BSWR	6.16	0.1021	94.9
BCWR	8.28	0.1357	95.7	BCWR	6.95	0.1267	95.5
BSWOR	2.54	0.0822	95.6	BSWOR	1.22	0.0766	94.3
BCWOR	2.49	0.1036	95.3	BCWOR	0.85	0.0982	93.9
Multiplicity for Median				Calibrated Multiplicity for Median			
	RB%	CV	CP		RB%	CV	CP
BSWR	7.67	0.1912	94.1	BSWR	7.43	0.1866	93.9
BCWR	7.06	0.1831	93.5	BCWR	6.72	0.1779	94.1
BSWOR	2.70	0.1716	93.4	BSWOR	2.25	0.1672	93.8
BCWOR	1.31	0.1624	92.9	BCWOR	1.01	0.1572	94.0

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