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Bandwidth Selection in Geographically Weighted Poisson Regression Model Using Firefly Optimization Algorithm with Application to Cancer Rate Data

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Abstract. Geographically Weighted Poisson Regression (GWPR) is an extension of the standard Poisson regression model designed to handle spatial count data by accounting for local associations among variables. However, the GWPR model faces several challenges that can affect its accuracy and reliability—one of the most critical being bandwidth selection. An inappropriate bandwidth may either overfit the model to noise or produce unrealistically low estimates. Specifically, a small bandwidth may capture excessive local variability, while a large bandwidth could smooth over meaningful local patterns. Meta-heuristic algorithms are optimization techniques designed to find approximate solutions to complex problems by efficiently exploring the solution space. The application of meta-heuristic algorithms for bandwidth selection in the GWPR model is relatively novel, as it introduces an optimization-based approach to this critical task. In this paper, the Firefly Algorithm (FA), a nature-inspired meta-heuristic method, is utilized to determine the optimal bandwidth value in the GWPR model. The FA algorithm searches for the bandwidth that minimizes prediction error, based on a defined objective function. Using cancer incidence data as a real-world case study, comparative analysis demonstrated that the proposed FA-based method outperforms traditional approaches in terms of pseudo-R^2 and Deviance metrics. The results suggest that employing meta-heuristic optimization—specifically the Firefly Algorithm—for bandwidth selection in GWPR models is a promising and effective strategy that enhances spatial modeling through the integration of advanced optimization techniques.

1. Introduction

Spatial data refers to observed phenomena that possess either an inherent or explicitly stated spatial reference. It includes both the content and entities of spatial information. Content data represents the actual observations tied to spatial locations, while change data refers to observations linked to a single spatial entity that vary over time. Spatial data objects can take the form of points, lines, areas, or surfaces to which content data are associated. Geographical modeling is a powerful

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analytical technique that provides deeper insights into spatial phenomena, particularly when the relationships between variables vary across different geographic locations [1, 2]. This approach enhances our understanding of how various processes or conditions change spatially. Among the most widely used tools in geographic modeling are spatial regression models, which are essential for analyzing spatially structured data where traditional regression assumptions (like independence) are violated due to spatial autocorrelation [3, 4]. Spatial regression models are widely applied in disciplines such as economics, urban and regional planning, environmental science, and public health [5–9]. The Geographically Weighted Regression (GWR) model is a spatial regression method appropriate when the response variable follows a normal distribution [10]. However, in many real-world applications—especially in the social, economic, and epidemiological domains—data take the form of counts, which are positive integers [11,12]. These types of data are often modeled using the Poisson distribution, which is well suited for handling count responses. The Poisson regression model links the count response variable and explanatory variables to estimate the underlying relationships. To address spatial variation in count data, the Geographically Weighted Poisson Regression (GWPR) model extends the Poisson regression framework by incorporating spatially varying coefficients, allowing it to capture local patterns more effectively [13, 14]. However, the performance of the GWPR model can be significantly influenced by the choice of bandwidth, a critical parameter that determines the degree of spatial smoothing. Improper bandwidth selection may lead to overfitting or oversmoothing, thereby reducing model accuracy and reliability. In this study, we propose the application of a nature-inspired meta-heuristic algorithm—the Firefly Algorithm (FA)—to optimize the bandwidth selection in the GWPR model. The FA is employed to search for the bandwidth value that minimizes prediction error, offering an efficient and adaptive solution to this challenging task. Through real-world cancer incidence data, we demonstrate that the FA-based bandwidth optimization method achieves superior predictive performance compared to traditional methods. These findings highlight the potential of combining advanced optimization techniques with spatial statistical modeling to improve the accuracy and applicability of spatial regression models.

2. The Description of GWPR Model

Count data are used frequently in many types of research fields from Epidemiology, social and economic investigations [15, 16]. The following type of data is positive integers. Poisson distribution is a familiar distribution that used in modeling such type of data. This permits the examination of Poisson regression (PR) model aimed at modeling the counts as the response variable and possibly the explanation variable.

Let y_i follows a Poisson distribution with ω_i, then

$$f\left(y_i\right) = \frac{e^{-\omega_i}{\omega_i}^{y_i}}{y_i!}, \quad y_i = 0, 1, \ldots ; \; i = 1, 2, \ldots, n. \tag{1}$$

In a PR model, $\ln(\omega_i) = \mathrm{x}_i^T \mathrm{fi}$ with $\mathrm{x}_i = (x_{i1}, ..., x_{ip})^T$and fi is a $(p+1) \times 1$ vector of unknown regression coefficients. According to this, the PR model can be as:

$$\begin{aligned} y_i &= \exp(\mathrm{x}_i^T \mathrm{fi}) \\ &= \exp(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_p x_{ip}) \end{aligned} \tag{2}$$

Using the maximum likelihood (ML) method for parameter estimation, the log-likelihood function is

$$\ell(\mathrm{fi}) = \sum_{i=1}^{n} \left\{ y_i \mathrm{x}_i^T \mathrm{fi} - \exp(\mathrm{x}_i^T \mathrm{fi}) - \ln y_i! \right\}. \tag{3}$$

The ML estimator of PR model coefficients, $\hat{\mathrm{fi}}_{PR}$ is

$$\hat{\mathrm{fi}}_{PR} = (\mathrm{X}^T \hat{\mathrm{Z}} \mathrm{X})^{-1} \mathrm{X}^T \hat{\mathrm{Z}} \hat{\mathrm{v}}, \tag{4}$$

where $\hat{\mathrm{Z}} = \mathrm{diag}(\hat{\omega}_i)$ and $\hat{\mathrm{v}}$ is a vector where i^{th} element equals to $\hat{v}_i = \ln(\hat{\omega}_i) + ((y_i - \hat{\omega}_i)/\hat{\omega}_i)$.

In practice, the relationships between variables might vary geographically. Unlike global regression (PR model), where the regression coefficients that arise in PR are fixed over space. GWPR model enables local variations in the estimation of coefficients [15–18]. In other words, the coefficients are estimated locally at spatial references data points using GWPR model. The GWR model is defined as [19,20]

$$\begin{aligned} y_{i,spatial} &= \exp(\mathrm{x}_i^T \mathrm{fi}(r_i, q_i)) \\ &= \exp(\beta_0(r_i, q_i) + \beta_1(r_i, q_i) x_{i1} + \beta_2(r_i, q_i) x_{i2} + ... + \beta_p(r_i, q_i) x_{ip}) \end{aligned} \tag{5}$$

where $\beta_j(r_i, q_i)$, $j = 1, 2, ..., p$ is the coefficients which are varying conditionals on the location and $(r_i, q_i)$is the two-dimensional coordinates of the i^{th} point in the geographical location. Thus, the spatial heterogeneity is handled by GWPR model in a way that permits by parameters to be location dependent, thus enables estimation of localized effects.

Based on locally weighted likelihood method which is maximizing the geographically weighted log-likelihood function, the estimated coefficient, $\hat{\mathrm{fi}}_{GWPR}$, at location i, can be obtained as

$$\hat{\mathrm{fi}}_{GWPR} = (\mathrm{X}^T \mathrm{W}(r_i, q_i) \mathrm{X})^{-1} \mathrm{X}^T \mathrm{W}(r_i, q_i) \mathrm{y}, \tag{6}$$

where $\mathrm{W}(r_i, q_i)$is an $n \times n$spatial weight matrix. Spatial weights are quantitative measures associated with observations to derive them based on the distance from the focal observation. They define how the observations within a close working range impact the auto regression characteristics of the prediction for the particular position. In GWPR model, these weights are obtained through a kernel function which describes the relative position between the data points. Several kernel functions available for weighting and were adopted for use in developing the GWPR model such as Gaussian, bi-square, box-car, tri-cube, exponential among them [21–24].

3. Bandwidth Selection

In general, Geographically Weighted Poisson Regression (GWPR) involves estimating a local regression equation at each spatial intersection, with support from observations at neighboring locations. Empirical evidence suggests that observations closer to the target intersection contribute more significantly to parameter estimation than those farther away. This influence decreases inversely with increasing distance. To capture the smoothed geographical variation in parameter estimates, the GWPR model employs a distance-based weighting scheme using a spatial kernel function [25].

The bandwidth of the kernel defines the extent to which weights are assigned. It can be specified in two ways: either fixed, based on a predefined distance, or adaptive, based on a specified number of nearest neighbors. The selection of bandwidth plays a crucial role in determining the performance of the GWPR model [26]. A small bandwidth, which includes only a few nearby observations, may result in unstable and noisy parameter estimates. Conversely, a large bandwidth may overly smooth the data, introducing bias and masking local variation [27].

The bandwidth essentially determines the neighborhood size considered in the weighting process for each observation. A smaller bandwidth is more sensitive to local changes, capturing fine-scale spatial heterogeneity. On the other hand, a larger bandwidth tends to generalize across space, potentially overlooking important local details. Adaptive bandwidth methods dynamically adjust the bandwidth based on the density of the data, offering a more nuanced and context-aware modeling approach [28, 29].

Bandwidth selection can be approached in two ways:

(1) Fixed bandwidth, where a constant value is applied uniformly across all observations.
(2) Adaptive bandwidth, where the bandwidth varies depending on data density, thus enabling the model to better capture spatial non-stationarity.

Several commonly used kernel functions are summarized in Table 1, each assigning weights to observations based on their Euclidean distance from the regression point being estimated. In addition to the choice of kernel function, determining an appropriate bandwidth value, which reflects the number of neighboring observations or spatial extent, is essential for the accuracy and reliability of GWPR modeling.

The idea behind estimating the bandwidth value in GWPR model is to determine the optimal extent of spatial influence that neighboring observations have on the regression estimates for a specific location. Bandwidth selection is crucial because it directly affects the model's ability to capture local variations in relationships between dependent and independent variables. Methods such as cross-validation (CV), generalized cross-validation (GCV), and information criteria like Akaike information criterion (AIC) or corrected AIC (CAIC) can help identify the optimal bandwidth, σ [29, 30].

Table 1. Kernel functions in GWPR model

Kernel	Mathematical form
Gaussian	$w_{ij} = \exp\left(-\frac{1}{2}\left(\frac{d_{ij}}{\sigma}\right)^2\right)$
Exponential	$w_{ij} = \exp\left(-\frac{\lvert d_{ij}\rvert}{\sigma}\right)$
Bi-square	$w_{ij} = \begin{cases} \left(1-\left(\frac{d_{ij}}{\sigma}\right)^2\right)^2 & if\lvert d_{ij}\rvert < \sigma, \\ 0 & otherwise. \end{cases}$
Tri-cube	$w_{ij} = \begin{cases} \left(1-\left(\frac{d_{ij}}{\sigma}\right)^3\right)^3 & if\lvert d_{ij}\rvert < \sigma, \\ 0 & otherwise. \end{cases}$
Box-car	$w_{ij} = \begin{cases} 1 & if\lvert d_{ij}\rvert < \sigma, \\ 0 & otherwise. \end{cases}$

Meta-heuristic algorithm are the refined methods of optimization employed to find good solutions to problems which are hard to solve conventionally. These algorithms are widely used where the solution space is large, non-linear, or not very well defined [31–33]. These algorithms are planned to come out of local optimal and aimed for global optima than local search hence more accurate than local searches. Further, the nature of these algorithms is that they can give good solutions at once especially in the search spaces of high dimensions, which can be hardly solved by the traditional optimization processes [34–36].

From this point, our proposed idea is to use meta-heuristic algorithms for estimating the bandwidth value in GWPR model which can offer a promising alternative to traditional methods. Through the application of these optimization techniques, our proposed idea is able to improve their probability of identifying optimal bandwidths that leads to accurate model representation and presentation. When the nature of spatial data analysis becomes more intricate, the implementation of complex optimization solutions to support modeling may be essential. In this paper, firefly optimization (FA) algorithm[36], which is swarm-based metaheuristic algorithm inspired from the behaviors of beluga whales, is employed to tune the optimal bandwidth value in the GWPR model.

The main components of the FA algorithm are:

1- Attractiveness-Based Movement: Fireflies are attracted to others with higher brightness (better fitness). The algorithm simulates this by moving less bright fireflies toward brighter ones in the search space. 2- Light Intensity, Attractiveness, and Randomness:

(1) Light Intensity (I): Represents the quality (fitness) of a solution. It decreases with distance due to absorption and is directly linked to the objective function value.

(2) Attractiveness (β): Determines how strongly one firefly is attracted to another. It is highest at close distances and decreases exponentially as distance increases.

(3) Randomization (α): Adds a stochastic component to each firefly's movement to ensure exploration and avoid local optima. The randomness can decrease over time for better convergence.

The following are the parameter combinations for our suggested methodology.

(1) The number of fireflies is 20 members and the number of iterations is t_{max}= 500.

(2) Every member's position is representing the bandwidth value of the kernel, σin Table 1 and it chosen at random. The members' starting positions are produced from a uniform distribution in the interval $[6, n]$where $n$represents the number of samples in the real data under the study.

(3) The definition of the fitness function is considered as the deviance criterion and it is defined as

$$\text{fitness} = \min D\left(y; \hat{y}\left(\hat{\text{fi}}_{GWPR}\right)\right) = 2\sum_{i=1}^{n}\left[y_i \log\left(\frac{y_i}{\hat{y}\left(\hat{\text{fi}}_{GWPR}\right)}\right) - y_i + \hat{y}\left(\hat{\text{fi}}_{GWPR}\right)\right], \quad (7)$$

(4) The best bandwidth value is obtaining after updating the positions according the FA algorithm until $t_{\max}$ is reached.

4. Evaluation Criteria

To compare and evaluate the performance of our proposed method, FA-GWPR, against other approaches, two model evaluation criteria were employed: pseudo-R^2 and Deviance. These metrics are defined as follows, respectively,

$$\text{pseudo} - R^2 = 1 - \frac{D\left(y, \hat{y}\left(\hat{\text{fi}}_{GWPR}\right)\right)}{D\left(y, \hat{y}\left(\hat{\text{fi}}_0\right)\right)} \quad (8)$$

$$\text{Deviance} = 2\sum_{i=1}^{n}\left[y_i \log\left(\frac{y_i}{\hat{y}\left(\hat{\text{fi}}_{GWPR}\right)}\right) - y_i + \hat{y}\left(\hat{\text{fi}}_{GWPR}\right)\right] \quad (9)$$

where $D\left(y, \hat{y}\left(\hat{\text{fi}}_{GWPR}\right)\right)$ is the deviance of the fitted GWPR model and $D\left(y, \hat{y}\left(\hat{\text{fi}}_0\right)\right)$ is the deviance of the Intercept-only model. The best value of the bandwidth would be the one with the highest value of $\text{pseudo} - R^2$and the lowest values of the Deviance.

5. Data description

A year frame data, 2022, were collected from the 18 Iraqi provinces. The datasets for this study were obtained from Authority of Statistics and Geographic Information System, Iraq (https://cosit.gov.iq/ar/). The data included nine types of information in each individual provinces: cancer rate (average per (10000) persons), as count data, representing the response variable. Unemployment rate (X1), Urbanization rate (X2), PM2.5 (X3), NO2 (X4), SO2 (X5), O3 (X6), CO (X7), and CH4 (X8). Variables X1 to X8 represent the explanatory variables. In Figure 1, the cancer rate of 18 Iraqi provinces is reported. The geographical pattern of the cancer rate suggests differences between northern part and the southern part of the provinces.

6. Results and Discussion

First, the Kolmogorov Smirnov (KS) test was applied in this study to assess the goodness of fit of the response variable to the Poisson distribution. The test yielded a statistic of 7.486 with a p-value of 0.80128, indicating that the Poisson distribution is an appropriate and well-fitting model for the response variable, which in this case is the cancer rate. Table 2 presents the coefficients of the Poisson Regression (PR) model, which serves as the global model in this analysis. According to the results, the variables Urbanization rate (X2), PM2.5 (X3), CO (X7), and CH4 (X8) had a statistically significant effect on the cancer rate. Specifically, the association between cancer rate and Urbanization rate (X2), PM2.5 (X3), and CH4 (X8) was positive, suggesting that increases in these variables are associated with an increased likelihood of cancer incidence. Conversely, CO (X7) exhibited a negative association with the cancer rate, indicating that decreases in CO levels are linked to increases in cancer rate. Meanwhile, variables such as Unemployment rate (X1), NO2 (X4), SO2 (X5), and O3 (X6) were found to have no statistically significant association with the cancer rate.

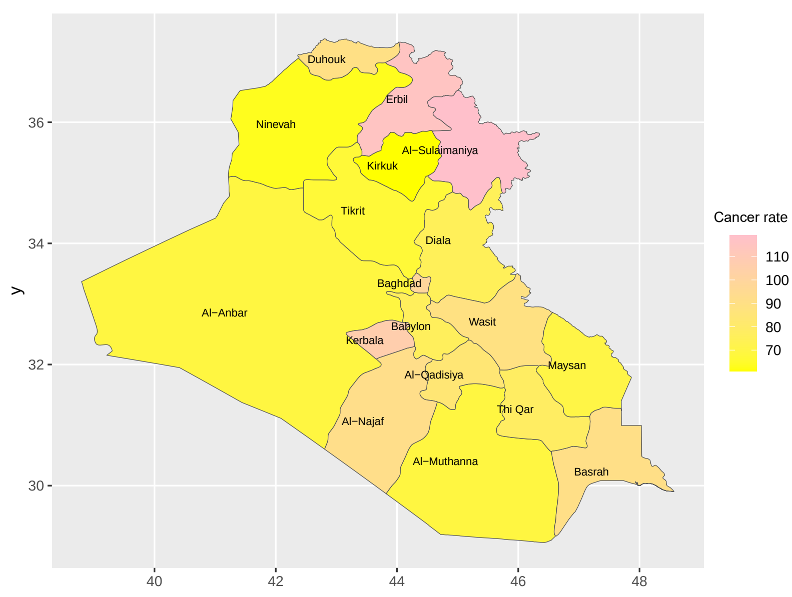

Figure 1. The spatial distribution of the cancer rate of 18 Iraqi provinces under study.

Second, to examine local variations in the relationship between the dependent variable and the predictors across the 18 locations within the study area, a spatial heterogeneity test was performed. Specifically, the Breusch–Pagan (BP) test was used to determine whether the variance of residuals

Table 2. PR model estimation

parameter	estimation	Std. error	t-value	p-value
Intercept	2.4689	0.5641	4.376	**0.0001**
X1	-0.0011	0.0083	-0.127	0.8989
X2	0.0079	0.0021	3.847	**0.0001**
X3	0.0080	0.0037	2.163	**0.0001**
X4	-0.9336	4.0006	-0.233	0.8154
X5	4.4307	3.7468	1.183	0.2370
X6	5.2732	3.8420	1.373	0.1699
X7	-0.9329	0.4526	-2.061	**0.0001**
X8	0.6388	0.2813	2.270	**0.0002**

is homoscedastic (constant) or heteroscedastic (varies) across locations. The null hypothesis states that the variances are equal across all locations, while the alternative hypothesis suggests that variance differs in one or more locations. Rejection of the null hypothesis indicates the presence of significant spatial heterogeneity. The results revealed a BP test statistic of 16.059 with a p-value of 0.00251, which is below the 0.05 threshold. This indicates significant spatial diversity among the 18 locations in the study area. To account for and model this spatial heterogeneity, the Geographically Weighted Poisson Regression (GWPR) model, a local modeling approach, was employed to explore spatially varying relationships between cancer rate and the eight explanatory variables.

Using the bi-square kernel weighting function, the GWPR model parameters were estimated based on four bandwidth selection methods: Cross-Validation (CV), Generalized Cross-Validation (GCV), Akaike Information Criterion (AIC), and our proposed optimization-based method, FA-GWPR. The results of these estimations are summarized in Tables 3 to 6, and are presented using five descriptive statistics: minimum (Min), first quartile (Q1), median (Med), third quartile (Q3), and maximum (Max) values. Additionally, Table 7 presents the evaluation criteria results along with the corresponding optimal bandwidth values.

We will first make two general observations from Tables 3 – 6 : (1) For the five statistics indicators of Min, Q1, Med, Q3, and Max, as to the direction (positive or negative correlation between cancer rate and each of the explanatory variables of FA-GWPR), the sign of the correlations with the corresponding ones of AIC, CV, and GCV methods are the same. For example, the parameters of PM2.5 (X3) in AIC, CV, and GCV are all positive. (2) The varying parameters of each significant variables in AIC, CV, and GCV always fall into the range of corresponding counterparts in FA-GWPR.

Table 3. Summary of GWPR parameters for AIC method

parameter	Min	Q1	Med	Q3	Max
Intercept	1.9957	2.3020	2.4390	3.3099	3.5212
X1	-0.0001	0.0035	0.0052	0.0101	0.0105
X2	0.0067	0.0069	0.0080	0.0087	0.0089
X3	0.0061	0.0086	0.0157	0.0179	0.0181
X4	-4.4812	-4.0111	-2.4221	-2.0230	-0.8564
X5	-1.4767	-1.1745	1.0510570	5.6959049	7.1754
X6	5.3699	5.9835	8.9095	10.2804	10.5664
X7	-1.2604	-1.1007	-0.9852	-0.9464	-0.7524
X8	-0.2115	-0.0846	0.4801	0.6683	0.8184

Table 4. Summary of GWPR parameters for CV method

parameter	Min	Q1	Med	Q3	Max
Intercept	1.5176	2.2307	2.6719	3.3554	3.5354
X1	0.0017	0.0074	0.0087	0.0101	0.0105
X2	0.0064	0.0069	0.0074	0.0088	0.0092
X3	0.0042	0.0079	0.0171	0.0179	0.0183
X4	-4.6813	-4.0229	-3.6008	-2.8366	-0.2338
X5	-1.8386	-1.2337	0.4256	8.2163	10.0032
X6	5.5287	7.3340	10.0703	10.2899	10.6670
X7	-1.2929	-1.1032	-1.0252	-0.9752	-0.6934
X8	-0.2192	0.1111	0.2694	0.6708	1.0107

Table 5. Summary of GWPR parameters for GCV method

parameter	Min	Q1	Med	Q3	Max
Intercept	1.4861	2.2081	2.9839	3.5412	3.8595
X1	0.0026	0.0074	0.0087	0.0099	0.0126
X2	0.0056	0.0065	0.0071	0.0088	0.0093
X3	0.0040	0.0084	0.0174	0.0179	0.0194
X4	-6.8642	-4.5407	-3.8301	-2.6757	-0.1301
X5	-2.7364	-2.5067	-0.4729	8.2856	10.3894
X6	5.1972	7.6457	9.5011	10.2436	10.7793
X7	-1.3083	-1.0795	-0.9803	-0.9447	-0.8651
X8	-0.3666	-0.2138	0.1067	0.6904	1.0251

Table 6. Summary of GWPR parameters for FA-GWPR method

parameter	Min	Q1	Med	Q3	Max
Intercept	1.3951	2.1922	3.0453	3.5400	4.7471
X1	0.0003	0.0070	0.0103	0.0117	0.0207
X2	0.0038	0.0053	0.0063	0.0089	0.0095
X3	0.0036	0.0093	0.0151	0.0201	0.0235
X4	-7.1094	-5.3747	-4.0124	-2.4889	0.1521
X5	-4.50461	-3.6698	-0.6544	8.2734	11.0500
X6	4.77963	6.4165	10.4743	11.2640	15.6820
X7	-1.34581	-1.1796	-1.0047	-0.9245	-0.7275
X8	-0.71641	-0.2523	-0.0138	0.7044	1.0654

Regarding GWPR model performance, the results in Table 7 show that both deviance and pseudo-R^2 obtained by our proposed method, FA-GWPR, is more accurate indicated that FA-GWPR had the highest pseudo-R^2 and least Deviance compared with AIC, CV, and GCV methods. The Deviance in the FA-GWPR is reduced by 51.27%, 32.22%, and 19.08% respectively as compared to the AIC, CV, and GCV. The results indicate that the GWPR model using our proposed method, FA-GWPR produces more accurate predictions for cancer rate in individual Iraqi provinces than those the AIC, CV, and GCV by capturing the spatial heterogeneity in the data.

Table 7. Summary of evaluation criteria and the best bandwidth for used methods

Methods	pseudo-R^2	Deviance	best bandwidth
AIC	0.9042	6.1676	18
CV	0.934	4.3845	17
GCV	0.9441	3.783	16
FA-GWPR	0.9562	3.0882	15

In Figure 2, following the use of FA-GWPR to choose the bandwidth, the map result displaying the spatial distribution of the predicted cancer rate in the GWPR model is provided. From figure 2 it can be noticed that the distribution of the predicted cancer rate as similar to the distribution and the real cancer rate as shown Figure 1.

The distributions of the parameters of the significant explanatory variables, Urbanization rate (X2), PM2.5 (X3), CO (X7), and CH4 (X8) over the 18 provinces of are shown in Figures 3 – 6. The parameters indicate fairly conspicuous trends in terms of spatial differentiation. The maps also show that the four parameter estimates using AIC, CV, GCV, and FA-GWPR are not equal to their corresponding location.

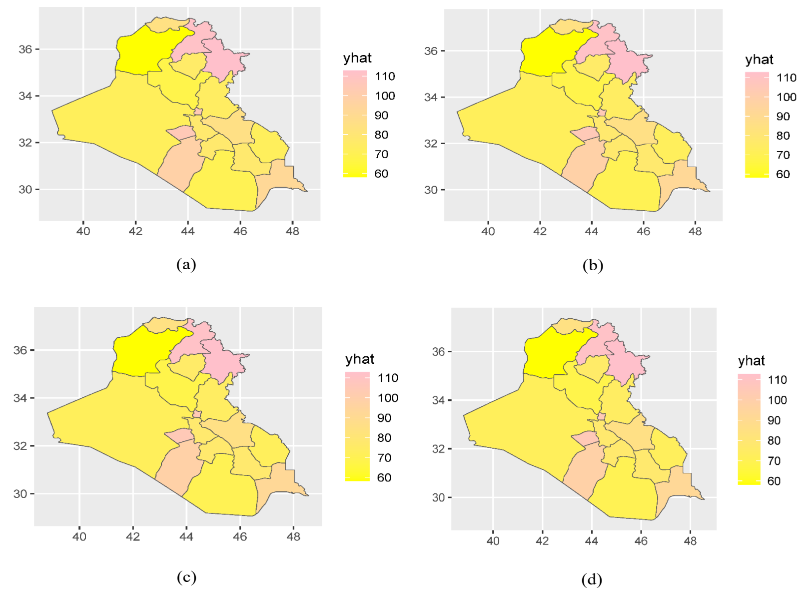

Figure 2. The cancer rate prediction (a) AIC, (b) CV, (c) GCV, and (d) FA-GWPR.

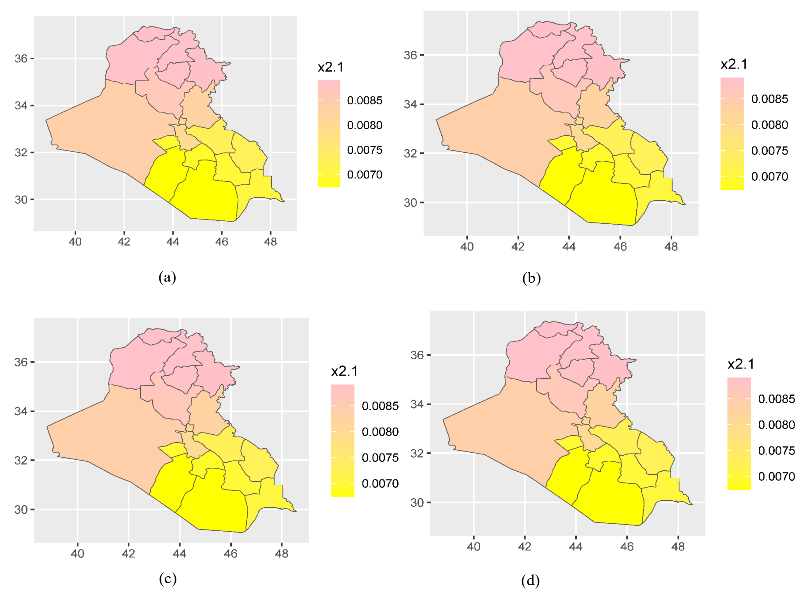

Figure 3. The significant Urbanization rate (X2) parameter estimates (a) AIC, (b) CV, (c) GCV, and (d) FA-GWPR.

7. Conclusion

The Geographically Weighted Poisson Regression (GWPR) model is a specialized statistical technique designed for analyzing spatially varying count data, capturing the localized relationships between variables across different geographic locations. A critical factor in the accuracy and reliability of the GWPR model is the selection of the bandwidth. If chosen poorly, the model

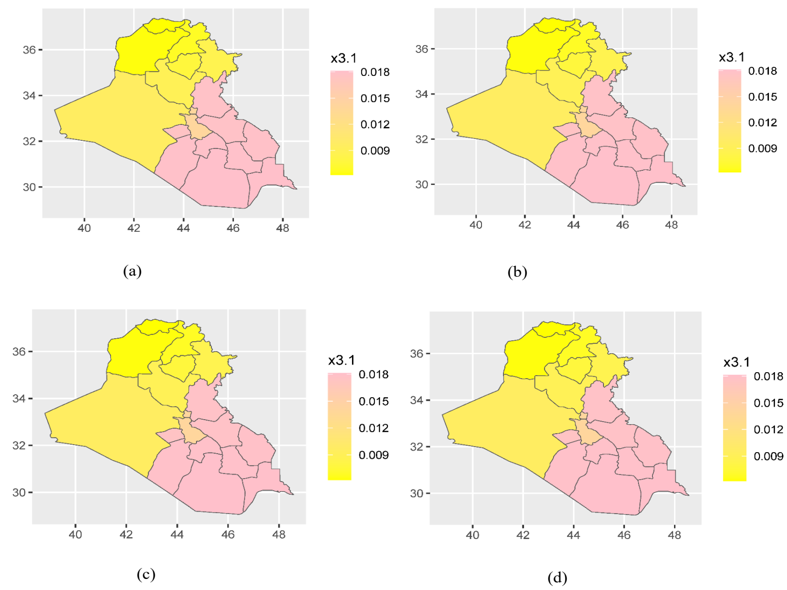

FIGURE 4. The significant PM2.5 (X3) parameter estimates (a) AIC, (b) CV, (c) GCV, and (d) FA-GWPR.

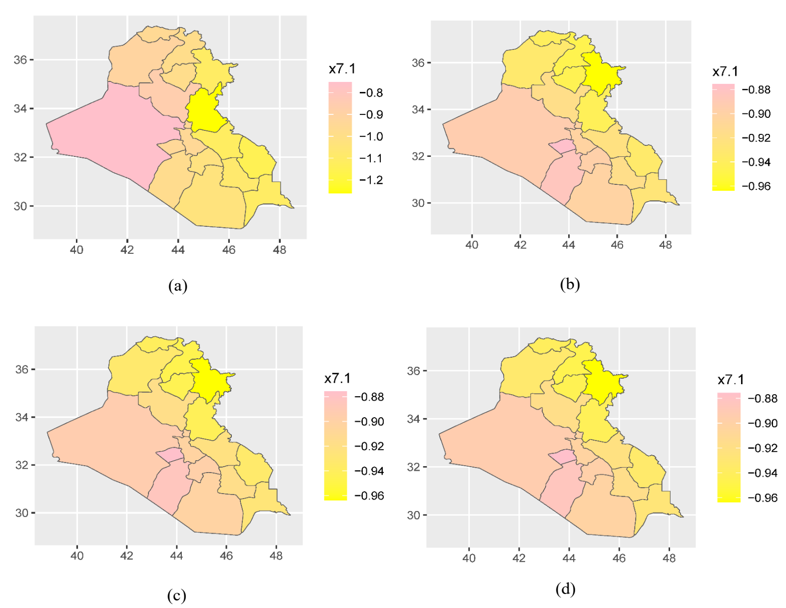

FIGURE 5. The significant CO (X7) parameter estimates (a) AIC, (b) CV, (c) GCV, and (d) FA-GWPR.

may either overfit the data by capturing noise or underfit by missing essential spatial patterns. This study introduced the use of a meta-heuristic optimization algorithm, specifically the Firefly Algorithm (FA), to determine the optimal bandwidth in the GWPR model. The proposed method, termed FA-GWPR, provides a promising alternative to traditional bandwidth selection techniques such as AIC, CV, and GCV. Through a real-world application focused on cancer rate estimation, the

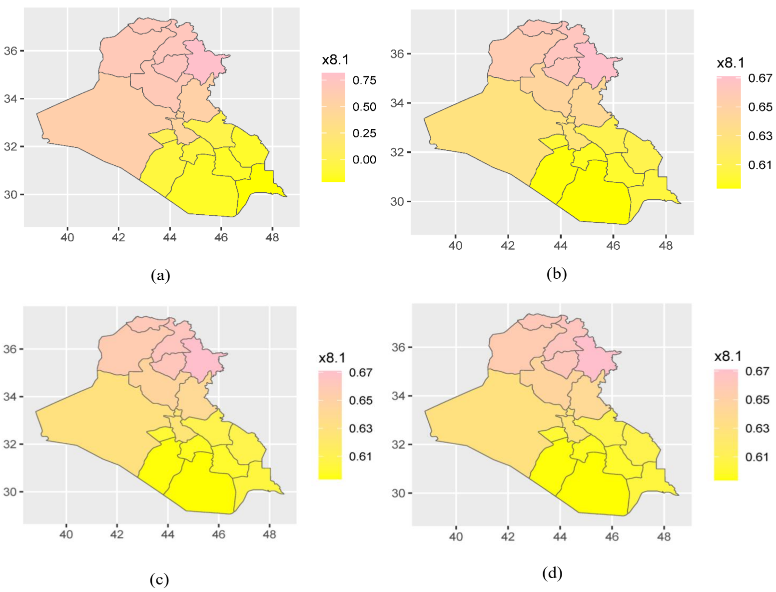

Figure 6. The significant CH4 (X8) parameter estimates (a) AIC, (b) CV, (c) GCV, and (d) FA-GWPR.

comparison and evaluation results demonstrated that FA-GWPR achieved superior performance in terms of pseudo-R^2 and Deviance metrics. Furthermore, the parameter estimates for significant variables obtained using AIC, CV, and GCV were consistently contained within the range of those estimated by FA-GWPR, highlighting the robustness and flexibility of the proposed method.

Competing interests: The authors declare that there is no conflict of interest regarding the publication of this paper.

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