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Development and Exploration of Cosine Rayleigh Distribution

Sule Omeiza Bashiru[1,*], Alhaji Modu Isa[2], Ahmed M. Gemeay[3]

[1]*Department of Mathematics and Statistics, Confluence University of Science and Technology, Osara, Kogi State, Nigeria*
bash0140@gmail.com
[2]*Department of Mathematics and Computer Science, Borno State University, Maiduguri, Nigeria*
alhajimoduisa@bosu.edu.ng
[3]*Department of Mathematics, Faculty of Science, Tanta University, Tanta 31527, Egypt*
ahmed.gemeay@science.tanta.edu.eg
[*]*Correspondence: bash0140@gmail.com*

Abstract. This study introduces a novel trigonometric extension of the Rayleigh distribution, the Cosine Rayleigh (CR) distribution. This new distribution is formulated by compounding the Rayleigh distribution with the Cosine G family of distributions. We derive the statistical properties of the CR distribution, including moments, hazard function, survival function, entropy measure, and order statistics. To estimate the parameter of the CR distribution, we employed sixteen different techniques, such as maximum likelihood, Anderson-Darling, Cramer-von Mises, maximum product of spacings, least squares, percentile, right-tail Anderson-Darling, weighted least squares, left-tail Anderson-Darling, minimum spacing absolute distance, minimum spacing absolute-log distance, Anderson-Darling left-tail second order, Kolmogorov, minimum spacing square distance, minimum spacing square-log distance, and minimum spacing Linex distance. A simulation study was conducted to evaluate the performance of the sixteen estimation methods. The results showed that all sixteen estimators produced consistent parameter estimates, with the maximum likelihood method emerging as the best in terms of accuracy. The practical utility of the CR distribution was demonstrated by fitting it to two real-life data sets and comparing its goodness-of-fit with other competing models, including the baseline Rayleigh distribution. The results indicated that the CR model provides a superior fit, highlighting its potential applicability in various fields involving lifetime data.

1. Introduction

Statistical distributions are commonly used to model datasets across various fields, including engineering, ecological studies, environmental science, actuarial science, and finance. However, traditional distributions often fail to adequately fit datasets that exhibit non-normal features. To

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address these limitations, researchers have developed novel families of distributions that modify classical ones. Introducing these generalized families aims to enhance the flexibility of classical distributions in modeling real-life datasets by incorporating additional features and improving their adaptability to a wider range of data characteristics.

In recent years, a surge in new families of distributions has been introduced in the literature. Some notable recent contributions include [1]- [9]. This study focuses on a trigonometric family called the Cosine Generalized family of distributions, introduced by [10]. This family's choice is motivated by its relative novelty in the field of statistics and reliability analysis and our interest in exploring its properties. The family enhances the flexibility of classical distributions without adding extra parameters, which simplifies property derivation and parameter estimation. It generally provides excellent fits for a variety of data types.

The cumulative distribution function (CDF) and the corresponding probability density function (PDF) of the Cosine G family of distributions are given in (1) and (2) below:

$$F(x;\xi) = 1 - \cos\left\{\frac{\pi}{2}H(x)\right\}, \tag{1}$$

$$f(x;\xi) = \frac{\pi}{2}h(x)\sin\left\{\frac{\pi}{2}H(x)\right\}. \tag{2}$$

This study focuses on the classical Rayleigh distribution as the baseline distribution. Proposed by [11], this continuous distribution is based on the amplitude of sound resulting from different significant sources. The Rayleigh distribution has been applied in several areas, including life-testing experiments, reliability analysis, applied statistics, and clinical studies. It is also a special case of the two-parameter Weibull distribution when the shape parameter is 2. The history and other useful features of the Rayleigh model can be found in [12]- [14]. The CDF and PDF of the one-parameter Rayleigh distribution are given in (3) and (4) below:

$$H(x;\sigma) = 1 - \exp\left(-\frac{x}{2\sigma^2}\right), \ x > 0, \sigma > 0. \tag{3}$$

and the corresponding PDF is given as:

$$h(x;\sigma) = \frac{x}{\sigma^2}\exp\left(-\frac{x}{2\sigma^2}\right), \ x > 0, \sigma > 0. \tag{4}$$

Several extensions of the Rayleigh distribution exist in the literature, including those cited in references [15]– [21]. However, to the best of our knowledge, the use of the Cosine Generalized family of distributions to extend the Rayleigh distribution remains unexplored despite recent advances in this area. Additionally, previous studies on Rayleigh distribution extensions have not employed a variety of parameter estimation methods, potentially limiting the robustness and accuracy of their findings. This paper aims to address these gaps by introducing a new extension of the Rayleigh distribution using the Cosine Generalized family and by introducing and comparing various parameter estimation methods that have not been utilized in earlier studies. The methods employed for parameter estimation include maximum likelihood, Anderson-Darling, Cramer-von Mises, maximum product of spacings, least squares, percentile, right-tail Anderson-Darling, weighted least squares, left-tail Anderson-Darling, minimum spacing absolute distance, minimum spacing absolute-log distance, Anderson-Darling left-tail second order, Kolmogorov, minimum spacing square distance, minimum spacing square-log distance, and minimum spacing Linex distance. We aim to provide researchers with a robust tool for analyzing and modeling data that do not conform to traditional probability distributions.

Our motivation for this study arises from the limitations of the Rayleigh distribution in modeling tail and peak behaviors, challenges that previous extensions have not fully addressed. To overcome these limitations, we propose a new, flexible extension within the Cosine Generalized family to better capture the complexities of real-world datasets. Additionally, we will evaluate various parameter estimation methods to enhance the accuracy and reliability of statistical analyses for non-traditional datasets.

The contributions of this study are summarized as follows:

(i) We introduce a new generalization of the Rayleigh distribution based on the Cosine Generalized family of distributions, called the Cosine Rayleigh (CR) distribution.
(ii) We discuss several statistical properties of the CR distribution, as outlined in Section 3.
(iii) We estimate the parameter of the model using sixteen different estimation techniques, as detailed in Section 4.
(iv) We investigate the behavior of parameter estimates using the sixteen estimation techniques through simulation, as detailed in Section 5.
(v) We apply this model to actual datasets in the fields of engineering and meteorology, demonstrating its practical utility and effectiveness in real-world scenarios, as illustrated in Section 6.

The content of the article is as follows: Section 2 presents the development of the CL distribution. Section 3 elaborates on the principal statistical characteristics of this model. The examination of sixteen estimation techniques is detailed in Section 4. . Simulation studies evaluating the consistency of the estimators are presented in Section 5. In Section 6, the proposed distribution is applied to two real datasets. Finally, Section 7 offers concluding remarks based on the findings of the study.

2. Model Development

In this section, we construct the CDF and PDF of the CR distribution and describe the density expansion.

2.1. Construction of the CDF and PDF of the CR distribution.

By substituting (3) into (1), the CDF of the CR distribution is given as:

$$F\left(x\right) = 1-\cos\left\{\frac{ß}{2}\left[1-\exp\left(-\frac{x}{2œ^{2}}\right)\right]\right\}. \tag{5}$$

with the corresponding PDF obtained by setting (3) and (4) into (2):

$$f\left(x\right) = \frac{\pi}{2}\frac{x}{\sigma^{2}}\exp\left(-\frac{x}{2\sigma^{2}}\right)\ \sin\left\{\frac{\pi}{2}\left[1-\exp\left(-\frac{x}{2\sigma^{2}}\right)\right]\right\}, x>0, \sigma>0, \tag{6}$$

where σ is a scale parameter.

The PDF and CDF plots of the CR model are shown in Figure 1 below.

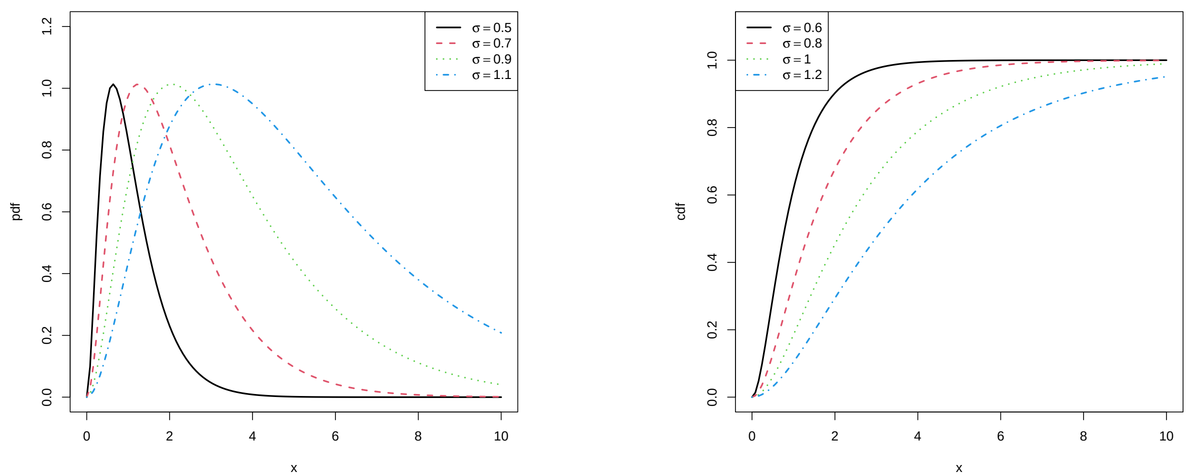

Figure 1. PDF and CDF Plots of the CR distribution

Observing Figure 1, the PDF plot of the CL distribution exhibits a right-skewed shape. Additionally, the CDF plot demonstrates convergence to one, with probability values ranging from zero to one. This convergence indicates that the novel CL distribution is a valid probability distribution.

2.2. **Important Representation.** The PDF and the CDF of the CR distribution can be expanded using Taylors Series expansion and binomial expansion as follows;

$$\cos\left(x\right) = \sum_{i=0}^{\infty}\frac{(-1)^{i}}{(2i)!}\ x^{2i}, \quad \sin\left(x\right) = \sum_{i=0}^{\infty}\frac{(-1)^{i}}{(2i+1)!}\ x^{2i+1}, \quad (1-x)^{n} = \sum_{i=1}^{\infty}(-1)^{i}\ \binom{n}{i}\ x^{i}.$$

Therefore,

$$\cos\left\{\frac{\pi}{2}\left[1-\exp\left(-\frac{x}{2\sigma^{2}}\right)\right]\right\} = \sum_{i=0}^{\infty}\frac{(-1)^{i}}{(2i+1)!}\frac{\pi^{2i+1}}{2^{2i+1}}\ \left[1-\exp\left(-\frac{x}{2\sigma^{2}}\right)\right]^{2i+1},$$

Using the binomial expansion, we get:

$$\left[1-\exp\left(-\frac{x}{2\sigma^{2}}\right)\right]^{2i+1} = \sum_{j=1}^{\infty}(-1)^{j}\ \binom{2i+1}{j}\left(\exp\left(\frac{x}{2\sigma^{2}}\right)\right)^{j},$$

$$f(x)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^{i+j}}{\sigma^2(2i+1)!}\,\frac{\pi^{2i+2}}{2^{2i+2}}\binom{2i+1}{j}\;x\;\left(\exp\left(\frac{x}{2\sigma^2}\right)\right)^{j+1}.$$

Therefore, the PDF of the CR distribution is given by:

$$f(x)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\aleph_{i,j}\;x\;\left[\exp\left(\frac{x}{2\sigma^2}\right)\right]^{j+1}, \tag{7}$$

where

$$\aleph_{i,j}=\frac{(-1)^{i+j}}{\sigma^2(2i+1)!}\,\frac{\pi^{2i+2}}{2^{2i+2}}\binom{2i+1}{j}.$$

The CDF can also be expanded as follows using the Taylor series:

$$1-\cos\left\{\frac{\pi}{2}\left[1-\exp\left(-\frac{x}{2\sigma^2}\right)\right]\right\}=1-\sum_{p=0}^{\infty}\frac{(-1)^p}{(2p)!}\frac{\pi^{2p}}{2^{2p}}\left[1-\exp\left(-\frac{x}{2\sigma^2}\right)\right]^{2p}.$$

Using binomial expansion:

$$\left[1-\exp\left(-\frac{x}{2\sigma^2}\right)\right]^{2p}=\sum_{q=0}^{\infty}(-1)^q\binom{2p}{q}\left[\exp\left(-\frac{x}{2\sigma^2}\right)\right]^q,$$

$$F(x)=1-\sum_{p=0}^{\infty}\frac{(-1)^{p+q}}{(2p)!}\frac{\pi^{2p}}{2^{2p}}\binom{2p}{q}\left[\exp\left(-\frac{x}{2\sigma^2}\right)\right]^q.$$

Therefore, the CDF in (5) can be written as:

$$F(x)=1-\sum_{p=0}^{\infty}\Upsilon_{p,q}\left[\exp\left(-\frac{x}{2\sigma^2}\right)\right]^q, \tag{8}$$

where

$$\Upsilon_{p,q}=\frac{(-1)^{p+q}}{(2p)!}\frac{\pi^{2p}}{2^{2p}}\binom{2p}{q}.$$

3. Statistical Properties

In this section, we will delve into the statistical properties of the CR distribution, including the survival function $S(x)$, hazard function $K(x)$, reverse hazard function $r(x)$, cumulative hazard function $R(x)$, quantile function $Q(u)$, moments, moment generating function, order statistic, and entropy.

3.1. **Survival Function.** The survival function represents the probability that the event of interest (e.g., failure, death, etc.) has not occurred by a specific time point.

The survival function of the CR distribution is given as:

$$S(x)=\cos\left\{\frac{\pi}{2}\left[1-\exp\left(-\frac{x}{2\sigma^2}\right)\right]\right\}. \tag{9}$$

3.2. **Hazard Function.** The hazard function represents the instantaneous rate at which an event occurs at a specific time, given that it has not occurred before that time. It is a fundamental concept in survival analysis and provides insight into the conditional likelihood of the event occurring in the next instant, assuming the individual or subject has survived up to that point.

The hazard function of the CR distribution is given as:

$$K(x) = \frac{\pi}{2}\frac{x}{\sigma^2}\exp\left(-\frac{x}{2\sigma^2}\right)\tan\left\{\frac{\pi}{2}\left[1-\exp\left(-\frac{x}{2\sigma^2}\right)\right]\right\}. \tag{10}$$

The hazard function plot of the CR distribution is shown in Figure 2 below.

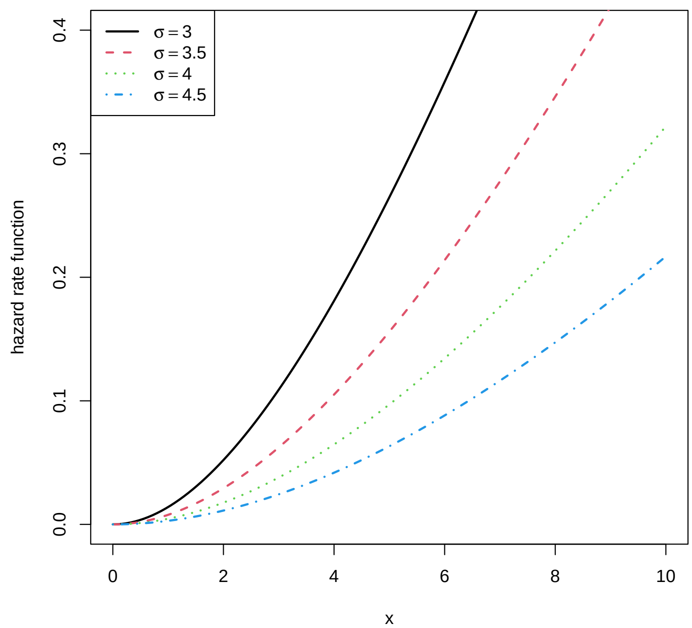

Figure 2. Hazard function plot of the CR distribution

3.3. **Reverse Hazard Function.** The reverse hazard function of the CR distribution is given as:

$$r(x) = \frac{\frac{\pi}{2}\frac{x}{\sigma^2}\exp\left(-\frac{x}{2\sigma^2}\right)\sin\left\{\frac{\pi}{2}\left[1-\exp\left(-\frac{x}{2\sigma^2}\right)\right]\right\}}{1-\cos\left\{\frac{\pi}{2}\left[1-\exp\left(-\frac{x}{2\sigma^2}\right)\right]\right\}}. \tag{11}$$

3.4. **Cumulative Hazard Function.** The cumulative hazard function of the CR distribution is given as:

$$R(x) = -\ln\left\{1-\cos\left[\frac{\pi}{2}\left(1-\exp\left(-\frac{x}{2\sigma^2}\right)\right)\right]\right\}. \tag{12}$$

3.5. **Quantile Function.** The quantile function, which is referred to as the inverse of the CDF, is a fundamental concept in statistics. For the CR distribution, the quantile function is given by:

$$Q(u) = -2\sigma^2\left[\ln\left(1-\frac{\cos^{-1}(1-u)}{\frac{\pi}{2}}\right)\right]. \tag{13}$$

3.6. **Moments.** A moment is a quantitative measure that provides insights into the characteristics of a probability distribution, including its central tendency, dispersion, skewness, and kurtosis. The moment is generally defined as:

$$\mu_r = \int_{-\infty}^{\infty} x^r f(x)\, dx,$$

using the important representation of the PDF in (7), we have:

$$\mu_r = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \aleph_{i,j} \int_{-\infty}^{\infty} x^{r+1} \left[\exp\left(\frac{x}{2\sigma^2}\right)\right]^{j+1} dx,$$

Therefore, the moment of the CR distribution is given as:

$$\mu_r = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \aleph_{i,j} \, \left(2\sigma^2\right)^{r+1} \frac{\Gamma(r+2)}{(j+1)^{r+2}}. \tag{14}$$

By utilizing the r^{th} raw moment of the CR distribution in (14), setting $r = 1$ yields the mean. The variance, skewness, and kurtosis are then computed using appropriate combinations of the first four raw moments.

3.7. **Moment Generating Function.** The moment generating function (MGF) of a random variable is the expected value of the exponential of that variable. It is expressed as:

$$M_x(t) = \int_{-\infty}^{\infty} \exp(tx) \;\, f(x)\, dx \quad ,$$

Utilizing the PDF representation in (7), we have

$$M_x(t) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \aleph_{i,j} \int_{0}^{\infty} x \;\, \exp(tx) \;\, \left[\exp\left(\frac{x}{2\sigma^2}\right)\right]^{j+1} dx \quad ,$$

Hence, the MGF of the CR distribution is given as:

$$M_x(t) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \aleph_{i,j} \, \frac{1}{\left(t + \frac{1}{2\sigma^2}\right)(j+1)}. \tag{15}$$

3.8. **Entropy.** Entropy serves as a measure of information or uncertainty in a random observation from a population. A higher entropy value corresponds to greater uncertainty in the data. The Renyi entropy of a continuous random variable X is defined as follows:

$$I_\theta(x) = \frac{1}{1-\theta} \log\left[\int_0^{\infty} f(x)^{\theta}\, d\, x\right],$$

The Renyi entropy of the CR model is given as:

$$I_\theta(x) = \frac{1}{1-\theta} \log\left[\int_0^{\infty} \left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \aleph_{i,j}\, x \left[\exp\left(\frac{x}{2\sigma^2}\right)\right]^{j+1}\right)^{\theta} d\, x\right], \tag{16}$$

where $\theta > 0$ and $\theta \neq 1$.

3.9. **Order Statistic.** Let $x_1, x_2, ...x_n$ be n independent random variable from the CR distribution and let $x_{(1)}, x_{(2)}, ...x_{(n)}$ be their corresponding order satistics. The i^{th} order statistics is given as:

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-1)!}[F(x)]^{i-1}[1-F(x)]^{n-i}f(x),$$

The order statistics of the CR distribution is given as:

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!}\left[1-\sum_{p=0}^{\infty}\gamma_{p,q}\left(\exp\left(-\frac{x}{2\sigma^2}\right)\right)^q\right]^{i-1}\left[\sum_{p=0}^{\infty}\gamma_{p,q}\left(\exp\left(-\frac{x}{2\sigma^2}\right)\right)^q\right]^{n-i} \times\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\aleph_{i,j}\, x\left(\exp\left(\frac{x}{2\sigma^2}\right)\right)^{j+1}. \tag{17}$$

4. Estimation Methods

This section discusses the 16 methods of estimation that were employed for estimating the parameter of the CR distribution.

4.1. **Method of maximum likelihood.** The purpose of parameter estimation using the Maximum Likelihood Estimation (MLE) method is to determine the parameter values of a theoretical distribution that maximize the likelihood function. MLE is chosen due to its desirable properties, such as efficiency, consistency, and asymptotic unbiasedness, particularly as the sample size increases. This approach involves adjusting the parameter values to maximize the probability of observing the given data under the assumed distribution.

The likelihood function $L(\sigma)$ is given by:

$$L(\sigma) = \prod_{i=1}^{n} f(x_i \mid \sigma)$$

The log-likelihood function$l(\sigma)$ is given by:

$$\ell(\sigma) = \sum_{i=1}^{n} \log f(x_i \mid \sigma)$$

The likelihood function and the log-likelihood function are both dependent on the parameter σ and the observed data x_i. The MLE $\hat{\sigma}$ of the parameter σ for the CR distribution is found by solving the equation:

$$\frac{\partial \ell(\sigma)}{\partial \sigma} = 0.$$

4.2. **Method of Anderson-Darling.** The Anderson-Darling estimation (ADE) method is employed due to its high sensitivity to deviations from the assumed distribution, especially in the tails. For the CL distribution, the ADE can be obtained by minimizing the following objective function with respect to the model parameter:

$$A(x_i) = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\log F(x_{i:n}) + \log S(x_{n-i-1:n})\right].$$

4.3. **Method of Cramer von Mises.** The purpose of parameter estimation using the Cramér–von Mises Estimation (CVME) method is to determine the parameter of a theoretical distribution by minimizing the Cramér–von Mises (CVM) statistic. This method is chosen for its sensitivity to variations in the cumulative distribution function (CDF) and its ability to detect deviations from the assumed model. The estimation process involves adjusting the parameter to achieve the best fit between the observed data and the hypothesized distribution. For the CR distribution, the CVME is obtained by minimizing the following function:

$$C(x_i) = \frac{1}{12n} + \sum_{i=1}^{n} \left[F(x_{i:n}) - \frac{2i-1}{2n} \right]^2 .$$

4.4. **Method of maximum product of spacings.** The maximum product of spacings estimation (MPSE) entails maximizing the product of observed spacings between ordered data points. For the CL distribution, the MPSE is obtained by maximizing the following objective function with respect to the model parameter:

$$\delta\left(x_i\right) = \frac{1}{n+1} \sum_{i=1}^{n+1} \log I_i(x_i),$$

where $I_i(x_i) = F(x_{i:n}) - F(x_{i-1:n})$, $F(x_{0:n}) = 0$ and $F(x_{n+1:n}) = 1$.

4.5. **Method of least squares.** The Least Squares Estimation (LSE) approach is a statistical technique used to estimate the parameter of a distribution by minimizing the sum of the squares of the differences between the observed and expected values. It is chosen for its simplicity and ease of interpretation. For the CR distribution, the LSE is obtained by minimizing the following function:

$$V(x_i) = \sum_{i=1}^{n} \left[F(x_{i:n}) - \frac{i}{n+1} \right]^2 .$$

4.6. **Method of percentile.** To estimate the parameter of the CR distribution, the Percentile Estimation (PCE) method is used by minimizing the following function:

$$PCE = \sum_{i=1}^{n} [x_{i:n} - Q(p_i)]^2, \ \ p_i = \frac{i}{n+1}.$$

4.7. **Method of right tail Anderson Darling.** The Right-Tail Anderson–Darling Estimation (RTADE) method is used to estimate the parameter of a statistical distribution by focusing on the goodness of fit in the right tail. This approach employs a modified version of the Anderson–Darling statistic tailored to emphasize right-tail behavior. The RTADE for the CR distribution is obtained by minimizing the following function:

$$R(x_i) = \frac{n}{2} - 2\sum_{i=1}^{n} F(x_{i:n}) - \frac{1}{n} \sum_{i=1}^{n} (2i-1) \log S(x_{i:n}).$$

4.8. **Method of weighted least squares.** Weighted Least Squares Estimation (WLSE) is a statistical method used to estimate the parameter of a probability distribution by assigning different weights to each observation. WLSE is applied when some data points are considered more important or reliable than others, allowing for more precise estimation. The WLSE for the CR distribution is obtained by minimizing the following function:

$$W(x_i) = \sum_{i=1}^{n} \frac{(n+1)^2(n+2)}{i(n-i+1)} \left[F(x_{i:n}) - \frac{i}{n+1} \right]^2.$$

4.9. **Method of left tail Anderson Darling.** The Left-Tail Anderson–Darling Estimation (LTADE) method is used for estimating the parameter of a distribution with emphasis on the left tail, utilizing the Anderson–Darling statistic. The Anderson–Darling statistic measures how well a proposed distribution fits a given dataset. By minimizing the following function involving the CDF of the CR distribution for ordered observations, the model parameter can be estimated:

$$L(x_i) = -\frac{3}{2}n + 2\sum_{i=1}^{n} F(x_{i:n}) - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\log F(x_{i:n}).$$

4.10. **Method of minimum spacing absolute distance.** This method is chosen for its simplicity and its ability to capture the distance between data points, which is useful for parameter estimation. Given an ordered sample denoted by $X_{1:n}, \ldots, X_{n:n}$ from the CR distribution, the minimum spacing absolute distance estimator can be obtained by minimizing the following function:

$$\zeta(x_i) = \sum_{i=1}^{n+1} |I_i - \frac{1}{n+1}|.$$

4.11. **Method of minimum spacing absolute-log distance.** This method extends the minimal spacing strategy by taking the logarithm of the distances, which can be advantageous when dealing with data of varied sizes. Using this approach, we can estimate the CR model parameter by minimizing the following expression:

$$\Upsilon(x_i) = \sum_{i=1}^{n+1} |\log I_i - \log \frac{1}{n+1}|.$$

4.12. **Method of Anderson Darling left tail second order.** This method is employed to capture second-order effects in the left tail of the distribution, allowing for a more detailed examination of model fit in that region. The Anderson–Darling left-tail second-order estimation can be used to estimate the parameter of the CR distribution by minimizing the following objective function:

$$LTS = 2\sum_{i=1}^{n} \log F(x_i) + \frac{1}{n}\sum_{i=1}^{n} \frac{(2i-1)}{F(x_i)}.$$

4.13. **Kolmogorov Method.** The Kolmogorov estimation (KE) method is adopted for its simplicity and effectiveness in measuring the maximum deviation between the empirical and theoretical cumulative distribution functions. The estimator for the parameter of the CR distribution is obtained by minimizing the following functionn:

$$KM = \underset{1\le i\le n}{MAX}\left[\frac{i}{n} - F(x_i), F(x_i) - \frac{i-1}{n}\right].$$

4.14. **Method of minimum spacing square distance.** The minimum spacing square distance estimation (MSSDE) method is employed to estimate the parameter of the proposed model. This is achieved by minimizing the following objective function:

$$\phi_{(x_i)} = \sum_{i=1}^{n+1}(I_i - \frac{1}{n+1})^2.$$

4.15. **Method of minimum spacing square-log distance.** The minimum spacing square-log distance estimation (MSSLDE) method is employed to estimate the parameter of the proposed model. This is accomplished by minimizing the following objective function:

$$\Psi_{(x_i)} = \sum_{i=1}^{n+1}\left(\log I_i - \log\frac{1}{n+1}\right)^2.$$

4.16. **Method of minimum spacing Linex distance.** The minimum spacing Linex distance estimation (MSLNDE) method is used to estimate the parameter of the proposed model. This is achieved by minimizing the following objective function:

$$\Delta_{(x_i)} = \sum_{i=1}^{n+1}\left[e^{I_i-\frac{1}{n+1}} - \left(I_i - \frac{1}{n+1}\right) - 1\right].$$

5. Numerical Simulation

This section evaluates the performance of various estimation methods used to estimate the parameter of the proposed model based on a large volume of simulated data. In the simulation study, the quantile function of the proposed model was used to generate random datasets for different sample sizes ($n = 15, 45, 90, 140, 200, 250, 300,$ and 400). This section will look at our model estimators' performance and behavior. Moreover, we will evaluate the effectiveness of various estimation techniques using a range of measures, such as average of bias ($|Bias(\widehat{\boldsymbol{\sigma}})| = \frac{1}{L}\sum_{i=1}^{L}|\widehat{\boldsymbol{\sigma}} - \boldsymbol{\sigma}|$), mean squared errors ($MSE = \frac{1}{L}\sum_{i=1}^{L}(\widehat{\boldsymbol{\sigma}} - \boldsymbol{\sigma})^2$), mean relative errors ($MRE = \frac{1}{L}\sum_{i=1}^{L}|\widehat{\boldsymbol{\sigma}} - \boldsymbol{\sigma}|/\boldsymbol{\sigma}$), average absolute difference ($D_{abs} = \frac{1}{n\,L}\sum_{i=1}^{L}\sum_{j=1}^{n}|F(x_{ij}|\boldsymbol{\sigma}) - F(x_{ij}|\widehat{\boldsymbol{\sigma}})|$), maximum absolute difference ($D_{max} = \frac{1}{L}\sum_{i=1}^{L}\max_j|F(x_{ij}|\boldsymbol{\sigma}) - F(x_{ij}|\widehat{\boldsymbol{\sigma}})|$), and average squared absolute error ($ASAE = \frac{1}{n}\sum_{i=1}^{n}\frac{|x_i - \hat{x}_i|}{x_n - x_1}$), where x_i are the ascending ordered observations.

The results of simulating the parameter of the proposed model using 16 estimation methods are presented in Tables 1 through 6. The data in Table 6 are graphically illustrated in Figures 3 through 6. It is important to emphasize that all parameter estimates for the proposed distribution are reliable and fairly close to their actual values. The estimation methods demonstrate high accuracy. As the sample size (n) increases, the estimated metrics generally decrease across all scenarios. All methods perform remarkably well in estimating the parameter of the proposed model. Based on a total score of 79.0, Table 7 shows that the maximum likelihood estimation (MLE) method performs best among the methods considered. The total rankings of all estimation procedures are also provided in Table 7.

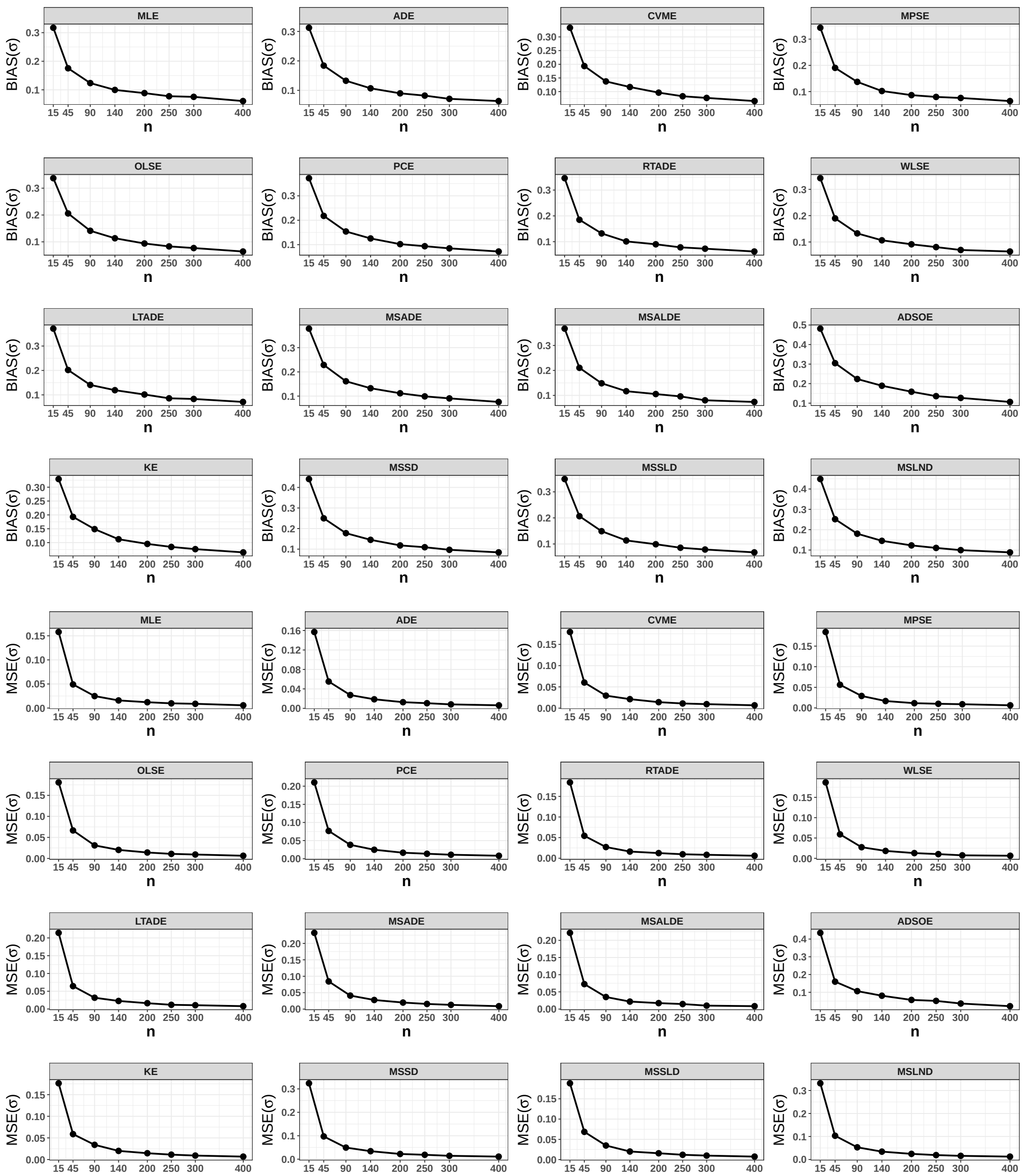

FIGURE 3. Graphical representation for BIAS and MSE values presented in Table 6.

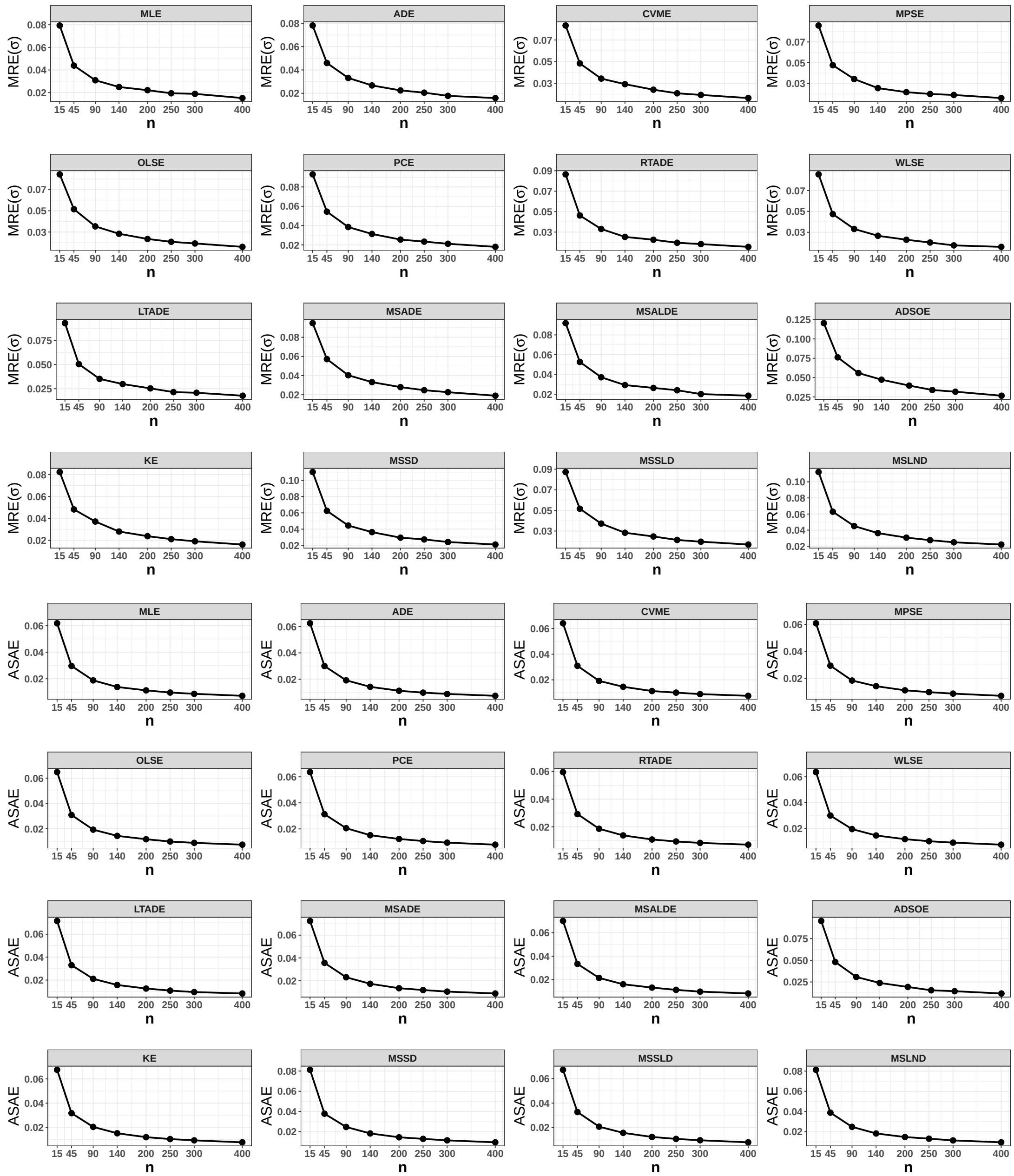

Figure 4. Graphical representation for MRE and ASAE values presented in Table 6.

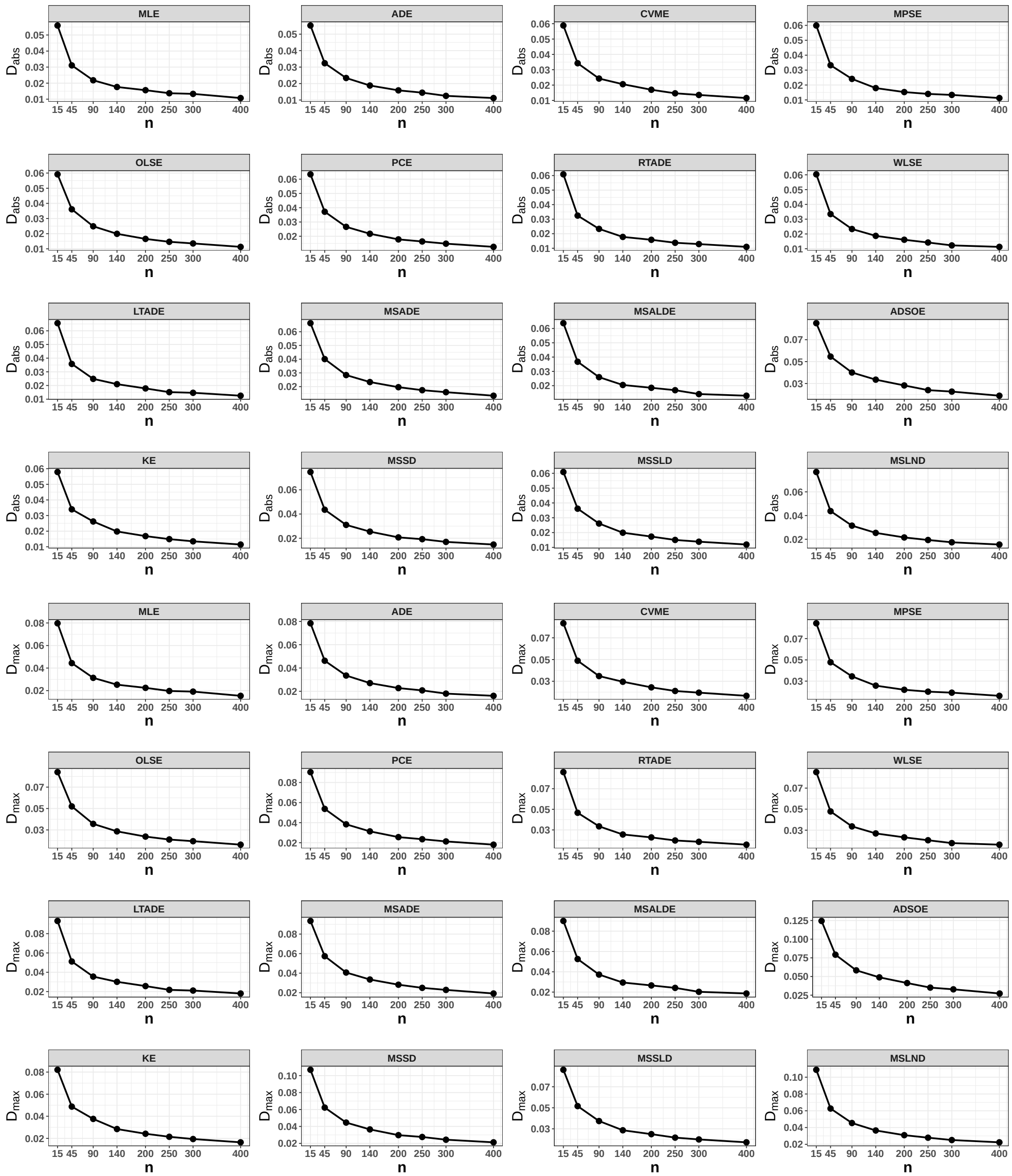

FIGURE 5. Graphical representation for D_{abs} and D_{max} values presented in Table 6.

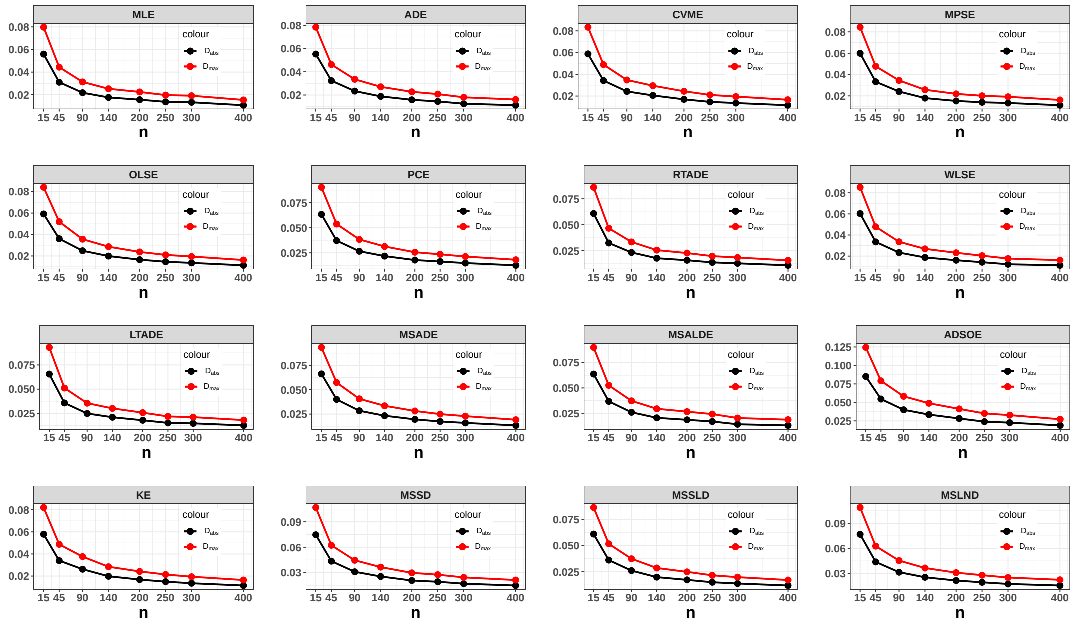

Figure 6. Comparison between D_{abs} and D_{max} values presented in Table 6.

Table 1. Numerical values of simulation measures for $\sigma = 0.4$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSD	MSSLD	MSLND
15	BIAS($\hat{\sigma}$)	$0.03091^{\{1\}}$	$0.03267^{\{4\}}$	$0.03407^{\{6\}}$	$0.0321^{\{3\}}$	$0.03495^{\{10\}}$	$0.03705^{\{11\}}$	$0.03113^{\{2\}}$	$0.03366^{\{5\}}$	$0.0346^{\{8\}}$	$0.03887^{\{13\}}$	$0.03804^{\{12\}}$	$0.05015^{\{16\}}$	$0.03409^{\{7\}}$	$0.0433^{\{14\}}$	$0.03469^{\{9\}}$	$0.04575^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00153^{\{1\}}$	$0.00174^{\{4\}}$	$0.00184^{\{6\}}$	$0.00166^{\{3\}}$	$0.00188^{\{7\}}$	$0.00219^{\{11\}}$	$0.00155^{\{2\}}$	$0.00179^{\{5\}}$	$0.00197^{\{10\}}$	$0.00243^{\{13\}}$	$0.00226^{\{12\}}$	$0.00478^{\{16\}}$	$0.00189^{\{8\}}$	$0.00304^{\{14\}}$	$0.00191^{\{9\}}$	$0.00329^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.07728^{\{1\}}$	$0.08167^{\{4\}}$	$0.08518^{\{6\}}$	$0.08024^{\{3\}}$	$0.08738^{\{10\}}$	$0.09262^{\{11\}}$	$0.07783^{\{2\}}$	$0.08415^{\{5\}}$	$0.0865^{\{8\}}$	$0.09718^{\{13\}}$	$0.0951^{\{12\}}$	$0.12536^{\{16\}}$	$0.08522^{\{7\}}$	$0.10824^{\{14\}}$	$0.08673^{\{9\}}$	$0.11437^{\{15\}}$
	D_{abs}	$0.05551^{\{2\}}$	$0.05806^{\{4\}}$	$0.06059^{\{7\}}$	$0.05593^{\{3\}}$	$0.06168^{\{10\}}$	$0.0633^{\{11\}}$	$0.05488^{\{1\}}$	$0.05938^{\{5\}}$	$0.06144^{\{9\}}$	$0.06689^{\{13\}}$	$0.0662^{\{12\}}$	$0.08847^{\{16\}}$	$0.06008^{\{6\}}$	$0.07417^{\{14\}}$	$0.0606^{\{8\}}$	$0.07822^{\{15\}}$
	D_{max}	$0.07815^{\{2\}}$	$0.08202^{\{4\}}$	$0.08569^{\{7\}}$	$0.07868^{\{3\}}$	$0.08722^{\{10\}}$	$0.08986^{\{11\}}$	$0.07735^{\{1\}}$	$0.08405^{\{5\}}$	$0.08687^{\{9\}}$	$0.09506^{\{13\}}$	$0.09324^{\{12\}}$	$0.13001^{\{16\}}$	$0.08466^{\{6\}}$	$0.1063^{\{14\}}$	$0.08614^{\{8\}}$	$0.11243^{\{15\}}$
	ASAE	$0.0624^{\{2\}}$	$0.06399^{\{4\}}$	$0.06307^{\{7\}}$	$0.06119^{\{3\}}$	$0.06353^{\{10\}}$	$0.06423^{\{11\}}$	$0.05888^{\{1\}}$	$0.06325^{\{5\}}$	$0.06853^{\{9\}}$	$0.07169^{\{13\}}$	$0.0683^{\{12\}}$	$0.09625^{\{16\}}$	$0.06758^{\{6\}}$	$0.08006^{\{14\}}$	$0.06651^{\{8\}}$	$0.08283^{\{15\}}$
	$\sum Ranks$	$10^{\{2\}}$	$27^{\{4\}}$	$36^{\{6\}}$	$17^{\{3\}}$	$53^{\{9\}}$	$63^{\{11\}}$	$9^{\{1\}}$	$30^{\{5\}}$	$56^{\{10\}}$	$78^{\{13\}}$	$71^{\{12\}}$	$96^{\{16\}}$	$44^{\{7\}}$	$84^{\{14\}}$	$52^{\{8\}}$	$90^{\{15\}}$
45	BIAS($\hat{\sigma}$)	$0.01749^{\{1\}}$	$0.01883^{\{3\}}$	$0.01986^{\{8\}}$	$0.01893^{\{4\}}$	$0.01903^{\{5\}}$	$0.02177^{\{11\}}$	$0.01835^{\{2\}}$	$0.01957^{\{6\}}$	$0.02109^{\{10\}}$	$0.0229^{\{13\}}$	$0.02199^{\{12\}}$	$0.02925^{\{16\}}$	$0.01964^{\{7\}}$	$0.02578^{\{14\}}$	$0.02039^{\{9\}}$	$0.026^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00049^{\{1\}}$	$0.00055^{\{3.5\}}$	$6e-04^{\{7\}}$	$0.00055^{\{3.5\}}$	$0.00057^{\{5\}}$	$0.00076^{\{12\}}$	$0.00053^{\{2\}}$	$0.00059^{\{6\}}$	$0.00072^{\{10\}}$	$0.00086^{\{13\}}$	$0.00074^{\{11\}}$	$0.00175^{\{16\}}$	$0.00061^{\{8\}}$	$0.00106^{\{14\}}$	$0.00068^{\{9\}}$	$0.00107^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.04371^{\{1\}}$	$0.04708^{\{3\}}$	$0.04966^{\{8\}}$	$0.04734^{\{4\}}$	$0.04758^{\{5\}}$	$0.05442^{\{11\}}$	$0.04587^{\{2\}}$	$0.04891^{\{6\}}$	$0.05273^{\{10\}}$	$0.05725^{\{13\}}$	$0.05497^{\{12\}}$	$0.07312^{\{16\}}$	$0.0491^{\{7\}}$	$0.06444^{\{14\}}$	$0.05098^{\{9\}}$	$0.065^{\{15\}}$
	D_{abs}	$0.03078^{\{1\}}$	$0.03333^{\{4\}}$	$0.03508^{\{8\}}$	$0.03312^{\{3\}}$	$0.03361^{\{5\}}$	$0.03731^{\{11\}}$	$0.03242^{\{2\}}$	$0.03447^{\{7\}}$	$0.03718^{\{10\}}$	$0.04021^{\{13\}}$	$0.03846^{\{12\}}$	$0.05159^{\{16\}}$	$0.03446^{\{6\}}$	$0.0447^{\{14\}}$	$0.03571^{\{9\}}$	$0.04541^{\{15\}}$
	D_{max}	$0.04419^{\{1\}}$	$0.0474^{\{3.5\}}$	$0.05017^{\{8\}}$	$0.0474^{\{3.5\}}$	$0.04813^{\{5\}}$	$0.05383^{\{11\}}$	$0.04628^{\{2\}}$	$0.04929^{\{6\}}$	$0.05332^{\{10\}}$	$0.05738^{\{13\}}$	$0.05495^{\{12\}}$	$0.07562^{\{16\}}$	$0.04951^{\{7\}}$	$0.06439^{\{14\}}$	$0.05119^{\{9\}}$	$0.06512^{\{15\}}$
	ASAE	$0.0292^{\{1\}}$	$0.03009^{\{3.5\}}$	$0.03121^{\{8\}}$	$0.02916^{\{3.5\}}$	$0.03012^{\{5\}}$	$0.03119^{\{11\}}$	$0.0287^{\{2\}}$	$0.02984^{\{6\}}$	$0.03347^{\{10\}}$	$0.03518^{\{13\}}$	$0.03406^{\{12\}}$	$0.04631^{\{16\}}$	$0.03206^{\{7\}}$	$0.03918^{\{14\}}$	$0.0322^{\{9\}}$	$0.03898^{\{15\}}$
	$\sum Ranks$	$8^{\{1\}}$	$22^{\{4\}}$	$56^{\{9\}}$	$20^{\{3\}}$	$31^{\{5\}}$	$62^{\{11\}}$	$11^{\{2\}}$	$35^{\{6\}}$	$60^{\{10\}}$	$77^{\{13\}}$	$70^{\{12\}}$	$95^{\{16\}}$	$43^{\{7\}}$	$84^{\{14\}}$	$54^{\{8\}}$	$88^{\{15\}}$
90	BIAS($\hat{\sigma}$)	$0.0127^{\{1\}}$	$0.01306^{\{2\}}$	$0.01378^{\{6\}}$	$0.01336^{\{3\}}$	$0.01384^{\{7\}}$	$0.01561^{\{11.5\}}$	$0.01365^{\{5\}}$	$0.01347^{\{4\}}$	$0.01477^{\{10\}}$	$0.01686^{\{13\}}$	$0.01561^{\{11.5\}}$	$0.02247^{\{16\}}$	$0.01407^{\{8\}}$	$0.01781^{\{14\}}$	$0.01415^{\{9\}}$	$0.0182^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00026^{\{1\}}$	$0.00028^{\{3\}}$	$3e-04^{\{7\}}$	$0.00028^{\{3\}}$	$3e-04^{\{7\}}$	$0.00039^{\{11.5\}}$	$0.00028^{\{3\}}$	$0.00029^{\{5\}}$	$0.00034^{\{10\}}$	$0.00045^{\{13\}}$	$0.00039^{\{11.5\}}$	$0.00094^{\{16\}}$	$3e-04^{\{7\}}$	$0.00051^{\{14\}}$	$0.00032^{\{9\}}$	$0.00054^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.03174^{\{1\}}$	$0.03266^{\{2\}}$	$0.03445^{\{6\}}$	$0.0334^{\{3\}}$	$0.03461^{\{7\}}$	$0.03903^{\{11.5\}}$	$0.03411^{\{5\}}$	$0.03367^{\{4\}}$	$0.03693^{\{10\}}$	$0.04216^{\{13\}}$	$0.03903^{\{11.5\}}$	$0.05619^{\{16\}}$	$0.03519^{\{8\}}$	$0.04452^{\{14\}}$	$0.03537^{\{9\}}$	$0.0455^{\{15\}}$
	D_{abs}	$0.02243^{\{1\}}$	$0.02306^{\{2\}}$	$0.02428^{\{6\}}$	$0.0234^{\{3\}}$	$0.02437^{\{7\}}$	$0.02701^{\{11\}}$	$0.02394^{\{5\}}$	$0.0237^{\{4\}}$	$0.02601^{\{10\}}$	$0.02958^{\{13\}}$	$0.02728^{\{12\}}$	$0.04027^{\{16\}}$	$0.02479^{\{8\}}$	$0.03114^{\{14\}}$	$0.0248^{\{9\}}$	$0.0317^{\{15\}}$
	D_{max}	$0.03216^{\{1\}}$	$0.03305^{\{2\}}$	$0.03484^{\{6\}}$	$0.03365^{\{3\}}$	$0.03497^{\{7\}}$	$0.03891^{\{11\}}$	$0.03443^{\{5\}}$	$0.03407^{\{4\}}$	$0.03739^{\{10\}}$	$0.04238^{\{13\}}$	$0.03918^{\{12\}}$	$0.05855^{\{16\}}$	$0.03566^{\{9\}}$	$0.04473^{\{14\}}$	$0.03554^{\{8\}}$	$0.04556^{\{15\}}$
	ASAE	$0.01886^{\{1\}}$	$0.01887^{\{2\}}$	$0.01948^{\{6\}}$	$0.01852^{\{3\}}$	$0.01924^{\{7\}}$	$0.02038^{\{11\}}$	$0.01824^{\{5\}}$	$0.01889^{\{4\}}$	$0.02123^{\{10\}}$	$0.0228^{\{13\}}$	$0.02125^{\{12\}}$	$0.03021^{\{16\}}$	$0.0202^{\{9\}}$	$0.02469^{\{14\}}$	$0.02055^{\{8\}}$	$0.02483^{\{15\}}$
	$\sum Ranks$	$8^{\{1\}}$	$15^{\{2\}}$	$46^{\{6\}}$	$17^{\{3\}}$	$49^{\{7\}}$	$62.5^{\{11\}}$	$24^{\{4\}}$	$26^{\{5\}}$	$58^{\{10\}}$	$75^{\{13\}}$	$67.5^{\{12\}}$	$93^{\{16\}}$	$56^{\{9\}}$	$81^{\{14\}}$	$51^{\{8\}}$	$87^{\{15\}}$
140	BIAS($\hat{\sigma}$)	$0.01022^{\{2\}}$	$0.01058^{\{3\}}$	$0.01123^{\{7\}}$	$0.0102^{\{1\}}$	$0.0106^{\{4\}}$	$0.01229^{\{11\}}$	$0.01068^{\{5\}}$	$0.011^{\{6\}}$	$0.01191^{\{10\}}$	$0.01357^{\{13\}}$	$0.01253^{\{12\}}$	$0.01795^{\{16\}}$	$0.01139^{\{8\}}$	$0.01401^{\{14\}}$	$0.0115^{\{9\}}$	$0.01487^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00017^{\{2\}}$	$0.00017^{\{2\}}$	$2e-04^{\{7.5\}}$	$0.00017^{\{2\}}$	$0.00018^{\{4.5\}}$	$0.00024^{\{11\}}$	$0.00018^{\{4.5\}}$	$0.00019^{\{6\}}$	$0.00022^{\{10\}}$	$0.00029^{\{13\}}$	$0.00025^{\{12\}}$	$0.00071^{\{16\}}$	$2e-04^{\{7.5\}}$	$0.00032^{\{14\}}$	$0.00021^{\{9\}}$	$0.00035^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.02556^{\{2\}}$	$0.02646^{\{3\}}$	$0.02809^{\{7\}}$	$0.02551^{\{1\}}$	$0.0265^{\{4\}}$	$0.03073^{\{11\}}$	$0.02669^{\{5\}}$	$0.02751^{\{6\}}$	$0.02978^{\{10\}}$	$0.03394^{\{13\}}$	$0.03132^{\{12\}}$	$0.04487^{\{16\}}$	$0.02846^{\{8\}}$	$0.03504^{\{14\}}$	$0.02875^{\{9\}}$	$0.03718^{\{15\}}$
	D_{abs}	$0.01801^{\{2\}}$	$0.01868^{\{3\}}$	$0.01978^{\{7\}}$	$0.01786^{\{1\}}$	$0.01869^{\{4\}}$	$0.02145^{\{11\}}$	$0.01879^{\{5\}}$	$0.01943^{\{6\}}$	$0.02104^{\{10\}}$	$0.02388^{\{13\}}$	$0.02197^{\{12\}}$	$0.03208^{\{16\}}$	$0.02005^{\{8\}}$	$0.02454^{\{14\}}$	$0.02023^{\{9\}}$	$0.0261^{\{15\}}$
	D_{max}	$0.02588^{\{2\}}$	$0.02679^{\{3\}}$	$0.02843^{\{7\}}$	$0.0257^{\{1\}}$	$0.02682^{\{4\}}$	$0.03086^{\{11\}}$	$0.027^{\{5\}}$	$0.02785^{\{6\}}$	$0.03018^{\{10\}}$	$0.03428^{\{13\}}$	$0.03156^{\{12\}}$	$0.04661^{\{16\}}$	$0.0288^{\{8\}}$	$0.03529^{\{14\}}$	$0.02902^{\{9\}}$	$0.03745^{\{15\}}$
	ASAE	$0.01413^{\{2\}}$	$0.01427^{\{3\}}$	$0.01442^{\{7\}}$	$0.014^{\{1\}}$	$0.01472^{\{4\}}$	$0.01555^{\{11\}}$	$0.01388^{\{5\}}$	$0.01442^{\{6\}}$	$0.01583^{\{10\}}$	$0.01712^{\{13\}}$	$0.01618^{\{12\}}$	$0.02302^{\{16\}}$	$0.01511^{\{8\}}$	$0.01816^{\{14\}}$	$0.01548^{\{9\}}$	$0.01833^{\{15\}}$
	$\sum Ranks$	$13^{\{2\}}$	$18^{\{3\}}$	$49.5^{\{7\}}$	$8^{\{1\}}$	$27.5^{\{5\}}$	$63^{\{11\}}$	$25.5^{\{4\}}$	$35^{\{6\}}$	$59^{\{10\}}$	$76^{\{13\}}$	$70^{\{12\}}$	$94^{\{16\}}$	$55.5^{\{9\}}$	$82^{\{14\}}$	$52^{\{8\}}$	$88^{\{15\}}$
200	BIAS($\hat{\sigma}$)	$0.00904^{\{4\}}$	$0.00907^{\{5\}}$	$0.00968^{\{9\}}$	$0.00862^{\{1\}}$	$0.00923^{\{6\}}$	$0.00964^{\{8\}}$	$0.00863^{\{2\}}$	$0.00894^{\{3\}}$	$0.00974^{\{10\}}$	$0.01145^{\{13\}}$	$0.01014^{\{12\}}$	$0.01518^{\{16\}}$	$0.00942^{\{7\}}$	$0.01214^{\{15\}}$	$0.00976^{\{11\}}$	$0.01181^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00013^{\{4.5\}}$	$0.00013^{\{4.5\}}$	$0.00015^{\{9.5\}}$	$0.00012^{\{2\}}$	$0.00014^{\{6.5\}}$	$0.00015^{\{9.5\}}$	$0.00012^{\{2\}}$	$0.00012^{\{2\}}$	$0.00015^{\{9.5\}}$	$0.00021^{\{13\}}$	$0.00016^{\{12\}}$	$0.00043^{\{16\}}$	$0.00014^{\{6.5\}}$	$0.00023^{\{15\}}$	$0.00015^{\{9.5\}}$	$0.00022^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.0226^{\{4\}}$	$0.02267^{\{5\}}$	$0.02419^{\{9\}}$	$0.02154^{\{1\}}$	$0.02308^{\{6\}}$	$0.0241^{\{8\}}$	$0.02157^{\{2\}}$	$0.02234^{\{3\}}$	$0.02436^{\{10\}}$	$0.02862^{\{13\}}$	$0.02535^{\{12\}}$	$0.03794^{\{16\}}$	$0.02355^{\{7\}}$	$0.03035^{\{15\}}$	$0.02439^{\{11\}}$	$0.02954^{\{14\}}$
	D_{abs}	$0.01595^{\{4\}}$	$0.01597^{\{5\}}$	$0.01702^{\{9\}}$	$0.01514^{\{1\}}$	$0.01628^{\{6\}}$	$0.01681^{\{8\}}$	$0.01517^{\{2\}}$	$0.01573^{\{3\}}$	$0.01718^{\{11\}}$	$0.02003^{\{13\}}$	$0.01783^{\{12\}}$	$0.02706^{\{16\}}$	$0.01658^{\{7\}}$	$0.02135^{\{15\}}$	$0.01713^{\{10\}}$	$0.0207^{\{14\}}$
	D_{max}	$0.02287^{\{4\}}$	$0.02297^{\{5\}}$	$0.02444^{\{9\}}$	$0.02176^{\{1\}}$	$0.02338^{\{6\}}$	$0.02424^{\{8\}}$	$0.02183^{\{2\}}$	$0.02259^{\{3\}}$	$0.02469^{\{11\}}$	$0.02885^{\{13\}}$	$0.02557^{\{12\}}$	$0.03914^{\{16\}}$	$0.02387^{\{7\}}$	$0.03072^{\{15\}}$	$0.02456^{\{10\}}$	$0.02977^{\{14\}}$
	ASAE	$0.01114^{\{4\}}$	$0.01134^{\{5\}}$	$0.01149^{\{9\}}$	$0.01116^{\{1\}}$	$0.01157^{\{6\}}$	$0.01226^{\{8\}}$	$0.01103^{\{2\}}$	$0.01131^{\{3\}}$	$0.01267^{\{11\}}$	$0.0136^{\{13\}}$	$0.01284^{\{12\}}$	$0.01849^{\{16\}}$	$0.01207^{\{7\}}$	$0.01473^{\{15\}}$	$0.01229^{\{10\}}$	$0.01447^{\{14\}}$
	$\sum Ranks$	$22.5^{\{4\}}$	$29.5^{\{5\}}$	$51.5^{\{9\}}$	$9^{\{1\}}$	$37.5^{\{6\}}$	$50.5^{\{8\}}$	$11^{\{2\}}$	$18^{\{3\}}$	$62.5^{\{11\}}$	$78^{\{13\}}$	$72^{\{12\}}$	$96^{\{16\}}$	$42.5^{\{7\}}$	$90^{\{15\}}$	$61.5^{\{10\}}$	$84^{\{14\}}$
250	BIAS($\hat{\sigma}$)	$0.00768^{\{1\}}$	$0.00781^{\{2\}}$	$0.0086^{\{8\}}$	$0.00787^{\{3\}}$	$0.00829^{\{6\}}$	$0.00926^{\{13\}}$	$0.00815^{\{5\}}$	$0.00793^{\{4\}}$	$0.0088^{\{9\}}$	$0.00924^{\{12\}}$	$0.00904^{\{11\}}$	$0.01263^{\{16\}}$	$0.00857^{\{7\}}$	$0.01098^{\{14\}}$	$0.00889^{\{10\}}$	$0.01134^{\{15\}}$
	MSE($\hat{\sigma}$)	$9e-05^{\{1\}}$	$1e-04^{\{3\}}$	$0.00012^{\{8.5\}}$	$1e-04^{\{3\}}$	$0.00011^{\{5.5\}}$	$0.00013^{\{11.5\}}$	$0.00011^{\{5.5\}}$	$1e-04^{\{3\}}$	$0.00012^{\{8.5\}}$	$0.00014^{\{13\}}$	$0.00013^{\{11.5\}}$	$3e-04^{\{16\}}$	$0.00012^{\{8.5\}}$	$0.00019^{\{14\}}$	$0.00012^{\{8.5\}}$	$2e-04^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.01921^{\{1\}}$	$0.01952^{\{2\}}$	$0.02149^{\{8\}}$	$0.01966^{\{3\}}$	$0.02071^{\{6\}}$	$0.02316^{\{13\}}$	$0.02038^{\{5\}}$	$0.01983^{\{4\}}$	$0.02199^{\{9\}}$	$0.02309^{\{12\}}$	$0.0226^{\{11\}}$	$0.03157^{\{16\}}$	$0.02144^{\{7\}}$	$0.02745^{\{14\}}$	$0.02222^{\{10\}}$	$0.02836^{\{15\}}$
	D_{abs}	$0.01356^{\{1\}}$	$0.01377^{\{2\}}$	$0.01514^{\{8\}}$	$0.01385^{\{3\}}$	$0.0146^{\{6\}}$	$0.01611^{\{12\}}$	$0.01434^{\{5\}}$	$0.01398^{\{4\}}$	$0.01548^{\{9\}}$	$0.01622^{\{13\}}$	$0.01588^{\{11\}}$	$0.02248^{\{16\}}$	$0.0151^{\{7\}}$	$0.01932^{\{14\}}$	$0.01564^{\{10\}}$	$0.0199^{\{15\}}$
	D_{max}	$0.01949^{\{1\}}$	$0.01977^{\{2\}}$	$0.02176^{\{8\}}$	$0.01987^{\{3\}}$	$0.02097^{\{6\}}$	$0.02326^{\{12\}}$	$0.02064^{\{5\}}$	$0.02008^{\{4\}}$	$0.02226^{\{9\}}$	$0.02333^{\{13\}}$	$0.02281^{\{11\}}$	$0.03232^{\{16\}}$	$0.0217^{\{7\}}$	$0.02775^{\{14\}}$	$0.02246^{\{10\}}$	$0.02859^{\{15\}}$
	ASAE	$0.00966^{\{1\}}$	$0.00977^{\{2\}}$	$0.01003^{\{8\}}$	$0.00975^{\{3\}}$	$0.01014^{\{6\}}$	$0.01074^{\{12\}}$	$0.00946^{\{5\}}$	$0.00992^{\{4\}}$	$0.01102^{\{9\}}$	$0.0114^{\{13\}}$	$0.011^{\{11\}}$	$0.01533^{\{16\}}$	$0.01064^{\{7\}}$	$0.01281^{\{14\}}$	$0.01068^{\{10\}}$	$0.01305^{\{15\}}$
	$\sum Ranks$	$22^{\{1\}}$	$24^{\{3\}}$	$42.5^{\{8\}}$	$27^{\{4\}}$	$32.5^{\{5\}}$	$67.5^{\{12\}}$	$22.5^{\{2\}}$	$33^{\{6\}}$	$52.5^{\{9\}}$	$72^{\{13\}}$	$62.5^{\{11\}}$	$95^{\{16\}}$	$40.5^{\{7\}}$	$80^{\{14\}}$	$53.5^{\{10\}}$	$89^{\{15\}}$
300	BIAS($\hat{\sigma}$)	$0.00726^{\{3\}}$	$0.00756^{\{6\}}$	$0.00734^{\{5\}}$	$0.00713^{\{1\}}$	$0.00765^{\{7\}}$	$0.00816^{\{10\}}$	$0.00731^{\{4\}}$	$0.00718^{\{2\}}$	$0.00792^{\{9\}}$	$0.00848^{\{13\}}$	$0.00827^{\{11\}}$	$0.01144^{\{16\}}$	$0.00783^{\{8\}}$	$0.00983^{\{14\}}$	$0.0083^{\{12\}}$	$0.00993^{\{15\}}$
	MSE($\hat{\sigma}$)	$8e-05^{\{2.5\}}$	$9e-05^{\{6\}}$	$8e-05^{\{2.5\}}$	$8e-05^{\{2.5\}}$	$9e-05^{\{6\}}$	$1e-04^{\{9.5\}}$	$9e-05^{\{6\}}$	$8e-05^{\{2.5\}}$	$1e-04^{\{9.5\}}$	$0.00011^{\{12.5\}}$	$0.00011^{\{12.5\}}$	$0.00025^{\{16\}}$	$1e-04^{\{9.5\}}$	$0.00016^{\{14.5\}}$	$1e-04^{\{9.5\}}$	$0.00016^{\{14.5\}}$
	MRE($\hat{\sigma}$)	$0.01814^{\{3\}}$	$0.0189^{\{6\}}$	$0.01836^{\{5\}}$	$0.01781^{\{1\}}$	$0.01913^{\{7\}}$	$0.0204^{\{10\}}$	$0.01828^{\{4\}}$	$0.01795^{\{2\}}$	$0.01981^{\{9\}}$	$0.02121^{\{13\}}$	$0.02068^{\{11\}}$	$0.02861^{\{16\}}$	$0.01959^{\{8\}}$	$0.02457^{\{14\}}$	$0.02075^{\{12\}}$	$0.02482^{\{15\}}$
	D_{abs}	$0.01283^{\{3\}}$	$0.01332^{\{6\}}$	$0.01295^{\{5\}}$	$0.01253^{\{1\}}$	$0.01347^{\{7\}}$	$0.01428^{\{10\}}$	$0.0129^{\{4\}}$	$0.01265^{\{2\}}$	$0.01395^{\{9\}}$	$0.01494^{\{13\}}$	$0.01453^{\{11\}}$	$0.02036^{\{16\}}$	$0.01379^{\{8\}}$	$0.0173^{\{14\}}$	$0.0146^{\{12\}}$	$0.01741^{\{15\}}$
	D_{max}	$0.01838^{\{3\}}$	$0.01914^{\{6\}}$	$0.0186^{\{5\}}$	$0.018^{\{1\}}$	$0.01939^{\{7\}}$	$0.02054^{\{10\}}$	$0.0185^{\{4\}}$	$0.01819^{\{2\}}$	$0.02006^{\{9\}}$	$0.02146^{\{13\}}$	$0.02085^{\{11\}}$	$0.02935^{\{16\}}$	$0.01985^{\{8\}}$	$0.02484^{\{14\}}$	$0.021^{\{12\}}$	$0.02506^{\{15\}}$
	ASAE	$0.00875^{\{3\}}$	$0.009^{\{6\}}$	$0.00882^{\{5\}}$	$0.00853^{\{1\}}$	$0.00898^{\{7\}}$	$0.00939^{\{10\}}$	$0.00866^{\{4\}}$	$0.00871^{\{2\}}$	$0.00956^{\{9\}}$	$0.01042^{\{13\}}$	$0.01004^{\{11\}}$	$0.01363^{\{16\}}$	$0.0094^{\{8\}}$	$0.01124^{\{14\}}$	$0.00975^{\{12\}}$	$0.01138^{\{15\}}$
	$\sum Ranks$	$27.5^{\{3\}}$	$46^{\{6\}}$	$36.5^{\{5\}}$	$16.5^{\{1\}}$	$49^{\{8\}}$	$55.5^{\{10\}}$	$33^{\{4\}}$	$22.5^{\{2\}}$	$53.5^{\{9\}}$	$66.5^{\{12.5\}}$	$57.5^{\{11\}}$	$85^{\{16\}}$	$48.5^{\{7\}}$	$73.5^{\{14\}}$	$66.5^{\{12.5\}}$	$78.5^{\{15\}}$
400	BIAS($\hat{\sigma}$)	$0.00605^{\{3\}}$	$0.00653^{\{4\}}$	$0.00683^{\{8\}}$	$0.006^{\{1\}}$	$0.00657^{\{5\}}$	$0.00704^{\{11\}}$	$0.00602^{\{2\}}$	$0.0066^{\{6\}}$	$0.00703^{\{10\}}$	$0.00748^{\{13\}}$	$0.00722^{\{12\}}$	$0.0104^{\{16\}}$	$0.00675^{\{7\}}$	$0.00892^{\{15\}}$	$0.00698^{\{9\}}$	$0.00881^{\{14\}}$
	MSE($\hat{\sigma}$)	$6e-05^{\{2\}}$	$7e-05^{\{6\}}$	$7e-05^{\{6\}}$	$6e-05^{\{2\}}$	$7e-05^{\{6\}}$	$8e-05^{\{10.5\}}$	$6e-05^{\{2\}}$	$7e-05^{\{6\}}$	$8e-05^{\{10.5\}}$	$9e-05^{\{13\}}$	$8e-05^{\{10.5\}}$	$0.00021^{\{16\}}$	$7e-05^{\{6\}}$	$0.00012^{\{14.5\}}$	$8e-05^{\{10.5\}}$	$0.00012^{\{14.5\}}$
	MRE($\hat{\sigma}$)	$0.01514^{\{3\}}$	$0.01632^{\{4\}}$	$0.01707^{\{8\}}$	$0.015^{\{1\}}$	$0.01644^{\{5\}}$	$0.01759^{\{11\}}$	$0.01505^{\{2\}}$	$0.0165^{\{6\}}$	$0.01757^{\{10\}}$	$0.01871^{\{13\}}$	$0.01806^{\{12\}}$	$0.02599^{\{16\}}$	$0.01689^{\{7\}}$	$0.02229^{\{15\}}$	$0.01744^{\{9\}}$	$0.02203^{\{14\}}$
	D_{abs}	$0.01067^{\{3\}}$	$0.01153^{\{4\}}$	$0.01202^{\{8\}}$	$0.01057^{\{1\}}$	$0.01159^{\{5\}}$	$0.01232^{\{10\}}$	$0.01062^{\{2\}}$	$0.01164^{\{6\}}$	$0.01237^{\{11\}}$	$0.01317^{\{13\}}$	$0.0127^{\{12\}}$	$0.01845^{\{16\}}$	$0.0119^{\{7\}}$	$0.01567^{\{15\}}$	$0.01228^{\{9\}}$	$0.01546^{\{14\}}$
	D_{max}	$0.01533^{\{3\}}$	$0.01654^{\{4\}}$	$0.01727^{\{8\}}$	$0.01518^{\{1\}}$	$0.01666^{\{5\}}$	$0.01773^{\{10\}}$	$0.01525^{\{2\}}$	$0.01673^{\{6\}}$	$0.01779^{\{11\}}$	$0.01891^{\{13\}}$	$0.01825^{\{12\}}$	$0.02659^{\{16\}}$	$0.01711^{\{7\}}$	$0.02252^{\{15\}}$	$0.01764^{\{9\}}$	$0.02224^{\{14\}}$
	ASAE	$0.00726^{\{3\}}$	$0.00738^{\{4\}}$	$0.0074^{\{8\}}$	$0.00721^{\{1\}}$	$0.00743^{\{5\}}$	$0.00788^{\{10\}}$	$0.00706^{\{2\}}$	$0.00727^{\{6\}}$	$0.00828^{\{11\}}$	$0.00874^{\{13\}}$	$0.00831^{\{12\}}$	$0.0114^{\{16\}}$	$0.00781^{\{7\}}$	$0.00942^{\{15\}}$	$0.00792^{\{9\}}$	$0.00967^{\{14\}}$
	$\sum Ranks$	$20^{\{3\}}$	$30^{\{4\}}$	$47^{\{8\}}$	$11^{\{1\}}$	$36^{\{5\}}$	$64.5^{\{10\}}$	$14^{\{2\}}$	$37^{\{6\}}$	$66.5^{\{11\}}$	$81^{\{15\}}$	$73.5^{\{13\}}$	$83^{\{16\}}$	$45^{\{7\}}$	$75.5^{\{14\}}$	$59.5^{\{9\}}$	$72.5^{\{12\}}$

Table 2. Numerical values of simulation measures for $\sigma = 0.8$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSD	MSSLD	MSLND
15	BIAS($\hat{\sigma}$)	$0.06373^{\{2\}}$	$0.06567^{\{3\}}$	$0.06701^{\{7\}}$	$0.06636^{\{4\}}$	$0.06918^{\{9\}}$	$0.07478^{\{12\}}$	$0.06355^{\{1\}}$	$0.06667^{\{6\}}$	$0.07059^{\{10\}}$	$0.08145^{\{13\}}$	$0.07087^{\{11\}}$	$0.0921^{\{16\}}$	$0.06656^{\{5\}}$	$0.08852^{\{15\}}$	$0.06878^{\{8\}}$	$0.0882^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00642^{\{1\}}$	$0.00664^{\{3\}}$	$0.00717^{\{7\}}$	$0.00713^{\{6\}}$	$0.00735^{\{8\}}$	$0.00909^{\{12\}}$	$0.00662^{\{2\}}$	$0.00701^{\{5\}}$	$0.00797^{\{10\}}$	$0.01064^{\{13\}}$	$0.008^{\{11\}}$	$0.01473^{\{16\}}$	$0.0068^{\{4\}}$	$0.01269^{\{15\}}$	$0.00768^{\{9\}}$	$0.01265^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.07966^{\{2\}}$	$0.08208^{\{3\}}$	$0.08377^{\{7\}}$	$0.08295^{\{4\}}$	$0.08647^{\{9\}}$	$0.09347^{\{12\}}$	$0.07944^{\{1\}}$	$0.08334^{\{6\}}$	$0.08824^{\{10\}}$	$0.10181^{\{13\}}$	$0.08859^{\{11\}}$	$0.11513^{\{16\}}$	$0.0832^{\{5\}}$	$0.11065^{\{15\}}$	$0.08597^{\{8\}}$	$0.11025^{\{14\}}$
	D_{abs}	$0.05636^{\{2\}}$	$0.05809^{\{4\}}$	$0.05929^{\{7\}}$	$0.05779^{\{3\}}$	$0.06129^{\{9\}}$	$0.06304^{\{12\}}$	$0.05572^{\{1\}}$	$0.05834^{\{5\}}$	$0.06257^{\{11\}}$	$0.06979^{\{13\}}$	$0.06199^{\{10\}}$	$0.08129^{\{16\}}$	$0.05889^{\{6\}}$	$0.07569^{\{14\}}$	$0.06064^{\{8\}}$	$0.07599^{\{15\}}$
	D_{max}	$0.08003^{\{2\}}$	$0.08197^{\{4\}}$	$0.08356^{\{7\}}$	$0.08139^{\{3\}}$	$0.08653^{\{9\}}$	$0.09011^{\{12\}}$	$0.07871^{\{1\}}$	$0.0829^{\{5\}}$	$0.08877^{\{11\}}$	$0.09978^{\{13\}}$	$0.08757^{\{10\}}$	$0.11796^{\{16\}}$	$0.08351^{\{6\}}$	$0.10838^{\{14\}}$	$0.08547^{\{8\}}$	$0.10886^{\{15\}}$
	ASAE	$0.06208^{\{2\}}$	$0.06342^{\{4\}}$	$0.0638^{\{7\}}$	$0.0617^{\{3\}}$	$0.06442^{\{9\}}$	$0.06391^{\{12\}}$	$0.06063^{\{1\}}$	$0.06173^{\{5\}}$	$0.07226^{\{11\}}$	$0.07184^{\{13\}}$	$0.06886^{\{10\}}$	$0.09378^{\{16\}}$	$0.06644^{\{6\}}$	$0.08194^{\{14\}}$	$0.06826^{\{8\}}$	$0.07952^{\{15\}}$
	$\sum Ranks$	$13^{\{2\}}$	$22^{\{3.5\}}$	$41^{\{7\}}$	$22^{\{3.5\}}$	$52^{\{9\}}$	$67^{\{12\}}$	$7^{\{1\}}$	$30^{\{5\}}$	$65^{\{11\}}$	$77^{\{13\}}$	$64^{\{10\}}$	$96^{\{16\}}$	$35^{\{6\}}$	$88^{\{15\}}$	$51^{\{8\}}$	$86^{\{14\}}$
45	BIAS($\hat{\sigma}$)	$0.03396^{\{1\}}$	$0.03583^{\{2\}}$	$0.03897^{\{5\}}$	$0.03804^{\{3\}}$	$0.03861^{\{4\}}$	$0.04344^{\{12\}}$	$0.04027^{\{7\}}$	$0.03907^{\{6\}}$	$0.0426^{\{10\}}$	$0.04548^{\{13\}}$	$0.04293^{\{11\}}$	$0.05766^{\{16\}}$	$0.04106^{\{8\}}$	$0.0531^{\{15\}}$	$0.04151^{\{9\}}$	$0.0501^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00184^{\{1\}}$	$0.00206^{\{2\}}$	$0.00236^{\{4\}}$	$0.00226^{\{3\}}$	$0.00237^{\{5.5\}}$	$0.003^{\{12\}}$	$0.00247^{\{7\}}$	$0.00237^{\{5.5\}}$	$0.00287^{\{10\}}$	$0.00331^{\{13\}}$	$0.00292^{\{11\}}$	$0.00642^{\{16\}}$	$0.00265^{\{8\}}$	$0.00445^{\{15\}}$	$0.00276^{\{9\}}$	$0.00397^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.04245^{\{1\}}$	$0.04479^{\{2\}}$	$0.04871^{\{5\}}$	$0.04755^{\{3\}}$	$0.04827^{\{4\}}$	$0.0543^{\{12\}}$	$0.05034^{\{7\}}$	$0.04884^{\{6\}}$	$0.05325^{\{10\}}$	$0.05685^{\{13\}}$	$0.05366^{\{11\}}$	$0.07208^{\{16\}}$	$0.05133^{\{8\}}$	$0.06638^{\{15\}}$	$0.05189^{\{9\}}$	$0.06263^{\{14\}}$
	D_{abs}	$0.03^{\{1\}}$	$0.03156^{\{2\}}$	$0.03424^{\{5\}}$	$0.03333^{\{3\}}$	$0.03388^{\{4\}}$	$0.03725^{\{10\}}$	$0.03564^{\{7\}}$	$0.03457^{\{6\}}$	$0.03771^{\{12\}}$	$0.03964^{\{13\}}$	$0.03751^{\{11\}}$	$0.05134^{\{16\}}$	$0.03631^{\{8\}}$	$0.04589^{\{15\}}$	$0.03634^{\{9\}}$	$0.04331^{\{14\}}$
	D_{max}	$0.04294^{\{1\}}$	$0.04508^{\{2\}}$	$0.04917^{\{5\}}$	$0.04754^{\{3\}}$	$0.04867^{\{4\}}$	$0.05373^{\{11\}}$	$0.05112^{\{7\}}$	$0.04936^{\{6\}}$	$0.05381^{\{12\}}$	$0.05695^{\{13\}}$	$0.05352^{\{10\}}$	$0.07508^{\{16\}}$	$0.05185^{\{8\}}$	$0.06599^{\{15\}}$	$0.05208^{\{9\}}$	$0.06234^{\{14\}}$
	ASAE	$0.02934^{\{1\}}$	$0.02953^{\{2\}}$	$0.03054^{\{5\}}$	$0.02923^{\{3\}}$	$0.03086^{\{4\}}$	$0.0313^{\{11\}}$	$0.02923^{\{7\}}$	$0.03034^{\{6\}}$	$0.03364^{\{12\}}$	$0.03565^{\{13\}}$	$0.03277^{\{10\}}$	$0.04758^{\{16\}}$	$0.03242^{\{8\}}$	$0.03823^{\{15\}}$	$0.03216^{\{9\}}$	$0.03865^{\{14\}}$
	$\sum Ranks$	$8^{\{1\}}$	$14^{\{2\}}$	$30^{\{5\}}$	$16^{\{3\}}$	$28.5^{\{4\}}$	$65^{\{10.5\}}$	$37^{\{7\}}$	$34.5^{\{6\}}$	$66^{\{12\}}$	$78^{\{13\}}$	$65^{\{10.5\}}$	$96^{\{16\}}$	$50^{\{8\}}$	$89^{\{15\}}$	$54^{\{9\}}$	$85^{\{14\}}$
90	BIAS($\hat{\sigma}$)	$0.02635^{\{3\}}$	$0.02611^{\{1\}}$	$0.02846^{\{7\}}$	$0.02899^{\{8\}}$	$0.02703^{\{4\}}$	$0.03063^{\{11\}}$	$0.02634^{\{2\}}$	$0.02726^{\{5\}}$	$0.0299^{\{9\}}$	$0.03473^{\{13\}}$	$0.03096^{\{12\}}$	$0.0449^{\{16\}}$	$0.02792^{\{6\}}$	$0.03492^{\{14\}}$	$0.02992^{\{10\}}$	$0.03791^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00109^{\{2\}}$	$0.00107^{\{1\}}$	$0.00129^{\{7.5\}}$	$0.00129^{\{7.5\}}$	$0.00114^{\{4\}}$	$0.00148^{\{11\}}$	$0.0011^{\{3\}}$	$0.00117^{\{5\}}$	$0.00143^{\{9.5\}}$	$0.00195^{\{14\}}$	$0.0015^{\{12\}}$	$0.00382^{\{16\}}$	$0.00121^{\{6\}}$	$0.00192^{\{13\}}$	$0.00143^{\{9.5\}}$	$0.00225^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.03294^{\{3\}}$	$0.03263^{\{1\}}$	$0.03557^{\{7\}}$	$0.03624^{\{8\}}$	$0.03379^{\{4\}}$	$0.03828^{\{11\}}$	$0.03292^{\{2\}}$	$0.03408^{\{5\}}$	$0.03737^{\{9\}}$	$0.04341^{\{13\}}$	$0.0387^{\{12\}}$	$0.05613^{\{16\}}$	$0.0349^{\{6\}}$	$0.04365^{\{14\}}$	$0.0374^{\{10\}}$	$0.04739^{\{15\}}$
	D_{abs}	$0.02336^{\{3\}}$	$0.02304^{\{1\}}$	$0.02506^{\{7\}}$	$0.02543^{\{8\}}$	$0.02395^{\{4.5\}}$	$0.02659^{\{11\}}$	$0.0232^{\{2\}}$	$0.02395^{\{4.5\}}$	$0.02632^{\{10\}}$	$0.0304^{\{13\}}$	$0.02711^{\{12\}}$	$0.0401^{\{16\}}$	$0.0247^{\{6\}}$	$0.03053^{\{14\}}$	$0.02623^{\{9\}}$	$0.03308^{\{15\}}$
	D_{max}	$0.03345^{\{3\}}$	$0.033^{\{1\}}$	$0.03596^{\{7\}}$	$0.03652^{\{8\}}$	$0.03417^{\{4\}}$	$0.03827^{\{11\}}$	$0.03325^{\{2\}}$	$0.03448^{\{5\}}$	$0.03774^{\{10\}}$	$0.04364^{\{13\}}$	$0.03888^{\{12\}}$	$0.05841^{\{16\}}$	$0.03536^{\{6\}}$	$0.04398^{\{14\}}$	$0.03771^{\{9\}}$	$0.04757^{\{15\}}$
	ASAE	$0.0187^{\{3\}}$	$0.01865^{\{1\}}$	$0.0196^{\{7\}}$	$0.01892^{\{8\}}$	$0.01923^{\{4\}}$	$0.02011^{\{11\}}$	$0.01835^{\{2\}}$	$0.01888^{\{5\}}$	$0.02105^{\{10\}}$	$0.02337^{\{13\}}$	$0.02136^{\{12\}}$	$0.03104^{\{16\}}$	$0.02023^{\{6\}}$	$0.02389^{\{14\}}$	$0.02069^{\{9\}}$	$0.0249^{\{15\}}$
	$\sum Ranks$	$17^{\{3\}}$	$7^{\{1\}}$	$42.5^{\{7\}}$	$44.5^{\{8\}}$	$26.5^{\{4\}}$	$63^{\{11\}}$	$12^{\{2\}}$	$28.5^{\{5\}}$	$58.5^{\{10\}}$	$79^{\{13\}}$	$72^{\{12\}}$	$96^{\{16\}}$	$39^{\{6\}}$	$83^{\{14\}}$	$57.5^{\{9\}}$	$90^{\{15\}}$
140	BIAS($\hat{\sigma}$)	$0.02079^{\{1\}}$	$0.0225^{\{6\}}$	$0.02271^{\{7\}}$	$0.02169^{\{4\}}$	$0.02183^{\{5\}}$	$0.02517^{\{12\}}$	$0.02102^{\{2\}}$	$0.0211^{\{3\}}$	$0.02393^{\{10\}}$	$0.02748^{\{13\}}$	$0.02502^{\{11\}}$	$0.03819^{\{16\}}$	$0.023^{\{8\}}$	$0.02861^{\{14\}}$	$0.02303^{\{9\}}$	$0.02872^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00068^{\{1\}}$	$0.00079^{\{6\}}$	$8e-04^{\{7\}}$	$0.00074^{\{4\}}$	$0.00075^{\{5\}}$	$0.00102^{\{12\}}$	$0.00069^{\{2\}}$	$0.00071^{\{3\}}$	$0.00089^{\{10\}}$	$0.0012^{\{13\}}$	$0.00097^{\{11\}}$	$0.00321^{\{16\}}$	$0.00082^{\{8\}}$	$0.00131^{\{15\}}$	$0.00088^{\{9\}}$	$0.00129^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.02599^{\{1\}}$	$0.02813^{\{6\}}$	$0.02838^{\{7\}}$	$0.02711^{\{4\}}$	$0.02729^{\{5\}}$	$0.03147^{\{12\}}$	$0.02627^{\{2\}}$	$0.02637^{\{3\}}$	$0.02991^{\{10\}}$	$0.03435^{\{13\}}$	$0.03128^{\{11\}}$	$0.04774^{\{16\}}$	$0.02875^{\{8\}}$	$0.03577^{\{14\}}$	$0.02879^{\{9\}}$	$0.03589^{\{15\}}$
	D_{abs}	$0.01838^{\{1\}}$	$0.01984^{\{6\}}$	$0.02^{\{7\}}$	$0.01902^{\{4\}}$	$0.01923^{\{5\}}$	$0.02183^{\{11\}}$	$0.01847^{\{2\}}$	$0.01861^{\{3\}}$	$0.02107^{\{10\}}$	$0.02414^{\{13\}}$	$0.02192^{\{12\}}$	$0.03423^{\{16\}}$	$0.02026^{\{8\}}$	$0.02506^{\{14\}}$	$0.02028^{\{9\}}$	$0.02512^{\{15\}}$
	D_{max}	$0.02635^{\{1\}}$	$0.02851^{\{6\}}$	$0.02871^{\{7\}}$	$0.02737^{\{4\}}$	$0.0276^{\{5\}}$	$0.0315^{\{11\}}$	$0.02655^{\{2\}}$	$0.0267^{\{3\}}$	$0.03026^{\{10\}}$	$0.03467^{\{13\}}$	$0.03151^{\{12\}}$	$0.04986^{\{16\}}$	$0.0291^{\{9\}}$	$0.03606^{\{14\}}$	$0.02906^{\{8\}}$	$0.0361^{\{15\}}$
	ASAE	$0.0142^{\{1\}}$	$0.01436^{\{6\}}$	$0.01433^{\{7\}}$	$0.0139^{\{4\}}$	$0.01458^{\{5\}}$	$0.01524^{\{11\}}$	$0.01377^{\{2\}}$	$0.01418^{\{3\}}$	$0.01553^{\{10\}}$	$0.01729^{\{13\}}$	$0.01639^{\{12\}}$	$0.02402^{\{16\}}$	$0.01525^{\{9\}}$	$0.01815^{\{14\}}$	$0.0157^{\{8\}}$	$0.01823^{\{15\}}$
	$\sum Ranks$	$9^{\{1\}}$	$36^{\{6\}}$	$49^{\{7.5\}}$	$22^{\{4\}}$	$32^{\{5\}}$	$65^{\{11\}}$	$11^{\{2\}}$	$18^{\{3\}}$	$59^{\{10\}}$	$77^{\{13\}}$	$68^{\{12\}}$	$95^{\{16\}}$	$49^{\{7.5\}}$	$84^{\{14\}}$	$54^{\{9\}}$	$88^{\{15\}}$
200	BIAS($\hat{\sigma}$)	$0.01756^{\{3\}}$	$0.01765^{\{4\}}$	$0.01836^{\{6\}}$	$0.01751^{\{2\}}$	$0.01944^{\{8\}}$	$0.01981^{\{11\}}$	$0.01697^{\{1\}}$	$0.01804^{\{5\}}$	$0.01968^{\{10\}}$	$0.02224^{\{13\}}$	$0.02067^{\{12\}}$	$0.02953^{\{16\}}$	$0.0187^{\{7\}}$	$0.02384^{\{14\}}$	$0.01959^{\{9\}}$	$0.02434^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00048^{\{2.5\}}$	$0.00049^{\{4\}}$	$0.00053^{\{6\}}$	$0.00048^{\{2.5\}}$	$0.00061^{\{9.5\}}$	$0.00063^{\{11\}}$	$0.00046^{\{1\}}$	$0.00051^{\{5\}}$	$6e-04^{\{8\}}$	$0.00077^{\{13\}}$	$0.00069^{\{12\}}$	$0.00165^{\{16\}}$	$0.00056^{\{7\}}$	$0.00091^{\{14\}}$	$0.00061^{\{9.5\}}$	$0.00092^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.02195^{\{3\}}$	$0.02206^{\{4\}}$	$0.02295^{\{6\}}$	$0.02189^{\{2\}}$	$0.0243^{\{8\}}$	$0.02477^{\{11\}}$	$0.02121^{\{1\}}$	$0.02255^{\{5\}}$	$0.02461^{\{10\}}$	$0.0278^{\{13\}}$	$0.02584^{\{12\}}$	$0.03692^{\{16\}}$	$0.02338^{\{7\}}$	$0.0298^{\{14\}}$	$0.02449^{\{9\}}$	$0.03043^{\{15\}}$
	D_{abs}	$0.0155^{\{3\}}$	$0.01554^{\{4\}}$	$0.01619^{\{6\}}$	$0.01534^{\{2\}}$	$0.01708^{\{8\}}$	$0.01725^{\{9.5\}}$	$0.01491^{\{1\}}$	$0.01592^{\{5\}}$	$0.01735^{\{11\}}$	$0.01953^{\{13\}}$	$0.01821^{\{12\}}$	$0.02637^{\{16\}}$	$0.01648^{\{7\}}$	$0.02086^{\{14\}}$	$0.01725^{\{9.5\}}$	$0.02132^{\{15\}}$
	D_{max}	$0.02226^{\{3\}}$	$0.02236^{\{4\}}$	$0.02326^{\{6\}}$	$0.02206^{\{2\}}$	$0.02459^{\{8\}}$	$0.02485^{\{10\}}$	$0.02145^{\{1\}}$	$0.02286^{\{5\}}$	$0.02493^{\{11\}}$	$0.02808^{\{13\}}$	$0.0261^{\{12\}}$	$0.03808^{\{16\}}$	$0.02373^{\{7\}}$	$0.03003^{\{14\}}$	$0.02473^{\{9\}}$	$0.03064^{\{15\}}$
	ASAE	$0.01115^{\{3\}}$	$0.01139^{\{4\}}$	$0.01166^{\{6\}}$	$0.01111^{\{2\}}$	$0.01173^{\{8\}}$	$0.0122^{\{10\}}$	$0.0109^{\{1\}}$	$0.01161^{\{5\}}$	$0.01277^{\{11\}}$	$0.01384^{\{13\}}$	$0.01289^{\{12\}}$	$0.01806^{\{16\}}$	$0.01218^{\{7\}}$	$0.01469^{\{14\}}$	$0.01228^{\{9\}}$	$0.01483^{\{15\}}$
	$\sum Ranks$	$17.5^{\{3\}}$	$24^{\{4\}}$	$36^{\{6\}}$	$12.5^{\{2\}}$	$47.5^{\{8\}}$	$60.5^{\{10\}}$	$6^{\{1\}}$	$30^{\{5\}}$	$69^{\{11\}}$	$77^{\{13\}}$	$71^{\{12\}}$	$95^{\{16\}}$	$43^{\{7\}}$	$83^{\{14\}}$	$55^{\{9\}}$	$89^{\{15\}}$
250	BIAS($\hat{\sigma}$)	$0.01608^{\{3\}}$	$0.0162^{\{4\}}$	$0.01732^{\{7\}}$	$0.01541^{\{1\}}$	$0.01685^{\{6\}}$	$0.01851^{\{12\}}$	$0.01596^{\{2\}}$	$0.01659^{\{5\}}$	$0.01764^{\{10\}}$	$0.0193^{\{13\}}$	$0.01796^{\{11\}}$	$0.02643^{\{16\}}$	$0.01759^{\{9\}}$	$0.02193^{\{14\}}$	$0.01753^{\{8\}}$	$0.02226^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00039^{\{2\}}$	$0.00042^{\{4.5\}}$	$0.00048^{\{7.5\}}$	$0.00038^{\{1\}}$	$0.00046^{\{6\}}$	$0.00053^{\{12\}}$	$4e-04^{\{3\}}$	$0.00042^{\{4.5\}}$	$5e-04^{\{9.5\}}$	$0.00059^{\{13\}}$	$0.00051^{\{11\}}$	$0.00124^{\{16\}}$	$0.00048^{\{7.5\}}$	$0.00074^{\{14\}}$	$5e-04^{\{9.5\}}$	$0.00075^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.0201^{\{3\}}$	$0.02025^{\{4\}}$	$0.02164^{\{7\}}$	$0.01926^{\{1\}}$	$0.02106^{\{6\}}$	$0.02314^{\{12\}}$	$0.01996^{\{2\}}$	$0.02074^{\{5\}}$	$0.02205^{\{10\}}$	$0.02413^{\{13\}}$	$0.02245^{\{11\}}$	$0.03304^{\{16\}}$	$0.02198^{\{9\}}$	$0.02741^{\{14\}}$	$0.02191^{\{8\}}$	$0.02783^{\{15\}}$
	D_{abs}	$0.01417^{\{3\}}$	$0.01428^{\{4\}}$	$0.01526^{\{7\}}$	$0.01353^{\{1\}}$	$0.01485^{\{6\}}$	$0.01619^{\{12\}}$	$0.01403^{\{2\}}$	$0.01464^{\{5\}}$	$0.01556^{\{10\}}$	$0.01695^{\{13\}}$	$0.0158^{\{11\}}$	$0.02347^{\{16\}}$	$0.0155^{\{9\}}$	$0.01927^{\{14\}}$	$0.01539^{\{8\}}$	$0.0195^{\{15\}}$
	D_{max}	$0.02034^{\{3\}}$	$0.02051^{\{4\}}$	$0.02194^{\{7\}}$	$0.01946^{\{1\}}$	$0.02136^{\{6\}}$	$0.02328^{\{12\}}$	$0.02016^{\{2\}}$	$0.021^{\{5\}}$	$0.02233^{\{10\}}$	$0.0244^{\{13\}}$	$0.02267^{\{11\}}$	$0.03389^{\{16\}}$	$0.02225^{\{9\}}$	$0.02766^{\{14\}}$	$0.02211^{\{8\}}$	$0.0281^{\{15\}}$
	ASAE	$0.00962^{\{3\}}$	$0.00972^{\{4\}}$	$0.01021^{\{7\}}$	$0.00963^{\{1\}}$	$0.00998^{\{6\}}$	$0.0107^{\{12\}}$	$0.00954^{\{2\}}$	$0.0101^{\{5\}}$	$0.01107^{\{10\}}$	$0.0118^{\{13\}}$	$0.0113^{\{11\}}$	$0.01526^{\{16\}}$	$0.01042^{\{9\}}$	$0.01271^{\{14\}}$	$0.0108^{\{8\}}$	$0.01269^{\{15\}}$
	$\sum Ranks$	$16^{\{2\}}$	$23.5^{\{4\}}$	$41.5^{\{7\}}$	$8^{\{1\}}$	$34^{\{6\}}$	$66^{\{11\}}$	$23^{\{3\}}$	$29.5^{\{5\}}$	$66.5^{\{12\}}$	$75^{\{13\}}$	$64^{\{10\}}$	$93^{\{16\}}$	$50.5^{\{8\}}$	$82^{\{14\}}$	$57.5^{\{9\}}$	$86^{\{15\}}$
300	BIAS($\hat{\sigma}$)	$0.01402^{\{2\}}$	$0.01525^{\{6\}}$	$0.01502^{\{5\}}$	$0.01446^{\{4\}}$	$0.0156^{\{7\}}$	$0.01693^{\{12\}}$	$0.01382^{\{1\}}$	$0.01436^{\{3\}}$	$0.01683^{\{11\}}$	$0.01793^{\{13\}}$	$0.01622^{\{10\}}$	$0.0252^{\{16\}}$	$0.01591^{\{8\}}$	$0.02078^{\{15\}}$	$0.01595^{\{9\}}$	$0.0194^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00032^{\{2.5\}}$	$0.00036^{\{5.5\}}$	$0.00036^{\{5.5\}}$	$0.00033^{\{4\}}$	$0.00039^{\{7.5\}}$	$0.00045^{\{12\}}$	$3e-04^{\{1\}}$	$0.00032^{\{2.5\}}$	$0.00043^{\{11\}}$	$0.00052^{\{13\}}$	$0.00042^{\{10\}}$	$0.00112^{\{16\}}$	$0.00039^{\{7.5\}}$	$0.00067^{\{15\}}$	$4e-04^{\{9\}}$	$6e-04^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.01752^{\{2\}}$	$0.01906^{\{6\}}$	$0.01878^{\{5\}}$	$0.01807^{\{4\}}$	$0.0195^{\{7\}}$	$0.02116^{\{12\}}$	$0.01727^{\{1\}}$	$0.01795^{\{3\}}$	$0.02103^{\{11\}}$	$0.02241^{\{13\}}$	$0.02027^{\{10\}}$	$0.03151^{\{16\}}$	$0.01989^{\{8\}}$	$0.02597^{\{15\}}$	$0.01993^{\{9\}}$	$0.02425^{\{14\}}$
	D_{abs}	$0.0124^{\{2\}}$	$0.01341^{\{6\}}$	$0.01325^{\{5\}}$	$0.01272^{\{4\}}$	$0.01372^{\{7\}}$	$0.01481^{\{11\}}$	$0.01216^{\{1\}}$	$0.01266^{\{3\}}$	$0.01482^{\{12\}}$	$0.01576^{\{13\}}$	$0.01426^{\{10\}}$	$0.02243^{\{16\}}$	$0.01402^{\{8\}}$	$0.01822^{\{15\}}$	$0.01404^{\{9\}}$	$0.01708^{\{14\}}$
	D_{max}	$0.01779^{\{2\}}$	$0.01929^{\{6\}}$	$0.01901^{\{5\}}$	$0.0183^{\{4\}}$	$0.01974^{\{7\}}$	$0.02132^{\{12\}}$	$0.01746^{\{1\}}$	$0.01819^{\{3\}}$	$0.0213^{\{11\}}$	$0.02265^{\{13\}}$	$0.02049^{\{10\}}$	$0.03235^{\{16\}}$	$0.02013^{\{8\}}$	$0.02622^{\{15\}}$	$0.02018^{\{9\}}$	$0.02454^{\{14\}}$
	ASAE	$0.00859^{\{2\}}$	$0.00892^{\{6\}}$	$0.00895^{\{5\}}$	$0.00857^{\{4\}}$	$0.00918^{\{7\}}$	$0.0094^{\{12\}}$	$0.00854^{\{1\}}$	$0.00889^{\{3\}}$	$0.00969^{\{11\}}$	$0.01062^{\{13\}}$	$0.00989^{\{10\}}$	$0.01413^{\{16\}}$	$0.00931^{\{8\}}$	$0.01135^{\{15\}}$	$0.00955^{\{9\}}$	$0.01121^{\{14\}}$
	$\sum Ranks$	$12.5^{\{1\}}$	$33.5^{\{6\}}$	$30.5^{\{5\}}$	$21^{\{4\}}$	$41.5^{\{7\}}$	$66^{\{12\}}$	$19^{\{3\}}$	$17.5^{\{2\}}$	$65^{\{11\}}$	$76^{\{13\}}$	$60^{\{9\}}$	$93^{\{16\}}$	$46.5^{\{8\}}$	$87^{\{15\}}$	$61^{\{10\}}$	$86^{\{14\}}$
400	BIAS($\hat{\sigma}$)	$0.01288^{\{5\}}$	$0.0126^{\{2\}}$	$0.01336^{\{7\}}$	$0.01277^{\{3\}}$	$0.01286^{\{4\}}$	$0.0144^{\{11\}}$	$0.01212^{\{1\}}$	$0.01295^{\{6\}}$	$0.01458^{\{12\}}$	$0.01617^{\{13\}}$	$0.01429^{\{10\}}$	$0.02132^{\{16\}}$	$0.01371^{\{8\}}$	$0.01719^{\{15\}}$	$0.01392^{\{9\}}$	$0.01675^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00026^{\{4.5\}}$	$0.00025^{\{2\}}$	$0.00027^{\{7\}}$	$0.00026^{\{4.5\}}$	$0.00026^{\{4.5\}}$	$0.00032^{\{10.5\}}$	$0.00024^{\{1\}}$	$0.00026^{\{4.5\}}$	$0.00033^{\{12\}}$	$4e-04^{\{13\}}$	$0.00032^{\{10.5\}}$	$0.00084^{\{16\}}$	$0.00029^{\{8\}}$	$0.00046^{\{15\}}$	$0.00031^{\{9\}}$	$0.00044^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.0161^{\{5\}}$	$0.01575^{\{2\}}$	$0.0167^{\{7\}}$	$0.01596^{\{3\}}$	$0.01608^{\{4\}}$	$0.018^{\{11\}}$	$0.01515^{\{1\}}$	$0.01619^{\{6\}}$	$0.01823^{\{12\}}$	$0.02021^{\{13\}}$	$0.01786^{\{10\}}$	$0.02665^{\{16\}}$	$0.01713^{\{8\}}$	$0.02149^{\{15\}}$	$0.0174^{\{9\}}$	$0.02094^{\{14\}}$
	D_{abs}	$0.01134^{\{4\}}$	$0.01111^{\{2\}}$	$0.01176^{\{7\}}$	$0.01123^{\{3\}}$	$0.01136^{\{5\}}$	$0.01256^{\{10\}}$	$0.01067^{\{1\}}$	$0.01139^{\{6\}}$	$0.01285^{\{12\}}$	$0.01421^{\{13\}}$	$0.01258^{\{11\}}$	$0.01893^{\{16\}}$	$0.0121^{\{8\}}$	$0.0151^{\{15\}}$	$0.01222^{\{9\}}$	$0.01472^{\{14\}}$
	D_{max}	$0.01632^{\{5\}}$	$0.01595^{\{2\}}$	$0.01693^{\{7\}}$	$0.01613^{\{3\}}$	$0.0163^{\{4\}}$	$0.01812^{\{11\}}$	$0.01533^{\{1\}}$	$0.0164^{\{6\}}$	$0.01848^{\{12\}}$	$0.02044^{\{13\}}$	$0.01805^{\{10\}}$	$0.02737^{\{16\}}$	$0.01737^{\{8\}}$	$0.02173^{\{15\}}$	$0.01758^{\{9\}}$	$0.02115^{\{14\}}$
	ASAE	$0.00733^{\{5\}}$	$0.00736^{\{2\}}$	$0.00762^{\{7\}}$	$0.00717^{\{3\}}$	$0.00737^{\{4\}}$	$0.00798^{\{11\}}$	$0.00697^{\{1\}}$	$0.0074^{\{6\}}$	$0.00834^{\{12\}}$	$0.00888^{\{13\}}$	$0.00823^{\{10\}}$	$0.01174^{\{16\}}$	$0.00778^{\{8\}}$	$0.00938^{\{15\}}$	$0.008^{\{9\}}$	$0.0094^{\{14\}}$
	$\sum Ranks$	$26.5^{\{4.5\}}$	$14^{\{2\}}$	$42^{\{7\}}$	$18.5^{\{3\}}$	$26.5^{\{4.5\}}$	$62.5^{\{10.5\}}$	$6^{\{1\}}$	$34.5^{\{6\}}$	$72^{\{12\}}$	$81^{\{13\}}$	$62.5^{\{10.5\}}$	$95^{\{16\}}$	$48^{\{8\}}$	$88^{\{15\}}$	$55^{\{9\}}$	$84^{\{14\}}$

Table 3. Numerical values of simulation measures for $\sigma = 1.5$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSD	MSSLD	MSLND
15	BIAS($\hat{\sigma}$)	$0.11925^{\{3\}}$	$0.11767^{\{2\}}$	$0.12245^{\{4\}}$	$0.12599^{\{6\}}$	$0.12713^{\{7\}}$	$0.13775^{\{12\}}$	$0.11598^{\{1\}}$	$0.13408^{\{10\}}$	$0.1366^{\{11\}}$	$0.14496^{\{13\}}$	$0.1281^{\{8\}}$	$0.18076^{\{16\}}$	$0.13124^{\{9\}}$	$0.17179^{\{15\}}$	$0.12533^{\{5\}}$	$0.1707^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.02231^{\{3\}}$	$0.02166^{\{2\}}$	$0.02409^{\{5\}}$	$0.02489^{\{6\}}$	$0.02518^{\{7\}}$	$0.0314^{\{12\}}$	$0.02138^{\{1\}}$	$0.02824^{\{10\}}$	$0.02965^{\{11\}}$	$0.03528^{\{13\}}$	$0.02625^{\{8\}}$	$0.05809^{\{16\}}$	$0.02699^{\{9\}}$	$0.04732^{\{15\}}$	$0.02399^{\{4\}}$	$0.04611^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.0795^{\{3\}}$	$0.07845^{\{2\}}$	$0.08163^{\{4\}}$	$0.084^{\{6\}}$	$0.08476^{\{7\}}$	$0.09184^{\{12\}}$	$0.07732^{\{1\}}$	$0.08939^{\{10\}}$	$0.09107^{\{11\}}$	$0.09664^{\{13\}}$	$0.0854^{\{8\}}$	$0.12051^{\{16\}}$	$0.08749^{\{9\}}$	$0.11453^{\{15\}}$	$0.08355^{\{5\}}$	$0.1138^{\{14\}}$
	D_{abs}	$0.05621^{\{3\}}$	$0.05562^{\{2\}}$	$0.05768^{\{4\}}$	$0.05779^{\{5\}}$	$0.05964^{\{8\}}$	$0.06198^{\{10\}}$	$0.05448^{\{1\}}$	$0.06326^{\{11\}}$	$0.06456^{\{12\}}$	$0.06743^{\{13\}}$	$0.05955^{\{7\}}$	$0.08471^{\{16\}}$	$0.06173^{\{9\}}$	$0.07872^{\{15\}}$	$0.05854^{\{6\}}$	$0.07731^{\{14\}}$
	D_{max}	$0.07974^{\{3\}}$	$0.0785^{\{2\}}$	$0.08122^{\{4\}}$	$0.08252^{\{5\}}$	$0.08475^{\{8\}}$	$0.08839^{\{10\}}$	$0.07706^{\{1\}}$	$0.08888^{\{11\}}$	$0.09158^{\{12\}}$	$0.09597^{\{13\}}$	$0.08384^{\{7\}}$	$0.12498^{\{16\}}$	$0.08727^{\{9\}}$	$0.11286^{\{15\}}$	$0.08298^{\{6\}}$	$0.11054^{\{14\}}$
	ASAE	$0.06526^{\{3\}}$	$0.06241^{\{2\}}$	$0.0652^{\{4\}}$	$0.0624^{\{5\}}$	$0.064^{\{8\}}$	$0.06475^{\{10\}}$	$0.06133^{\{1\}}$	$0.06276^{\{11\}}$	$0.07301^{\{12\}}$	$0.0718^{\{13\}}$	$0.06907^{\{7\}}$	$0.09945^{\{16\}}$	$0.06706^{\{9\}}$	$0.07993^{\{15\}}$	$0.06641^{\{6\}}$	$0.07915^{\{14\}}$
	$\sum Ranks$	$23^{\{3\}}$	$13^{\{2\}}$	$28^{\{4\}}$	$30^{\{5\}}$	$42^{\{7\}}$	$62^{\{11\}}$	$6^{\{1\}}$	$56^{\{10\}}$	$70^{\{12\}}$	$77^{\{13\}}$	$49^{\{8\}}$	$96^{\{16\}}$	$55^{\{9\}}$	$90^{\{15\}}$	$35^{\{6\}}$	$84^{\{14\}}$
45	BIAS($\hat{\sigma}$)	$0.06671^{\{1\}}$	$0.06888^{\{2\}}$	$0.07179^{\{5\}}$	$0.07026^{\{3\}}$	$0.07378^{\{6\}}$	$0.08038^{\{11\}}$	$0.07126^{\{4\}}$	$0.07393^{\{7\}}$	$0.08292^{\{12\}}$	$0.08834^{\{13\}}$	$0.07975^{\{10\}}$	$0.11949^{\{16\}}$	$0.07508^{\{9\}}$	$0.09778^{\{15\}}$	$0.07493^{\{8\}}$	$0.09737^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00704^{\{1\}}$	$0.00775^{\{2\}}$	$0.00846^{\{5\}}$	$0.00786^{\{3\}}$	$0.00875^{\{8\}}$	$0.0103^{\{11\}}$	$0.0079^{\{4\}}$	$0.00848^{\{6\}}$	$0.01069^{\{12\}}$	$0.0122^{\{13\}}$	$0.00988^{\{10\}}$	$0.03034^{\{16\}}$	$0.00863^{\{7\}}$	$0.015^{\{14\}}$	$0.00927^{\{9\}}$	$0.01535^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.04447^{\{1\}}$	$0.04592^{\{2\}}$	$0.04786^{\{5\}}$	$0.04684^{\{3\}}$	$0.04918^{\{6\}}$	$0.05359^{\{11\}}$	$0.04751^{\{4\}}$	$0.04929^{\{7\}}$	$0.05528^{\{12\}}$	$0.05889^{\{13\}}$	$0.05317^{\{10\}}$	$0.07966^{\{16\}}$	$0.05005^{\{9\}}$	$0.06519^{\{15\}}$	$0.04995^{\{8\}}$	$0.06491^{\{14\}}$
	D_{abs}	$0.03148^{\{1\}}$	$0.03233^{\{2\}}$	$0.03391^{\{5\}}$	$0.03281^{\{3\}}$	$0.03477^{\{7\}}$	$0.03687^{\{10\}}$	$0.0335^{\{4\}}$	$0.03466^{\{6\}}$	$0.03897^{\{12\}}$	$0.04134^{\{13\}}$	$0.03719^{\{11\}}$	$0.0562^{\{16\}}$	$0.0354^{\{9\}}$	$0.04542^{\{15\}}$	$0.0351^{\{8\}}$	$0.04501^{\{14\}}$
	D_{max}	$0.04503^{\{1\}}$	$0.04631^{\{2\}}$	$0.04843^{\{5\}}$	$0.04694^{\{3\}}$	$0.04952^{\{6\}}$	$0.05316^{\{10\}}$	$0.0479^{\{4\}}$	$0.04964^{\{7\}}$	$0.05592^{\{12\}}$	$0.0594^{\{13\}}$	$0.05321^{\{11\}}$	$0.08301^{\{16\}}$	$0.05068^{\{9\}}$	$0.06517^{\{15\}}$	$0.05003^{\{8\}}$	$0.06474^{\{14\}}$
	ASAE	$0.02942^{\{1\}}$	$0.02983^{\{2\}}$	$0.0306^{\{5\}}$	$0.02916^{\{3\}}$	$0.03112^{\{6\}}$	$0.03076^{\{10\}}$	$0.02874^{\{4\}}$	$0.03011^{\{7\}}$	$0.03379^{\{12\}}$	$0.0359^{\{13\}}$	$0.03274^{\{11\}}$	$0.04758^{\{16\}}$	$0.03223^{\{9\}}$	$0.03818^{\{15\}}$	$0.03271^{\{8\}}$	$0.03868^{\{14\}}$
	$\sum Ranks$	$8^{\{1\}}$	$14^{\{2\}}$	$31^{\{5\}}$	$17^{\{3\}}$	$41^{\{7\}}$	$60^{\{10\}}$	$21^{\{4\}}$	$38^{\{6\}}$	$72^{\{12\}}$	$78^{\{13\}}$	$63^{\{11\}}$	$96^{\{16\}}$	$52^{\{9\}}$	$88^{\{15\}}$	$51^{\{8\}}$	$86^{\{14\}}$
90	BIAS($\hat{\sigma}$)	$0.04671^{\{1\}}$	$0.05054^{\{4\}}$	$0.05157^{\{5\}}$	$0.04832^{\{2\}}$	$0.05235^{\{6\}}$	$0.05671^{\{12\}}$	$0.04862^{\{3\}}$	$0.0525^{\{7\}}$	$0.05258^{\{8\}}$	$0.05981^{\{13\}}$	$0.05487^{\{10\}}$	$0.08519^{\{16\}}$	$0.05426^{\{9\}}$	$0.06916^{\{15\}}$	$0.05634^{\{11\}}$	$0.06874^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.0035^{\{1\}}$	$0.00413^{\{4\}}$	$0.0042^{\{5\}}$	$0.00373^{\{2\}}$	$0.00436^{\{7\}}$	$0.00503^{\{11.5\}}$	$0.00381^{\{3\}}$	$0.00431^{\{6\}}$	$0.00441^{\{8\}}$	$0.00555^{\{13\}}$	$0.00478^{\{10\}}$	$0.01357^{\{16\}}$	$0.00461^{\{9\}}$	$0.00768^{\{15\}}$	$0.00503^{\{11.5\}}$	$0.00731^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.03114^{\{1\}}$	$0.03369^{\{4\}}$	$0.03438^{\{5\}}$	$0.03222^{\{2\}}$	$0.0349^{\{6\}}$	$0.03781^{\{12\}}$	$0.03241^{\{3\}}$	$0.035^{\{7\}}$	$0.03505^{\{8\}}$	$0.03987^{\{13\}}$	$0.03658^{\{10\}}$	$0.0568^{\{16\}}$	$0.03617^{\{9\}}$	$0.04611^{\{15\}}$	$0.03756^{\{11\}}$	$0.04583^{\{14\}}$
	D_{abs}	$0.02195^{\{1\}}$	$0.02379^{\{4\}}$	$0.02424^{\{5\}}$	$0.02259^{\{2\}}$	$0.02455^{\{6\}}$	$0.02614^{\{11\}}$	$0.02288^{\{3\}}$	$0.02467^{\{7\}}$	$0.0247^{\{8\}}$	$0.02795^{\{13\}}$	$0.02568^{\{10\}}$	$0.04047^{\{16\}}$	$0.02552^{\{9\}}$	$0.03218^{\{15\}}$	$0.02636^{\{12\}}$	$0.03203^{\{14\}}$
	D_{max}	$0.03154^{\{1\}}$	$0.03412^{\{4\}}$	$0.03475^{\{5\}}$	$0.03247^{\{2\}}$	$0.03529^{\{6\}}$	$0.03771^{\{11\}}$	$0.03277^{\{3\}}$	$0.0354^{\{7\}}$	$0.03541^{\{8\}}$	$0.04016^{\{13\}}$	$0.03685^{\{10\}}$	$0.05901^{\{16\}}$	$0.03662^{\{9\}}$	$0.04627^{\{15\}}$	$0.03775^{\{12\}}$	$0.04607^{\{14\}}$
	ASAE	$0.01846^{\{1\}}$	$0.01912^{\{4\}}$	$0.01943^{\{5\}}$	$0.01849^{\{2\}}$	$0.01898^{\{6\}}$	$0.02022^{\{11\}}$	$0.01813^{\{3\}}$	$0.01898^{\{7\}}$	$0.02126^{\{8\}}$	$0.02253^{\{13\}}$	$0.02115^{\{10\}}$	$0.03067^{\{16\}}$	$0.01994^{\{9\}}$	$0.02483^{\{15\}}$	$0.02089^{\{12\}}$	$0.02482^{\{14\}}$
	$\sum Ranks$	$7^{\{1\}}$	$26^{\{4\}}$	$32^{\{5\}}$	$13^{\{2\}}$	$35^{\{6\}}$	$66.5^{\{11\}}$	$16^{\{3\}}$	$39^{\{7\}}$	$52^{\{8\}}$	$78^{\{13\}}$	$61^{\{10\}}$	$96^{\{16\}}$	$53^{\{9\}}$	$90^{\{15\}}$	$67.5^{\{12\}}$	$84^{\{14\}}$
140	BIAS($\hat{\sigma}$)	$0.03758^{\{1\}}$	$0.04058^{\{4\}}$	$0.0413^{\{5\}}$	$0.04008^{\{3\}}$	$0.04206^{\{8\}}$	$0.04462^{\{11\}}$	$0.03765^{\{2\}}$	$0.04166^{\{7\}}$	$0.0443^{\{10\}}$	$0.0512^{\{13\}}$	$0.04487^{\{12\}}$	$0.06863^{\{16\}}$	$0.04165^{\{6\}}$	$0.0531^{\{14\}}$	$0.04281^{\{9\}}$	$0.05523^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00225^{\{1\}}$	$0.0026^{\{4\}}$	$0.00271^{\{5\}}$	$0.00245^{\{3\}}$	$0.00279^{\{8\}}$	$0.00314^{\{11\}}$	$0.00231^{\{2\}}$	$0.00274^{\{6.5\}}$	$0.00306^{\{10\}}$	$0.00415^{\{13\}}$	$0.00324^{\{12\}}$	$0.00986^{\{16\}}$	$0.00274^{\{6.5\}}$	$0.0046^{\{14\}}$	$0.00294^{\{9\}}$	$0.00481^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.02505^{\{1\}}$	$0.02705^{\{4\}}$	$0.02753^{\{5\}}$	$0.02672^{\{3\}}$	$0.02804^{\{8\}}$	$0.02974^{\{11\}}$	$0.0251^{\{2\}}$	$0.02777^{\{6.5\}}$	$0.02954^{\{10\}}$	$0.03413^{\{13\}}$	$0.02992^{\{12\}}$	$0.04575^{\{16\}}$	$0.02777^{\{6.5\}}$	$0.0354^{\{14\}}$	$0.02854^{\{9\}}$	$0.03682^{\{15\}}$
	D_{abs}	$0.01763^{\{1\}}$	$0.01904^{\{4\}}$	$0.01943^{\{5\}}$	$0.01877^{\{3\}}$	$0.01973^{\{8\}}$	$0.0207^{\{10\}}$	$0.01765^{\{2\}}$	$0.01959^{\{6\}}$	$0.02085^{\{11\}}$	$0.02401^{\{13\}}$	$0.02097^{\{12\}}$	$0.0328^{\{16\}}$	$0.0196^{\{7\}}$	$0.02479^{\{14\}}$	$0.02002^{\{9\}}$	$0.0258^{\{15\}}$
	D_{max}	$0.02535^{\{1\}}$	$0.0274^{\{4\}}$	$0.02789^{\{5\}}$	$0.02695^{\{3\}}$	$0.02835^{\{8\}}$	$0.02982^{\{10\}}$	$0.02536^{\{2\}}$	$0.02815^{\{7\}}$	$0.02991^{\{11\}}$	$0.03449^{\{13\}}$	$0.03012^{\{12\}}$	$0.04758^{\{16\}}$	$0.02808^{\{6\}}$	$0.03568^{\{14\}}$	$0.02878^{\{9\}}$	$0.03708^{\{15\}}$
	ASAE	$0.01388^{\{1\}}$	$0.01438^{\{4\}}$	$0.01442^{\{5\}}$	$0.0141^{\{3\}}$	$0.01469^{\{8\}}$	$0.01534^{\{10\}}$	$0.01386^{\{2\}}$	$0.01416^{\{7\}}$	$0.01573^{\{11\}}$	$0.01693^{\{13\}}$	$0.0161^{\{12\}}$	$0.02335^{\{16\}}$	$0.0152^{\{6\}}$	$0.01824^{\{14\}}$	$0.0157^{\{9\}}$	$0.01873^{\{15\}}$
	$\sum Ranks$	$7^{\{1\}}$	$25^{\{4\}}$	$31^{\{5\}}$	$18^{\{3\}}$	$47^{\{8\}}$	$62^{\{10\}}$	$11^{\{2\}}$	$37^{\{6\}}$	$63^{\{11\}}$	$78^{\{13\}}$	$72^{\{12\}}$	$96^{\{16\}}$	$40^{\{7\}}$	$84^{\{14\}}$	$55^{\{9\}}$	$90^{\{15\}}$
200	BIAS($\hat{\sigma}$)	$0.032^{\{1\}}$	$0.03277^{\{3\}}$	$0.03555^{\{6\}}$	$0.03364^{\{4\}}$	$0.03601^{\{7\}}$	$0.03883^{\{12\}}$	$0.03485^{\{5\}}$	$0.03259^{\{2\}}$	$0.03644^{\{10\}}$	$0.04323^{\{13\}}$	$0.03808^{\{11\}}$	$0.05677^{\{16\}}$	$0.0362^{\{9\}}$	$0.04451^{\{14\}}$	$0.03613^{\{8\}}$	$0.04749^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.0016^{\{1\}}$	$0.00165^{\{2\}}$	$0.00201^{\{6\}}$	$0.00176^{\{4\}}$	$0.00202^{\{7\}}$	$0.0024^{\{12\}}$	$0.00185^{\{5\}}$	$0.00166^{\{3\}}$	$0.0021^{\{10\}}$	$0.00295^{\{13\}}$	$0.0023^{\{11\}}$	$0.00686^{\{16\}}$	$0.00205^{\{8\}}$	$0.00312^{\{14\}}$	$0.00207^{\{9\}}$	$0.00347^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.02134^{\{1\}}$	$0.02185^{\{3\}}$	$0.0237^{\{6\}}$	$0.02243^{\{4\}}$	$0.024^{\{7\}}$	$0.02588^{\{12\}}$	$0.02323^{\{5\}}$	$0.02173^{\{2\}}$	$0.02429^{\{10\}}$	$0.02882^{\{13\}}$	$0.02539^{\{11\}}$	$0.03785^{\{16\}}$	$0.02413^{\{9\}}$	$0.02967^{\{14\}}$	$0.02409^{\{8\}}$	$0.03166^{\{15\}}$
	D_{abs}	$0.01506^{\{1\}}$	$0.01541^{\{3\}}$	$0.01672^{\{6\}}$	$0.01576^{\{4\}}$	$0.01692^{\{7\}}$	$0.01809^{\{12\}}$	$0.01639^{\{5\}}$	$0.01528^{\{2\}}$	$0.0171^{\{10\}}$	$0.02026^{\{13\}}$	$0.0178^{\{11\}}$	$0.02708^{\{16\}}$	$0.01699^{\{9\}}$	$0.02083^{\{14\}}$	$0.01697^{\{8\}}$	$0.02217^{\{15\}}$
	D_{max}	$0.02164^{\{1\}}$	$0.02217^{\{3\}}$	$0.02398^{\{6\}}$	$0.02264^{\{4\}}$	$0.02431^{\{7\}}$	$0.026^{\{12\}}$	$0.02353^{\{5\}}$	$0.02201^{\{2\}}$	$0.02461^{\{10\}}$	$0.02909^{\{13\}}$	$0.02556^{\{11\}}$	$0.03917^{\{16\}}$	$0.02443^{\{9\}}$	$0.02993^{\{14\}}$	$0.02434^{\{8\}}$	$0.03185^{\{15\}}$
	ASAE	$0.01109^{\{1\}}$	$0.0114^{\{3\}}$	$0.0116^{\{6\}}$	$0.01118^{\{4\}}$	$0.01156^{\{7\}}$	$0.01232^{\{12\}}$	$0.01092^{\{5\}}$	$0.01141^{\{2\}}$	$0.01248^{\{10\}}$	$0.01395^{\{13\}}$	$0.01258^{\{11\}}$	$0.01814^{\{16\}}$	$0.012^{\{9\}}$	$0.01474^{\{14\}}$	$0.01232^{\{8\}}$	$0.01473^{\{15\}}$
	$\sum Ranks$	$7^{\{1\}}$	$18^{\{3\}}$	$37^{\{6\}}$	$23^{\{4\}}$	$41^{\{7\}}$	$69^{\{12\}}$	$26^{\{5\}}$	$16^{\{2\}}$	$61^{\{10\}}$	$78^{\{13\}}$	$67^{\{11\}}$	$96^{\{16\}}$	$52^{\{9\}}$	$85^{\{14\}}$	$51^{\{8\}}$	$89^{\{15\}}$
250	BIAS($\hat{\sigma}$)	$0.02758^{\{1\}}$	$0.02991^{\{3\}}$	$0.03018^{\{4\}}$	$0.03047^{\{6\}}$	$0.03098^{\{7\}}$	$0.03508^{\{12\}}$	$0.03037^{\{5\}}$	$0.02963^{\{2\}}$	$0.03476^{\{11\}}$	$0.03755^{\{13\}}$	$0.03416^{\{10\}}$	$0.05271^{\{16\}}$	$0.03129^{\{8\}}$	$0.04037^{\{15\}}$	$0.03223^{\{9\}}$	$0.03824^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00118^{\{1\}}$	$0.00142^{\{4.5\}}$	$0.00141^{\{3\}}$	$0.00147^{\{6.5\}}$	$0.00147^{\{6.5\}}$	$0.00199^{\{12\}}$	$0.00142^{\{4.5\}}$	$0.00138^{\{2\}}$	$0.00193^{\{11\}}$	$0.00219^{\{13\}}$	$0.00185^{\{10\}}$	$0.00668^{\{16\}}$	$0.00155^{\{8\}}$	$0.00261^{\{15\}}$	$0.00166^{\{9\}}$	$0.00237^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.01839^{\{1\}}$	$0.01994^{\{3\}}$	$0.02012^{\{4\}}$	$0.02032^{\{6\}}$	$0.02066^{\{7\}}$	$0.02339^{\{12\}}$	$0.02025^{\{5\}}$	$0.01975^{\{2\}}$	$0.02318^{\{11\}}$	$0.02503^{\{13\}}$	$0.02278^{\{10\}}$	$0.03514^{\{16\}}$	$0.02086^{\{8\}}$	$0.02691^{\{15\}}$	$0.02149^{\{9\}}$	$0.02549^{\{14\}}$
	D_{abs}	$0.01299^{\{1\}}$	$0.01407^{\{3\}}$	$0.01421^{\{4\}}$	$0.01427^{\{5\}}$	$0.0146^{\{7\}}$	$0.01635^{\{11.5\}}$	$0.01429^{\{6\}}$	$0.0139^{\{2\}}$	$0.01635^{\{11.5\}}$	$0.01759^{\{13\}}$	$0.01599^{\{10\}}$	$0.02501^{\{16\}}$	$0.01471^{\{8\}}$	$0.01892^{\{15\}}$	$0.01511^{\{9\}}$	$0.01796^{\{14\}}$
	D_{max}	$0.01864^{\{1\}}$	$0.02018^{\{3\}}$	$0.0204^{\{4\}}$	$0.0205^{\{5\}}$	$0.02092^{\{7\}}$	$0.02353^{\{12\}}$	$0.02054^{\{6\}}$	$0.01998^{\{2\}}$	$0.02348^{\{11\}}$	$0.02529^{\{13\}}$	$0.02298^{\{10\}}$	$0.03632^{\{16\}}$	$0.02114^{\{8\}}$	$0.02718^{\{15\}}$	$0.02171^{\{9\}}$	$0.02579^{\{14\}}$
	ASAE	$0.00963^{\{1\}}$	$0.00996^{\{3\}}$	$0.00998^{\{4\}}$	$0.00975^{\{5\}}$	$0.00993^{\{7\}}$	$0.0107^{\{12\}}$	$0.00973^{\{6\}}$	$0.0097^{\{2\}}$	$0.0109^{\{11\}}$	$0.01193^{\{13\}}$	$0.01096^{\{10\}}$	$0.01631^{\{16\}}$	$0.01056^{\{8\}}$	$0.01257^{\{15\}}$	$0.01087^{\{9\}}$	$0.01268^{\{14\}}$
	$\sum Ranks$	$6^{\{1\}}$	$22.5^{\{3\}}$	$26^{\{4\}}$	$32.5^{\{6\}}$	$39.5^{\{7\}}$	$68.5^{\{12\}}$	$29.5^{\{5\}}$	$12^{\{2\}}$	$66.5^{\{11\}}$	$78^{\{13\}}$	$62^{\{10\}}$	$96^{\{16\}}$	$48^{\{8\}}$	$89^{\{15\}}$	$55^{\{9\}}$	$85^{\{14\}}$
300	BIAS($\hat{\sigma}$)	$0.02639^{\{1\}}$	$0.02735^{\{4\}}$	$0.02799^{\{6\}}$	$0.02723^{\{3\}}$	$0.02815^{\{8\}}$	$0.03118^{\{12\}}$	$0.02665^{\{2\}}$	$0.02766^{\{5\}}$	$0.03088^{\{11\}}$	$0.03334^{\{13\}}$	$0.02979^{\{9\}}$	$0.04816^{\{16\}}$	$0.02809^{\{7\}}$	$0.03807^{\{15\}}$	$0.03031^{\{10\}}$	$0.0363^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00107^{\{1\}}$	$0.00118^{\{4\}}$	$0.00125^{\{8\}}$	$0.00117^{\{3\}}$	$0.00123^{\{6\}}$	$0.00159^{\{12\}}$	$0.00115^{\{2\}}$	$0.00122^{\{5\}}$	$0.00152^{\{11\}}$	$0.00182^{\{13\}}$	$0.00139^{\{9\}}$	$0.00579^{\{16\}}$	$0.00124^{\{7\}}$	$0.00223^{\{15\}}$	$0.00144^{\{10\}}$	$0.00213^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.01759^{\{1\}}$	$0.01823^{\{4\}}$	$0.01866^{\{6\}}$	$0.01815^{\{3\}}$	$0.01877^{\{8\}}$	$0.02079^{\{12\}}$	$0.01776^{\{2\}}$	$0.01844^{\{5\}}$	$0.02058^{\{11\}}$	$0.02222^{\{13\}}$	$0.01986^{\{9\}}$	$0.0321^{\{16\}}$	$0.01873^{\{7\}}$	$0.02538^{\{15\}}$	$0.02021^{\{10\}}$	$0.0242^{\{14\}}$
	D_{abs}	$0.01243^{\{1\}}$	$0.01285^{\{4\}}$	$0.01316^{\{6\}}$	$0.01275^{\{3\}}$	$0.01321^{\{8\}}$	$0.0145^{\{12\}}$	$0.01253^{\{2\}}$	$0.013^{\{5\}}$	$0.01448^{\{11\}}$	$0.01564^{\{13\}}$	$0.01397^{\{9\}}$	$0.02294^{\{16\}}$	$0.0132^{\{7\}}$	$0.0178^{\{15\}}$	$0.01423^{\{10\}}$	$0.01702^{\{14\}}$
	D_{max}	$0.01784^{\{1\}}$	$0.01846^{\{4\}}$	$0.01892^{\{6\}}$	$0.01834^{\{3\}}$	$0.01901^{\{8\}}$	$0.02092^{\{12\}}$	$0.01796^{\{2\}}$	$0.01869^{\{5\}}$	$0.02087^{\{11\}}$	$0.02247^{\{13\}}$	$0.02009^{\{9\}}$	$0.0333^{\{16\}}$	$0.01897^{\{7\}}$	$0.02558^{\{15\}}$	$0.02042^{\{10\}}$	$0.02447^{\{14\}}$
	ASAE	$0.00847^{\{1\}}$	$0.00886^{\{4\}}$	$0.00912^{\{6\}}$	$0.00867^{\{3\}}$	$0.00887^{\{8\}}$	$0.00963^{\{12\}}$	$0.00845^{\{2\}}$	$0.00888^{\{5\}}$	$0.00996^{\{11\}}$	$0.01066^{\{13\}}$	$0.00995^{\{9\}}$	$0.01476^{\{16\}}$	$0.00924^{\{7\}}$	$0.01134^{\{15\}}$	$0.00958^{\{10\}}$	$0.01106^{\{14\}}$
	$\sum Ranks$	$7^{\{1\}}$	$24^{\{4\}}$	$39^{\{6\}}$	$18^{\{3\}}$	$43^{\{7.5\}}$	$70^{\{12\}}$	$11^{\{2\}}$	$31^{\{5\}}$	$67^{\{11\}}$	$78^{\{13\}}$	$56^{\{9\}}$	$96^{\{16\}}$	$43^{\{7.5\}}$	$90^{\{15\}}$	$59^{\{10\}}$	$84^{\{14\}}$
400	BIAS($\hat{\sigma}$)	$0.02391^{\{3\}}$	$0.02436^{\{5\}}$	$0.02445^{\{7\}}$	$0.02227^{\{1\}}$	$0.02385^{\{2\}}$	$0.02617^{\{10\}}$	$0.0242^{\{4\}}$	$0.02443^{\{6\}}$	$0.02656^{\{11\}}$	$0.02929^{\{13\}}$	$0.02754^{\{12\}}$	$0.04098^{\{16\}}$	$0.02563^{\{8\}}$	$0.03294^{\{15\}}$	$0.02592^{\{9\}}$	$0.03164^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00089^{\{2\}}$	$0.00092^{\{5.5\}}$	$0.00094^{\{7\}}$	$0.00077^{\{1\}}$	$0.00091^{\{3.5\}}$	$0.00109^{\{10\}}$	$0.00091^{\{3.5\}}$	$0.00092^{\{5.5\}}$	$0.00111^{\{11\}}$	$0.00139^{\{13\}}$	$0.00115^{\{12\}}$	$0.00429^{\{16\}}$	$0.00103^{\{8\}}$	$0.00172^{\{15\}}$	$0.00106^{\{9\}}$	$0.00156^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.01594^{\{3\}}$	$0.01624^{\{5\}}$	$0.0163^{\{7\}}$	$0.01484^{\{1\}}$	$0.0159^{\{2\}}$	$0.01745^{\{10\}}$	$0.01613^{\{4\}}$	$0.01629^{\{6\}}$	$0.01771^{\{11\}}$	$0.01953^{\{13\}}$	$0.01836^{\{12\}}$	$0.02732^{\{16\}}$	$0.01709^{\{8\}}$	$0.02196^{\{15\}}$	$0.01728^{\{9\}}$	$0.02109^{\{14\}}$
	D_{abs}	$0.01122^{\{3\}}$	$0.01145^{\{5\}}$	$0.0115^{\{7\}}$	$0.01045^{\{1\}}$	$0.01119^{\{2\}}$	$0.01223^{\{10\}}$	$0.01134^{\{4\}}$	$0.01147^{\{6\}}$	$0.01249^{\{11\}}$	$0.01373^{\{13\}}$	$0.01291^{\{12\}}$	$0.01949^{\{16\}}$	$0.01204^{\{8\}}$	$0.01544^{\{15\}}$	$0.01214^{\{9\}}$	$0.01483^{\{14\}}$
	D_{max}	$0.01614^{\{3\}}$	$0.01645^{\{5\}}$	$0.01652^{\{7\}}$	$0.01503^{\{1\}}$	$0.01609^{\{2\}}$	$0.01759^{\{10\}}$	$0.01631^{\{4\}}$	$0.0165^{\{6\}}$	$0.01795^{\{11\}}$	$0.01973^{\{13\}}$	$0.01856^{\{12\}}$	$0.02823^{\{16\}}$	$0.01731^{\{8\}}$	$0.02223^{\{15\}}$	$0.01745^{\{9\}}$	$0.02131^{\{14\}}$
	ASAE	$0.00715^{\{3\}}$	$0.00728^{\{5\}}$	$0.00751^{\{7\}}$	$0.00716^{\{1\}}$	$0.0074^{\{2\}}$	$0.00792^{\{10\}}$	$0.00718^{\{4\}}$	$0.00737^{\{6\}}$	$0.00801^{\{11\}}$	$0.00862^{\{13\}}$	$0.00827^{\{12\}}$	$0.01207^{\{16\}}$	$0.00777^{\{8\}}$	$0.00946^{\{15\}}$	$0.00791^{\{9\}}$	$0.00943^{\{14\}}$
	$\sum Ranks$	$15^{\{2\}}$	$29.5^{\{5\}}$	$42^{\{7\}}$	$7^{\{1\}}$	$17.5^{\{3\}}$	$60^{\{10\}}$	$22.5^{\{4\}}$	$34.5^{\{6\}}$	$66^{\{11\}}$	$78^{\{13\}}$	$72^{\{12\}}$	$96^{\{16\}}$	$48^{\{8\}}$	$90^{\{15\}}$	$54^{\{9\}}$	$84^{\{14\}}$

TABLE 4. Numerical values of simulation measures for $\sigma = 2.0$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSD	MSSLD	MSLND
15	BIAS($\hat{\sigma}$)	$0.15403^{\{1\}}$	$0.16229^{\{4\}}$	$0.17284^{\{7\}}$	$0.16564^{\{5\}}$	$0.17586^{\{9\}}$	$0.18633^{\{12\}}$	$0.15556^{\{2\}}$	$0.15632^{\{3\}}$	$0.17868^{\{10\}}$	$0.19975^{\{13\}}$	$0.18532^{\{11\}}$	$0.25528^{\{16\}}$	$0.17036^{\{6\}}$	$0.2257^{\{15\}}$	$0.17341^{\{8\}}$	$0.22351^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.0369^{\{1\}}$	$0.04192^{\{4\}}$	$0.04662^{\{7\}}$	$0.04327^{\{5\}}$	$0.04896^{\{9\}}$	$0.05574^{\{11\}}$	$0.03924^{\{2\}}$	$0.03987^{\{3\}}$	$0.04994^{\{10\}}$	$0.06577^{\{13\}}$	$0.05659^{\{12\}}$	$0.12626^{\{16\}}$	$0.0454^{\{6\}}$	$0.08251^{\{15\}}$	$0.04706^{\{8\}}$	$0.07999^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.07702^{\{1\}}$	$0.08114^{\{4\}}$	$0.08642^{\{7\}}$	$0.08282^{\{5\}}$	$0.08793^{\{9\}}$	$0.09317^{\{12\}}$	$0.07778^{\{2\}}$	$0.07816^{\{3\}}$	$0.08934^{\{10\}}$	$0.09987^{\{13\}}$	$0.09266^{\{11\}}$	$0.12764^{\{16\}}$	$0.08518^{\{6\}}$	$0.11285^{\{15\}}$	$0.08671^{\{8\}}$	$0.11175^{\{14\}}$
	D_{abs}	$0.05478^{\{2\}}$	$0.0574^{\{4\}}$	$0.06152^{\{8\}}$	$0.05745^{\{5\}}$	$0.06225^{\{9\}}$	$0.06322^{\{11\}}$	$0.05413^{\{1\}}$	$0.05551^{\{3\}}$	$0.06301^{\{10\}}$	$0.06969^{\{13\}}$	$0.06375^{\{12\}}$	$0.08955^{\{16\}}$	$0.06034^{\{6\}}$	$0.07731^{\{15\}}$	$0.06073^{\{7\}}$	$0.07589^{\{14\}}$
	D_{max}	$0.07772^{\{2\}}$	$0.08093^{\{4\}}$	$0.08727^{\{8\}}$	$0.08151^{\{5\}}$	$0.08846^{\{9\}}$	$0.08992^{\{11\}}$	$0.07672^{\{1\}}$	$0.07848^{\{3\}}$	$0.08927^{\{10\}}$	$0.09857^{\{13\}}$	$0.09044^{\{12\}}$	$0.1324^{\{16\}}$	$0.08552^{\{6\}}$	$0.11047^{\{15\}}$	$0.08605^{\{7\}}$	$0.10917^{\{14\}}$
	ASAE	$0.06212^{\{2\}}$	$0.06323^{\{4\}}$	$0.06451^{\{8\}}$	$0.0613^{\{5\}}$	$0.0634^{\{9\}}$	$0.06408^{\{11\}}$	$0.06065^{\{1\}}$	$0.0621^{\{3\}}$	$0.06972^{\{10\}}$	$0.07335^{\{13\}}$	$0.06732^{\{12\}}$	$0.09815^{\{16\}}$	$0.06742^{\{6\}}$	$0.07887^{\{15\}}$	$0.06624^{\{7\}}$	$0.08081^{\{14\}}$
	$\sum Ranks$	$11^{\{2\}}$	$25^{\{4\}}$	$45^{\{7\}}$	$27^{\{5\}}$	$51^{\{9\}}$	$64^{\{11\}}$	$9^{\{1\}}$	$18^{\{3\}}$	$62^{\{10\}}$	$78^{\{13\}}$	$68^{\{12\}}$	$96^{\{16\}}$	$41^{\{6\}}$	$89^{\{15\}}$	$47^{\{8\}}$	$85^{\{14\}}$
45	BIAS($\hat{\sigma}$)	$0.09193^{\{1\}}$	$0.09462^{\{3\}}$	$0.09663^{\{7\}}$	$0.09553^{\{5\}}$	$0.09622^{\{6\}}$	$0.10861^{\{12\}}$	$0.09467^{\{4\}}$	$0.09207^{\{2\}}$	$0.1016^{\{10\}}$	$0.11814^{\{13\}}$	$0.1059^{\{11\}}$	$0.15792^{\{16\}}$	$0.10024^{\{9\}}$	$0.12537^{\{15\}}$	$0.09895^{\{8\}}$	$0.12384^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.01289^{\{1\}}$	$0.01446^{\{6\}}$	$0.01473^{\{7\}}$	$0.01407^{\{4\}}$	$0.01431^{\{5\}}$	$0.01875^{\{12\}}$	$0.01399^{\{3\}}$	$0.01351^{\{2\}}$	$0.01608^{\{10\}}$	$0.02234^{\{13\}}$	$0.01804^{\{11\}}$	$0.04746^{\{16\}}$	$0.01567^{\{8\}}$	$0.02507^{\{15\}}$	$0.01572^{\{9\}}$	$0.02426^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.04597^{\{1\}}$	$0.04731^{\{3\}}$	$0.04831^{\{7\}}$	$0.04776^{\{5\}}$	$0.04811^{\{6\}}$	$0.0543^{\{12\}}$	$0.04734^{\{4\}}$	$0.04603^{\{2\}}$	$0.0508^{\{10\}}$	$0.05907^{\{13\}}$	$0.05295^{\{11\}}$	$0.07896^{\{16\}}$	$0.05012^{\{9\}}$	$0.06269^{\{15\}}$	$0.04947^{\{8\}}$	$0.06192^{\{14\}}$
	D_{abs}	$0.03241^{\{1\}}$	$0.03334^{\{3\}}$	$0.034^{\{7\}}$	$0.03339^{\{4\}}$	$0.03387^{\{6\}}$	$0.03772^{\{12\}}$	$0.03349^{\{5\}}$	$0.03254^{\{2\}}$	$0.03596^{\{10\}}$	$0.04155^{\{13\}}$	$0.03716^{\{11\}}$	$0.05631^{\{16\}}$	$0.03541^{\{9\}}$	$0.04347^{\{15\}}$	$0.03455^{\{8\}}$	$0.04302^{\{14\}}$
	D_{max}	$0.04656^{\{1\}}$	$0.04757^{\{3\}}$	$0.04873^{\{7\}}$	$0.04791^{\{5\}}$	$0.04856^{\{6\}}$	$0.05407^{\{12\}}$	$0.04781^{\{4\}}$	$0.0467^{\{2\}}$	$0.05128^{\{10\}}$	$0.05946^{\{13\}}$	$0.05291^{\{11\}}$	$0.08239^{\{16\}}$	$0.05055^{\{9\}}$	$0.06245^{\{15\}}$	$0.04955^{\{8\}}$	$0.06173^{\{14\}}$
	ASAE	$0.0302^{\{1\}}$	$0.02961^{\{3\}}$	$0.03102^{\{7\}}$	$0.02948^{\{5\}}$	$0.03108^{\{6\}}$	$0.03143^{\{12\}}$	$0.02869^{\{4\}}$	$0.03003^{\{2\}}$	$0.03391^{\{10\}}$	$0.03589^{\{13\}}$	$0.03329^{\{11\}}$	$0.04757^{\{16\}}$	$0.03151^{\{9\}}$	$0.03806^{\{15\}}$	$0.03225^{\{8\}}$	$0.03775^{\{14\}}$
	$\sum Ranks$	$10^{\{1\}}$	$21^{\{3.5\}}$	$41^{\{7\}}$	$25^{\{5\}}$	$36^{\{6\}}$	$68^{\{12\}}$	$21^{\{3.5\}}$	$14^{\{2\}}$	$62^{\{10\}}$	$78^{\{13\}}$	$66^{\{11\}}$	$96^{\{16\}}$	$53^{\{9\}}$	$90^{\{15\}}$	$51^{\{8\}}$	$84^{\{14\}}$
90	BIAS($\hat{\sigma}$)	$0.06488^{\{1\}}$	$0.06589^{\{2\}}$	$0.07173^{\{8\}}$	$0.06628^{\{3\}}$	$0.06829^{\{5\}}$	$0.07522^{\{10\}}$	$0.06984^{\{6\}}$	$0.06759^{\{4\}}$	$0.07654^{\{11\}}$	$0.08387^{\{13\}}$	$0.07786^{\{12\}}$	$0.10837^{\{16\}}$	$0.07111^{\{7\}}$	$0.08968^{\{14\}}$	$0.07515^{\{9\}}$	$0.09115^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00656^{\{1\}}$	$0.00684^{\{3\}}$	$0.00804^{\{7\}}$	$0.0068^{\{2\}}$	$0.00736^{\{5\}}$	$0.00902^{\{11\}}$	$0.00769^{\{6\}}$	$0.00701^{\{4\}}$	$0.009^{\{10\}}$	$0.01121^{\{13\}}$	$0.00974^{\{12\}}$	$0.02294^{\{16\}}$	$0.00808^{\{8\}}$	$0.01292^{\{14\}}$	$0.00871^{\{9\}}$	$0.01341^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.03244^{\{1\}}$	$0.03294^{\{2\}}$	$0.03587^{\{8\}}$	$0.03314^{\{3\}}$	$0.03414^{\{5\}}$	$0.03761^{\{10\}}$	$0.03492^{\{6\}}$	$0.03379^{\{4\}}$	$0.03827^{\{11\}}$	$0.04194^{\{13\}}$	$0.03893^{\{12\}}$	$0.05418^{\{16\}}$	$0.03555^{\{7\}}$	$0.04484^{\{14\}}$	$0.03757^{\{9\}}$	$0.04558^{\{15\}}$
	D_{abs}	$0.02284^{\{1\}}$	$0.0233^{\{2\}}$	$0.02527^{\{8\}}$	$0.02332^{\{3\}}$	$0.0241^{\{5\}}$	$0.02617^{\{9\}}$	$0.02458^{\{6\}}$	$0.02388^{\{4\}}$	$0.02695^{\{11\}}$	$0.02943^{\{13\}}$	$0.02732^{\{12\}}$	$0.03849^{\{16\}}$	$0.02512^{\{7\}}$	$0.03127^{\{14\}}$	$0.02634^{\{10\}}$	$0.03206^{\{15\}}$
	D_{max}	$0.03281^{\{1\}}$	$0.03339^{\{3\}}$	$0.03629^{\{8\}}$	$0.03337^{\{2\}}$	$0.03448^{\{5\}}$	$0.03756^{\{9\}}$	$0.03529^{\{6\}}$	$0.03416^{\{4\}}$	$0.0388^{\{11\}}$	$0.04235^{\{13\}}$	$0.03903^{\{12\}}$	$0.05579^{\{16\}}$	$0.03601^{\{7\}}$	$0.04507^{\{14\}}$	$0.03786^{\{10\}}$	$0.04606^{\{15\}}$
	ASAE	$0.01874^{\{1\}}$	$0.01878^{\{3\}}$	$0.01941^{\{8\}}$	$0.01897^{\{2\}}$	$0.01945^{\{5\}}$	$0.02083^{\{9\}}$	$0.0185^{\{6\}}$	$0.01876^{\{4\}}$	$0.02098^{\{11\}}$	$0.02288^{\{13\}}$	$0.02119^{\{12\}}$	$0.02984^{\{16\}}$	$0.02039^{\{7\}}$	$0.02484^{\{14\}}$	$0.02095^{\{10\}}$	$0.02461^{\{15\}}$
	$\sum Ranks$	$7^{\{1\}}$	$16^{\{2\}}$	$45^{\{8\}}$	$18^{\{3\}}$	$32^{\{6\}}$	$58^{\{10\}}$	$31^{\{5\}}$	$23^{\{4\}}$	$65^{\{11\}}$	$78^{\{13\}}$	$72^{\{12\}}$	$96^{\{16\}}$	$44^{\{7\}}$	$85^{\{14\}}$	$57^{\{9\}}$	$89^{\{15\}}$
140	BIAS($\hat{\sigma}$)	$0.05133^{\{1\}}$	$0.05421^{\{4\}}$	$0.05673^{\{8\}}$	$0.05233^{\{3\}}$	$0.05608^{\{6\}}$	$0.05874^{\{9\}}$	$0.05182^{\{2\}}$	$0.05568^{\{5\}}$	$0.05974^{\{11\}}$	$0.06498^{\{13\}}$	$0.06044^{\{12\}}$	$0.08565^{\{16\}}$	$0.05654^{\{7\}}$	$0.07234^{\{14\}}$	$0.05965^{\{10\}}$	$0.07437^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00403^{\{1\}}$	$0.0047^{\{4\}}$	$0.00507^{\{7\}}$	$0.00428^{\{3\}}$	$0.00484^{\{6\}}$	$0.00548^{\{9\}}$	$0.00423^{\{2\}}$	$0.0048^{\{5\}}$	$0.00563^{\{10\}}$	$0.00682^{\{13\}}$	$0.00601^{\{12\}}$	$0.0129^{\{16\}}$	$0.0051^{\{8\}}$	$0.00827^{\{14\}}$	$0.0057^{\{11\}}$	$0.00907^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.02566^{\{1\}}$	$0.02711^{\{4\}}$	$0.02836^{\{8\}}$	$0.02617^{\{3\}}$	$0.02804^{\{6\}}$	$0.02937^{\{9\}}$	$0.02591^{\{2\}}$	$0.02784^{\{5\}}$	$0.02987^{\{11\}}$	$0.03249^{\{13\}}$	$0.03022^{\{12\}}$	$0.04283^{\{16\}}$	$0.02827^{\{7\}}$	$0.03617^{\{14\}}$	$0.02983^{\{10\}}$	$0.03719^{\{15\}}$
	D_{abs}	$0.01808^{\{1\}}$	$0.01908^{\{4\}}$	$0.02002^{\{8\}}$	$0.01843^{\{3\}}$	$0.01977^{\{6\}}$	$0.02044^{\{9\}}$	$0.0182^{\{2\}}$	$0.01959^{\{5\}}$	$0.02108^{\{11\}}$	$0.0229^{\{13\}}$	$0.02115^{\{12\}}$	$0.03053^{\{16\}}$	$0.01994^{\{7\}}$	$0.02535^{\{14\}}$	$0.02094^{\{10\}}$	$0.02607^{\{15\}}$
	D_{max}	$0.02599^{\{1\}}$	$0.02739^{\{4\}}$	$0.02871^{\{8\}}$	$0.02643^{\{3\}}$	$0.02835^{\{6\}}$	$0.02946^{\{9\}}$	$0.02616^{\{2\}}$	$0.02819^{\{5\}}$	$0.03025^{\{11\}}$	$0.03292^{\{13\}}$	$0.0304^{\{12\}}$	$0.04402^{\{16\}}$	$0.02859^{\{7\}}$	$0.03643^{\{14\}}$	$0.03002^{\{10\}}$	$0.03749^{\{15\}}$
	ASAE	$0.01384^{\{1\}}$	$0.01413^{\{4\}}$	$0.0147^{\{8\}}$	$0.01418^{\{3\}}$	$0.01458^{\{6\}}$	$0.01504^{\{9\}}$	$0.0138^{\{2\}}$	$0.01436^{\{5\}}$	$0.01599^{\{11\}}$	$0.01709^{\{13\}}$	$0.01613^{\{12\}}$	$0.0225^{\{16\}}$	$0.01538^{\{7\}}$	$0.01797^{\{14\}}$	$0.01542^{\{10\}}$	$0.01856^{\{15\}}$
	$\sum Ranks$	$7^{\{1\}}$	$23^{\{4\}}$	$46^{\{8\}}$	$19^{\{3\}}$	$36^{\{6\}}$	$53^{\{9\}}$	$11^{\{2\}}$	$30^{\{5\}}$	$65^{\{11\}}$	$78^{\{13\}}$	$72^{\{12\}}$	$96^{\{16\}}$	$45^{\{7\}}$	$84^{\{14\}}$	$61^{\{10\}}$	$90^{\{15\}}$
200	BIAS($\hat{\sigma}$)	$0.04309^{\{1\}}$	$0.04618^{\{7\}}$	$0.04569^{\{4\}}$	$0.0446^{\{3\}}$	$0.04415^{\{2\}}$	$0.05039^{\{11\}}$	$0.04575^{\{5\}}$	$0.04607^{\{6\}}$	$0.04838^{\{9\}}$	$0.05556^{\{13\}}$	$0.05168^{\{12\}}$	$0.07311^{\{16\}}$	$0.04897^{\{10\}}$	$0.06113^{\{14\}}$	$0.04737^{\{8\}}$	$0.06229^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00293^{\{1\}}$	$0.00328^{\{5\}}$	$0.00337^{\{7\}}$	$0.00303^{\{2\}}$	$0.00308^{\{3\}}$	$0.00401^{\{11\}}$	$0.00323^{\{4\}}$	$0.00335^{\{6\}}$	$0.00372^{\{9\}}$	$0.00505^{\{13\}}$	$0.00421^{\{12\}}$	$0.00986^{\{16\}}$	$0.00375^{\{10\}}$	$0.00611^{\{15\}}$	$0.00357^{\{8\}}$	$0.00606^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.02154^{\{1\}}$	$0.02309^{\{7\}}$	$0.02285^{\{4\}}$	$0.0223^{\{3\}}$	$0.02208^{\{2\}}$	$0.0252^{\{11\}}$	$0.02288^{\{5\}}$	$0.02304^{\{6\}}$	$0.02419^{\{9\}}$	$0.02778^{\{13\}}$	$0.02584^{\{12\}}$	$0.03656^{\{16\}}$	$0.02449^{\{10\}}$	$0.03057^{\{14\}}$	$0.02368^{\{8\}}$	$0.03115^{\{15\}}$
	D_{abs}	$0.01521^{\{1\}}$	$0.0163^{\{7\}}$	$0.01609^{\{4\}}$	$0.01574^{\{3\}}$	$0.01557^{\{2\}}$	$0.01761^{\{11\}}$	$0.01614^{\{5\}}$	$0.01625^{\{6\}}$	$0.01704^{\{9\}}$	$0.01948^{\{13\}}$	$0.01815^{\{12\}}$	$0.02599^{\{16\}}$	$0.01725^{\{10\}}$	$0.02138^{\{14\}}$	$0.01663^{\{8\}}$	$0.02182^{\{15\}}$
	D_{max}	$0.02185^{\{1\}}$	$0.02337^{\{7\}}$	$0.02316^{\{4.5\}}$	$0.02255^{\{3\}}$	$0.02236^{\{2\}}$	$0.02535^{\{11\}}$	$0.02316^{\{4.5\}}$	$0.02334^{\{6\}}$	$0.02448^{\{9\}}$	$0.02805^{\{13\}}$	$0.02603^{\{12\}}$	$0.03757^{\{16\}}$	$0.0248^{\{10\}}$	$0.03079^{\{14\}}$	$0.02386^{\{8\}}$	$0.03134^{\{15\}}$
	ASAE	$0.01083^{\{1\}}$	$0.01145^{\{7\}}$	$0.01158^{\{4.5\}}$	$0.01132^{\{3\}}$	$0.0116^{\{2\}}$	$0.01229^{\{11\}}$	$0.01093^{\{4.5\}}$	$0.01126^{\{6\}}$	$0.01236^{\{9\}}$	$0.01362^{\{13\}}$	$0.0128^{\{12\}}$	$0.01811^{\{16\}}$	$0.012^{\{10\}}$	$0.01495^{\{14\}}$	$0.01217^{\{8\}}$	$0.01469^{\{15\}}$
	$\sum Ranks$	$6^{\{1\}}$	$38^{\{7\}}$	$29.5^{\{5\}}$	$18^{\{2.5\}}$	$18^{\{2.5\}}$	$65^{\{11\}}$	$25.5^{\{4\}}$	$33^{\{6\}}$	$56^{\{9\}}$	$78^{\{13\}}$	$72^{\{12\}}$	$96^{\{16\}}$	$58^{\{10\}}$	$86^{\{14\}}$	$49^{\{8\}}$	$88^{\{15\}}$
250	BIAS($\hat{\sigma}$)	$0.03793^{\{1\}}$	$0.04141^{\{6\}}$	$0.04009^{\{5\}}$	$0.03968^{\{3.5\}}$	$0.04283^{\{8\}}$	$0.04681^{\{12\}}$	$0.03954^{\{2\}}$	$0.03968^{\{3.5\}}$	$0.04328^{\{9\}}$	$0.0498^{\{13\}}$	$0.04668^{\{11\}}$	$0.0647^{\{16\}}$	$0.04241^{\{7\}}$	$0.0536^{\{14\}}$	$0.04351^{\{10\}}$	$0.05456^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.0023^{\{1\}}$	$0.00274^{\{6\}}$	$0.00258^{\{5\}}$	$0.00244^{\{2\}}$	$0.00287^{\{8\}}$	$0.00343^{\{11\}}$	$0.00249^{\{4\}}$	$0.00248^{\{3\}}$	$0.00291^{\{9\}}$	$0.00391^{\{13\}}$	$0.00347^{\{12\}}$	$0.00781^{\{16\}}$	$0.00285^{\{7\}}$	$0.00454^{\{14\}}$	$0.00304^{\{10\}}$	$0.00458^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.01897^{\{1\}}$	$0.02071^{\{6\}}$	$0.02005^{\{5\}}$	$0.01984^{\{3.5\}}$	$0.02141^{\{8\}}$	$0.0234^{\{12\}}$	$0.01977^{\{2\}}$	$0.01984^{\{3.5\}}$	$0.02164^{\{9\}}$	$0.0249^{\{13\}}$	$0.02334^{\{11\}}$	$0.03235^{\{16\}}$	$0.0212^{\{7\}}$	$0.0268^{\{14\}}$	$0.02176^{\{10\}}$	$0.02728^{\{15\}}$
	D_{abs}	$0.01336^{\{1\}}$	$0.01462^{\{6\}}$	$0.01409^{\{5\}}$	$0.01393^{\{2\}}$	$0.01509^{\{8\}}$	$0.01638^{\{11\}}$	$0.01394^{\{3\}}$	$0.01396^{\{4\}}$	$0.01526^{\{9\}}$	$0.01749^{\{13\}}$	$0.0164^{\{12\}}$	$0.02302^{\{16\}}$	$0.01496^{\{7\}}$	$0.01884^{\{14\}}$	$0.01535^{\{10\}}$	$0.01915^{\{15\}}$
	D_{max}	$0.0192^{\{1\}}$	$0.02099^{\{6\}}$	$0.02028^{\{5\}}$	$0.02002^{\{2.5\}}$	$0.02166^{\{8\}}$	$0.02356^{\{11\}}$	$0.02002^{\{2.5\}}$	$0.02007^{\{4\}}$	$0.02193^{\{9\}}$	$0.02517^{\{13\}}$	$0.02357^{\{12\}}$	$0.03326^{\{16\}}$	$0.02147^{\{7\}}$	$0.02707^{\{14\}}$	$0.02203^{\{10\}}$	$0.02753^{\{15\}}$
	ASAE	$0.00978^{\{1\}}$	$0.01005^{\{6\}}$	$0.00988^{\{5\}}$	$0.00971^{\{2.5\}}$	$0.01012^{\{8\}}$	$0.01061^{\{11\}}$	$0.00941^{\{2.5\}}$	$0.0099^{\{4\}}$	$0.01065^{\{9\}}$	$0.01194^{\{13\}}$	$0.01102^{\{12\}}$	$0.01591^{\{16\}}$	$0.01054^{\{7\}}$	$0.01283^{\{14\}}$	$0.01054^{\{10\}}$	$0.01268^{\{15\}}$
	$\sum Ranks$	$8^{\{1\}}$	$36^{\{6\}}$	$29^{\{5\}}$	$15.5^{\{3\}}$	$47^{\{8\}}$	$67^{\{11\}}$	$14.5^{\{2\}}$	$23^{\{4\}}$	$56^{\{9\}}$	$78^{\{13\}}$	$70^{\{12\}}$	$96^{\{16\}}$	$44^{\{7\}}$	$85^{\{14\}}$	$58^{\{10\}}$	$89^{\{15\}}$
300	BIAS($\hat{\sigma}$)	$0.0356^{\{1\}}$	$0.03706^{\{5\}}$	$0.0385^{\{7\}}$	$0.03659^{\{3.5\}}$	$0.03707^{\{6\}}$	$0.04093^{\{11\}}$	$0.03659^{\{3.5\}}$	$0.03615^{\{2\}}$	$0.04001^{\{10\}}$	$0.04645^{\{13\}}$	$0.04278^{\{12\}}$	$0.06106^{\{16\}}$	$0.0391^{\{8\}}$	$0.05045^{\{15\}}$	$0.03994^{\{9\}}$	$0.04983^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00204^{\{1\}}$	$0.00221^{\{6\}}$	$0.00241^{\{7.5\}}$	$0.00206^{\{2\}}$	$0.00218^{\{5\}}$	$0.00257^{\{11\}}$	$0.00212^{\{3.5\}}$	$0.00212^{\{3.5\}}$	$0.00248^{\{9\}}$	$0.00334^{\{13\}}$	$0.00286^{\{12\}}$	$0.00664^{\{16\}}$	$0.00241^{\{7.5\}}$	$0.0039^{\{14.5\}}$	$0.0025^{\{10\}}$	$0.0039^{\{14.5\}}$
	MRE($\hat{\sigma}$)	$0.0178^{\{1\}}$	$0.01853^{\{5\}}$	$0.01925^{\{7\}}$	$0.01829^{\{3.5\}}$	$0.01854^{\{6\}}$	$0.02046^{\{11\}}$	$0.01829^{\{3.5\}}$	$0.01807^{\{2\}}$	$0.02001^{\{10\}}$	$0.02323^{\{13\}}$	$0.02139^{\{12\}}$	$0.03053^{\{16\}}$	$0.01955^{\{8\}}$	$0.02522^{\{15\}}$	$0.01997^{\{9\}}$	$0.02492^{\{14\}}$
	D_{abs}	$0.01257^{\{1\}}$	$0.01307^{\{6\}}$	$0.01358^{\{7\}}$	$0.01286^{\{3\}}$	$0.01306^{\{5\}}$	$0.01429^{\{11\}}$	$0.01288^{\{4\}}$	$0.01276^{\{2\}}$	$0.01408^{\{10\}}$	$0.01634^{\{13\}}$	$0.01502^{\{12\}}$	$0.02168^{\{16\}}$	$0.01377^{\{8\}}$	$0.01771^{\{15\}}$	$0.01403^{\{9\}}$	$0.01754^{\{14\}}$
	D_{max}	$0.01803^{\{1\}}$	$0.01879^{\{6\}}$	$0.01945^{\{7\}}$	$0.01848^{\{3\}}$	$0.01878^{\{5\}}$	$0.02058^{\{11\}}$	$0.01851^{\{4\}}$	$0.0183^{\{2\}}$	$0.02023^{\{10\}}$	$0.02347^{\{13\}}$	$0.02156^{\{12\}}$	$0.03128^{\{16\}}$	$0.0198^{\{8\}}$	$0.02543^{\{15\}}$	$0.02016^{\{9\}}$	$0.0252^{\{14\}}$
	ASAE	$0.00876^{\{1\}}$	$0.00881^{\{6\}}$	$0.00892^{\{7\}}$	$0.00863^{\{3\}}$	$0.00911^{\{5\}}$	$0.00942^{\{11\}}$	$0.00834^{\{4\}}$	$0.00883^{\{2\}}$	$0.00969^{\{10\}}$	$0.01034^{\{13\}}$	$0.00996^{\{12\}}$	$0.01408^{\{16\}}$	$0.00922^{\{8\}}$	$0.01122^{\{15\}}$	$0.00946^{\{9\}}$	$0.01125^{\{14\}}$
	$\sum Ranks$	$8^{\{1\}}$	$32^{\{5\}}$	$41.5^{\{7\}}$	$17^{\{3\}}$	$34^{\{6\}}$	$64^{\{11\}}$	$19.5^{\{4\}}$	$16.5^{\{2\}}$	$60^{\{10\}}$	$78^{\{13\}}$	$72^{\{12\}}$	$96^{\{16\}}$	$47.5^{\{8\}}$	$88.5^{\{15\}}$	$56^{\{9\}}$	$85.5^{\{14\}}$
400	BIAS($\hat{\sigma}$)	$0.02955^{\{1\}}$	$0.03233^{\{7\}}$	$0.03284^{\{8\}}$	$0.03065^{\{2\}}$	$0.03215^{\{5\}}$	$0.03722^{\{12\}}$	$0.03169^{\{4\}}$	$0.03102^{\{3\}}$	$0.03454^{\{9\}}$	$0.03998^{\{13\}}$	$0.03546^{\{11\}}$	$0.05039^{\{16\}}$	$0.03221^{\{6\}}$	$0.04481^{\{15\}}$	$0.03479^{\{10\}}$	$0.04153^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00142^{\{1\}}$	$0.00167^{\{7\}}$	$0.00175^{\{8\}}$	$0.00147^{\{2\}}$	$0.00163^{\{6\}}$	$0.00222^{\{12\}}$	$0.00159^{\{4\}}$	$0.00152^{\{3\}}$	$0.00188^{\{9\}}$	$0.00252^{\{13\}}$	$0.00198^{\{11\}}$	$0.00506^{\{16\}}$	$0.00161^{\{5\}}$	$0.00318^{\{15\}}$	$0.00192^{\{10\}}$	$0.00277^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.01478^{\{1\}}$	$0.01617^{\{7\}}$	$0.01642^{\{8\}}$	$0.01533^{\{2\}}$	$0.01608^{\{5\}}$	$0.01861^{\{12\}}$	$0.01585^{\{4\}}$	$0.01551^{\{3\}}$	$0.01727^{\{9\}}$	$0.01999^{\{13\}}$	$0.01773^{\{11\}}$	$0.0252^{\{16\}}$	$0.0161^{\{6\}}$	$0.0224^{\{15\}}$	$0.0174^{\{10\}}$	$0.02077^{\{14\}}$
	D_{abs}	$0.01042^{\{1\}}$	$0.01138^{\{7\}}$	$0.01158^{\{8\}}$	$0.01078^{\{2\}}$	$0.01133^{\{5\}}$	$0.013^{\{12\}}$	$0.01115^{\{4\}}$	$0.01095^{\{3\}}$	$0.01216^{\{9\}}$	$0.01408^{\{13\}}$	$0.01247^{\{11\}}$	$0.01787^{\{16\}}$	$0.01135^{\{6\}}$	$0.01578^{\{15\}}$	$0.01226^{\{10\}}$	$0.01465^{\{14\}}$
	D_{max}	$0.01497^{\{1\}}$	$0.01637^{\{7\}}$	$0.01665^{\{8\}}$	$0.0155^{\{2\}}$	$0.01626^{\{5\}}$	$0.01874^{\{12\}}$	$0.01603^{\{4\}}$	$0.01571^{\{3\}}$	$0.01749^{\{9\}}$	$0.02022^{\{13\}}$	$0.01792^{\{11\}}$	$0.02581^{\{16\}}$	$0.01631^{\{6\}}$	$0.02267^{\{15\}}$	$0.01761^{\{10\}}$	$0.02104^{\{14\}}$
	ASAE	$0.00722^{\{1\}}$	$0.00735^{\{7\}}$	$0.00749^{\{8\}}$	$0.00723^{\{2\}}$	$0.00759^{\{5\}}$	$0.00786^{\{12\}}$	$0.00711^{\{4\}}$	$0.00732^{\{3\}}$	$0.00827^{\{9\}}$	$0.00885^{\{13\}}$	$0.00815^{\{11\}}$	$0.01152^{\{16\}}$	$0.00793^{\{6\}}$	$0.00957^{\{15\}}$	$0.00796^{\{10\}}$	$0.00941^{\{14\}}$
	$\sum Ranks$	$7^{\{1\}}$	$40^{\{7\}}$	$46^{\{8\}}$	$13^{\{2\}}$	$33^{\{5\}}$	$68^{\{12\}}$	$21^{\{4\}}$	$19^{\{3\}}$	$57^{\{9\}}$	$78^{\{13\}}$	$66^{\{11\}}$	$96^{\{16\}}$	$38^{\{6\}}$	$90^{\{15\}}$	$60^{\{10\}}$	$84^{\{14\}}$

Table 5. Numerical values of simulation measures for $\sigma = 2.5$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSD	MSSLD	MSLND
15	BIAS($\hat{\sigma}$)	$0.19498^{\{1\}}$	$0.20448^{\{2\}}$	$0.21187^{\{6\}}$	$0.20726^{\{4\}}$	$0.21603^{\{7\}}$	$0.22361^{\{11\}}$	$0.21139^{\{5\}}$	$0.2062^{\{3\}}$	$0.22083^{\{9\}}$	$0.24727^{\{13\}}$	$0.22656^{\{12\}}$	$0.3076^{\{16\}}$	$0.22118^{\{10\}}$	$0.26741^{\{14\}}$	$0.21819^{\{8\}}$	$0.27918^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.05984^{\{1\}}$	$0.06768^{\{3\}}$	$0.07051^{\{5\}}$	$0.06752^{\{2\}}$	$0.0742^{\{7\}}$	$0.08125^{\{11\}}$	$0.07157^{\{6\}}$	$0.06825^{\{4\}}$	$0.07871^{\{10\}}$	$0.0951^{\{13\}}$	$0.08514^{\{12\}}$	$0.17313^{\{16\}}$	$0.07623^{\{8\}}$	$0.11364^{\{14\}}$	$0.07762^{\{9\}}$	$0.12352^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.07799^{\{1\}}$	$0.08179^{\{2\}}$	$0.08475^{\{6\}}$	$0.0829^{\{4\}}$	$0.08641^{\{7\}}$	$0.08944^{\{11\}}$	$0.08456^{\{5\}}$	$0.08248^{\{3\}}$	$0.08833^{\{9\}}$	$0.09891^{\{13\}}$	$0.09063^{\{12\}}$	$0.12304^{\{16\}}$	$0.08847^{\{10\}}$	$0.10696^{\{14\}}$	$0.08728^{\{8\}}$	$0.11167^{\{15\}}$
	D_{abs}	$0.05542^{\{1\}}$	$0.05798^{\{2\}}$	$0.05964^{\{6\}}$	$0.05815^{\{3\}}$	$0.06039^{\{8\}}$	$0.0603^{\{7\}}$	$0.05962^{\{5\}}$	$0.05825^{\{4\}}$	$0.06258^{\{11\}}$	$0.06944^{\{13\}}$	$0.06261^{\{12\}}$	$0.0877^{\{16\}}$	$0.0624^{\{10\}}$	$0.07406^{\{14\}}$	$0.06089^{\{9\}}$	$0.07627^{\{15\}}$
	D_{max}	$0.07813^{\{1\}}$	$0.08188^{\{2\}}$	$0.08468^{\{6\}}$	$0.082^{\{3\}}$	$0.0862^{\{8\}}$	$0.08623^{\{9\}}$	$0.08421^{\{5\}}$	$0.08215^{\{4\}}$	$0.08834^{\{10\}}$	$0.09755^{\{13\}}$	$0.08884^{\{12\}}$	$0.12704^{\{16\}}$	$0.08846^{\{11\}}$	$0.10531^{\{14\}}$	$0.08569^{\{7\}}$	$0.10927^{\{15\}}$
	ASAE	$0.06278^{\{1\}}$	$0.06344^{\{2\}}$	$0.0649^{\{6\}}$	$0.06137^{\{3\}}$	$0.06373^{\{8\}}$	$0.06379^{\{9\}}$	$0.05937^{\{5\}}$	$0.06404^{\{4\}}$	$0.07126^{\{10\}}$	$0.07076^{\{13\}}$	$0.06761^{\{12\}}$	$0.09563^{\{16\}}$	$0.06749^{\{11\}}$	$0.0786^{\{14\}}$	$0.06736^{\{7\}}$	$0.07967^{\{15\}}$
	$\sum Ranks$	$8^{\{1\}}$	$15^{\{2\}}$	$37^{\{6\}}$	$18^{\{3\}}$	$42^{\{7\}}$	$55^{\{9\}}$	$27^{\{5\}}$	$25^{\{4\}}$	$62^{\{11\}}$	$77^{\{13\}}$	$71^{\{12\}}$	$96^{\{16\}}$	$59^{\{10\}}$	$84^{\{14\}}$	$50^{\{8\}}$	$90^{\{15\}}$
45	BIAS($\hat{\sigma}$)	$0.11373^{\{2\}}$	$0.1192^{\{6\}}$	$0.11715^{\{4\}}$	$0.11669^{\{3\}}$	$0.12236^{\{7\}}$	$0.12908^{\{8\}}$	$0.11162^{\{1\}}$	$0.11871^{\{5\}}$	$0.12914^{\{9\}}$	$0.1431^{\{13\}}$	$0.13492^{\{12\}}$	$0.2005^{\{16\}}$	$0.12927^{\{10\}}$	$0.15974^{\{14\}}$	$0.13395^{\{11\}}$	$0.16286^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.02031^{\{2\}}$	$0.02248^{\{6\}}$	$0.02147^{\{4\}}$	$0.02144^{\{3\}}$	$0.02312^{\{7\}}$	$0.02656^{\{10\}}$	$0.02007^{\{1\}}$	$0.02209^{\{5\}}$	$0.02607^{\{8\}}$	$0.03261^{\{13\}}$	$0.0285^{\{12\}}$	$0.08312^{\{16\}}$	$0.02613^{\{9\}}$	$0.0408^{\{14\}}$	$0.02821^{\{11\}}$	$0.04097^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.04549^{\{2\}}$	$0.04768^{\{6\}}$	$0.04686^{\{4\}}$	$0.04668^{\{3\}}$	$0.04894^{\{7\}}$	$0.05163^{\{8\}}$	$0.04465^{\{1\}}$	$0.04749^{\{5\}}$	$0.05165^{\{9\}}$	$0.05724^{\{13\}}$	$0.05397^{\{12\}}$	$0.0802^{\{16\}}$	$0.05171^{\{10\}}$	$0.06389^{\{14\}}$	$0.05358^{\{11\}}$	$0.06514^{\{15\}}$
	D_{abs}	$0.03228^{\{2\}}$	$0.03377^{\{6\}}$	$0.03319^{\{4\}}$	$0.03254^{\{3\}}$	$0.03454^{\{7\}}$	$0.03562^{\{8\}}$	$0.03143^{\{1\}}$	$0.03343^{\{5\}}$	$0.03626^{\{9\}}$	$0.04013^{\{13\}}$	$0.03773^{\{12\}}$	$0.05727^{\{16\}}$	$0.03655^{\{10\}}$	$0.04434^{\{14\}}$	$0.03747^{\{11\}}$	$0.0454^{\{15\}}$
	D_{max}	$0.04619^{\{2\}}$	$0.04828^{\{6\}}$	$0.04728^{\{4\}}$	$0.0466^{\{3\}}$	$0.04929^{\{7\}}$	$0.05134^{\{8\}}$	$0.04506^{\{1\}}$	$0.04785^{\{5\}}$	$0.05212^{\{9\}}$	$0.05756^{\{13\}}$	$0.05409^{\{12\}}$	$0.08404^{\{16\}}$	$0.0524^{\{10\}}$	$0.06368^{\{14\}}$	$0.05369^{\{11\}}$	$0.06527^{\{15\}}$
	ASAE	$0.02942^{\{2\}}$	$0.02974^{\{6\}}$	$0.03073^{\{4\}}$	$0.02949^{\{3\}}$	$0.03024^{\{7\}}$	$0.03131^{\{8\}}$	$0.02887^{\{1\}}$	$0.0305^{\{5\}}$	$0.03365^{\{9\}}$	$0.0361^{\{13\}}$	$0.03405^{\{12\}}$	$0.05074^{\{16\}}$	$0.03228^{\{10\}}$	$0.03844^{\{14\}}$	$0.03207^{\{11\}}$	$0.0377^{\{15\}}$
	$\sum Ranks$	$12^{\{2\}}$	$34^{\{6\}}$	$27^{\{4\}}$	$18^{\{3\}}$	$40^{\{7\}}$	$50^{\{8\}}$	$6^{\{1\}}$	$31^{\{5\}}$	$55^{\{9\}}$	$78^{\{13\}}$	$72^{\{12\}}$	$96^{\{16\}}$	$59^{\{10\}}$	$85^{\{14\}}$	$64^{\{11\}}$	$89^{\{15\}}$
90	BIAS($\hat{\sigma}$)	$0.07999^{\{2\}}$	$0.08687^{\{7\}}$	$0.08631^{\{6\}}$	$0.08097^{\{3\}}$	$0.08885^{\{9\}}$	$0.09852^{\{12\}}$	$0.08185^{\{4\}}$	$0.0788^{\{1\}}$	$0.09522^{\{10\}}$	$0.10372^{\{13\}}$	$0.09529^{\{11\}}$	$0.13624^{\{16\}}$	$0.08752^{\{8\}}$	$0.11777^{\{15\}}$	$0.08559^{\{5\}}$	$0.11289^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00988^{\{1\}}$	$0.01161^{\{6\}}$	$0.01158^{\{5\}}$	$0.01036^{\{3\}}$	$0.01271^{\{9\}}$	$0.01525^{\{12\}}$	$0.01052^{\{4\}}$	$0.01026^{\{2\}}$	$0.01433^{\{11\}}$	$0.01712^{\{13\}}$	$0.01429^{\{10\}}$	$0.03494^{\{16\}}$	$0.01198^{\{8\}}$	$0.02167^{\{15\}}$	$0.01182^{\{7\}}$	$0.02024^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.032^{\{2\}}$	$0.03475^{\{7\}}$	$0.03452^{\{6\}}$	$0.03239^{\{3\}}$	$0.03554^{\{9\}}$	$0.03941^{\{12\}}$	$0.03274^{\{4\}}$	$0.03152^{\{1\}}$	$0.03809^{\{10\}}$	$0.04149^{\{13\}}$	$0.03811^{\{11\}}$	$0.0545^{\{16\}}$	$0.03501^{\{8\}}$	$0.04711^{\{15\}}$	$0.03424^{\{5\}}$	$0.04515^{\{14\}}$
	D_{abs}	$0.02258^{\{2\}}$	$0.02445^{\{7\}}$	$0.02431^{\{6\}}$	$0.0227^{\{3\}}$	$0.0251^{\{9\}}$	$0.02725^{\{12\}}$	$0.02311^{\{4\}}$	$0.02224^{\{1\}}$	$0.02689^{\{11\}}$	$0.02904^{\{13\}}$	$0.02669^{\{10\}}$	$0.03912^{\{16\}}$	$0.02475^{\{8\}}$	$0.03286^{\{15\}}$	$0.02396^{\{5\}}$	$0.03144^{\{14\}}$
	D_{max}	$0.0324^{\{2\}}$	$0.03522^{\{7\}}$	$0.03491^{\{6\}}$	$0.03267^{\{3\}}$	$0.03595^{\{9\}}$	$0.03934^{\{12\}}$	$0.03309^{\{4\}}$	$0.03188^{\{1\}}$	$0.03869^{\{11\}}$	$0.04179^{\{13\}}$	$0.03822^{\{10\}}$	$0.0567^{\{16\}}$	$0.03541^{\{8\}}$	$0.0474^{\{15\}}$	$0.03441^{\{5\}}$	$0.04533^{\{14\}}$
	ASAE	$0.0189^{\{2\}}$	$0.01928^{\{7\}}$	$0.0199^{\{6\}}$	$0.01893^{\{3\}}$	$0.02008^{\{9\}}$	$0.02045^{\{12\}}$	$0.01793^{\{4\}}$	$0.01972^{\{1\}}$	$0.02112^{\{11\}}$	$0.02234^{\{13\}}$	$0.02127^{\{10\}}$	$0.03007^{\{16\}}$	$0.02031^{\{8\}}$	$0.02513^{\{15\}}$	$0.02032^{\{5\}}$	$0.02464^{\{14\}}$
	$\sum Ranks$	$11^{\{1.5\}}$	$38^{\{7\}}$	$35^{\{5\}}$	$18^{\{3\}}$	$52^{\{9\}}$	$70^{\{12\}}$	$21^{\{4\}}$	$11^{\{1.5\}}$	$64^{\{10.5\}}$	$78^{\{13\}}$	$64^{\{10.5\}}$	$96^{\{16\}}$	$48^{\{8\}}$	$90^{\{15\}}$	$36^{\{6\}}$	$84^{\{14\}}$
140	BIAS($\hat{\sigma}$)	$0.06133^{\{1\}}$	$0.06661^{\{4\}}$	$0.06997^{\{7\}}$	$0.06654^{\{3\}}$	$0.06935^{\{6\}}$	$0.07197^{\{8\}}$	$0.06588^{\{2\}}$	$0.06703^{\{5\}}$	$0.07383^{\{11\}}$	$0.08372^{\{13\}}$	$0.07349^{\{10\}}$	$0.1105^{\{16\}}$	$0.07211^{\{9\}}$	$0.08774^{\{14\}}$	$0.07576^{\{12\}}$	$0.08984^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00602^{\{1\}}$	$0.00706^{\{3\}}$	$0.00749^{\{6\}}$	$0.00707^{\{4\}}$	$0.00766^{\{7\}}$	$0.00843^{\{9\}}$	$0.00689^{\{2\}}$	$0.00717^{\{5\}}$	$0.00852^{\{10\}}$	$0.01109^{\{13\}}$	$0.00896^{\{12\}}$	$0.02196^{\{16\}}$	$0.00806^{\{8\}}$	$0.01252^{\{15\}}$	$0.00895^{\{11\}}$	$0.01244^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.02453^{\{1\}}$	$0.02664^{\{4\}}$	$0.02799^{\{7\}}$	$0.02662^{\{3\}}$	$0.02774^{\{6\}}$	$0.02879^{\{8\}}$	$0.02635^{\{2\}}$	$0.02681^{\{5\}}$	$0.02953^{\{11\}}$	$0.03349^{\{13\}}$	$0.02939^{\{10\}}$	$0.0442^{\{16\}}$	$0.02884^{\{9\}}$	$0.03509^{\{14\}}$	$0.0303^{\{12\}}$	$0.03594^{\{15\}}$
	D_{abs}	$0.0173^{\{1\}}$	$0.01875^{\{4\}}$	$0.01972^{\{7\}}$	$0.01871^{\{3\}}$	$0.01959^{\{6\}}$	$0.02008^{\{8\}}$	$0.01858^{\{2\}}$	$0.01889^{\{5\}}$	$0.02086^{\{11\}}$	$0.02352^{\{13\}}$	$0.02064^{\{10\}}$	$0.03153^{\{16\}}$	$0.02041^{\{9\}}$	$0.02451^{\{14\}}$	$0.02126^{\{12\}}$	$0.02522^{\{15\}}$
	D_{max}	$0.02482^{\{1\}}$	$0.02694^{\{4\}}$	$0.02837^{\{7\}}$	$0.02687^{\{3\}}$	$0.0281^{\{6\}}$	$0.0289^{\{8\}}$	$0.02665^{\{2\}}$	$0.02708^{\{5\}}$	$0.02991^{\{11\}}$	$0.03375^{\{13\}}$	$0.02958^{\{10\}}$	$0.04567^{\{16\}}$	$0.02917^{\{9\}}$	$0.03533^{\{14\}}$	$0.0305^{\{12\}}$	$0.03622^{\{15\}}$
	ASAE	$0.01397^{\{1\}}$	$0.01463^{\{4\}}$	$0.01482^{\{7\}}$	$0.01423^{\{3\}}$	$0.01448^{\{6\}}$	$0.01518^{\{8\}}$	$0.01399^{\{2\}}$	$0.01437^{\{5\}}$	$0.01582^{\{11\}}$	$0.01728^{\{13\}}$	$0.01593^{\{10\}}$	$0.02292^{\{16\}}$	$0.0151^{\{9\}}$	$0.0179^{\{14\}}$	$0.01537^{\{12\}}$	$0.01824^{\{15\}}$
	$\sum Ranks$	$6^{\{1\}}$	$25^{\{4\}}$	$41^{\{7\}}$	$19^{\{3\}}$	$36^{\{6\}}$	$50^{\{8\}}$	$12^{\{2\}}$	$29^{\{5\}}$	$65^{\{11\}}$	$78^{\{13\}}$	$64^{\{10\}}$	$96^{\{16\}}$	$52^{\{9\}}$	$85^{\{14\}}$	$69^{\{12\}}$	$89^{\{15\}}$
200	BIAS($\hat{\sigma}$)	$0.05327^{\{1\}}$	$0.05671^{\{6\}}$	$0.05576^{\{5\}}$	$0.0547^{\{3\}}$	$0.05747^{\{8\}}$	$0.06332^{\{12\}}$	$0.05535^{\{4\}}$	$0.05412^{\{2\}}$	$0.0612^{\{9\}}$	$0.06759^{\{13\}}$	$0.06265^{\{10\}}$	$0.09291^{\{16\}}$	$0.05743^{\{7\}}$	$0.07296^{\{14\}}$	$0.06266^{\{11\}}$	$0.07646^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00452^{\{1\}}$	$0.00493^{\{6\}}$	$0.0049^{\{5\}}$	$0.00463^{\{2\}}$	$0.0053^{\{8\}}$	$0.00646^{\{12\}}$	$0.00485^{\{4\}}$	$0.00472^{\{3\}}$	$0.00591^{\{9\}}$	$0.00736^{\{13\}}$	$0.00621^{\{11\}}$	$0.01537^{\{16\}}$	$0.00528^{\{7\}}$	$0.00824^{\{14\}}$	$0.00619^{\{10\}}$	$0.00963^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.02131^{\{1\}}$	$0.02268^{\{6\}}$	$0.0223^{\{5\}}$	$0.02188^{\{3\}}$	$0.02299^{\{8\}}$	$0.02533^{\{12\}}$	$0.02214^{\{4\}}$	$0.02165^{\{2\}}$	$0.02448^{\{9\}}$	$0.02704^{\{13\}}$	$0.02506^{\{10.5\}}$	$0.03717^{\{16\}}$	$0.02297^{\{7\}}$	$0.02918^{\{14\}}$	$0.02506^{\{10.5\}}$	$0.03058^{\{15\}}$
	D_{abs}	$0.015^{\{1\}}$	$0.016^{\{6\}}$	$0.0157^{\{5\}}$	$0.01541^{\{3\}}$	$0.01619^{\{8\}}$	$0.01762^{\{11\}}$	$0.0156^{\{4\}}$	$0.01524^{\{2\}}$	$0.01721^{\{9\}}$	$0.01908^{\{13\}}$	$0.01764^{\{12\}}$	$0.02647^{\{16\}}$	$0.01618^{\{7\}}$	$0.02044^{\{14\}}$	$0.01758^{\{10\}}$	$0.0214^{\{15\}}$
	D_{max}	$0.02161^{\{1\}}$	$0.02295^{\{6\}}$	$0.02257^{\{5\}}$	$0.02212^{\{3\}}$	$0.02324^{\{7.5\}}$	$0.02539^{\{12\}}$	$0.0224^{\{4\}}$	$0.02189^{\{2\}}$	$0.02478^{\{9\}}$	$0.02736^{\{13\}}$	$0.02529^{\{11\}}$	$0.03824^{\{16\}}$	$0.02324^{\{7.5\}}$	$0.02946^{\{14\}}$	$0.02528^{\{10\}}$	$0.0308^{\{15\}}$
	ASAE	$0.0113^{\{1\}}$	$0.01166^{\{6\}}$	$0.01171^{\{5\}}$	$0.01111^{\{3\}}$	$0.01155^{\{7.5\}}$	$0.0122^{\{12\}}$	$0.01106^{\{4\}}$	$0.0115^{\{2\}}$	$0.01281^{\{9\}}$	$0.01375^{\{13\}}$	$0.01284^{\{11\}}$	$0.01801^{\{16\}}$	$0.01199^{\{7.5\}}$	$0.01446^{\{14\}}$	$0.01274^{\{10\}}$	$0.01469^{\{15\}}$
	$\sum Ranks$	$8^{\{1\}}$	$36^{\{6\}}$	$32^{\{5\}}$	$16^{\{3\}}$	$44.5^{\{8\}}$	$68^{\{12\}}$	$21^{\{4\}}$	$15^{\{2\}}$	$56^{\{9\}}$	$78^{\{13\}}$	$66.5^{\{11\}}$	$96^{\{16\}}$	$43.5^{\{7\}}$	$84^{\{14\}}$	$61.5^{\{10\}}$	$90^{\{15\}}$
250	BIAS($\hat{\sigma}$)	$0.04845^{\{1\}}$	$0.05048^{\{4\}}$	$0.05176^{\{6\}}$	$0.04878^{\{2\}}$	$0.05203^{\{7\}}$	$0.05413^{\{11\}}$	$0.04982^{\{3\}}$	$0.0511^{\{5\}}$	$0.05707^{\{12\}}$	$0.06322^{\{13\}}$	$0.05337^{\{8\}}$	$0.0829^{\{16\}}$	$0.05401^{\{10\}}$	$0.0665^{\{15\}}$	$0.05372^{\{9\}}$	$0.06403^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00358^{\{1\}}$	$0.004^{\{4\}}$	$0.00414^{\{5\}}$	$0.00361^{\{2\}}$	$0.00422^{\{7\}}$	$0.00474^{\{11\}}$	$0.00386^{\{3\}}$	$0.00419^{\{6\}}$	$0.00504^{\{12\}}$	$0.00616^{\{13\}}$	$0.00455^{\{8\}}$	$0.01316^{\{16\}}$	$0.00459^{\{9\}}$	$0.00704^{\{15\}}$	$0.00466^{\{10\}}$	$0.00665^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.01938^{\{1\}}$	$0.02019^{\{4\}}$	$0.0207^{\{6\}}$	$0.01951^{\{2\}}$	$0.02081^{\{7\}}$	$0.02165^{\{11\}}$	$0.01993^{\{3\}}$	$0.02044^{\{5\}}$	$0.02283^{\{12\}}$	$0.02529^{\{13\}}$	$0.02135^{\{8\}}$	$0.03316^{\{16\}}$	$0.0216^{\{10\}}$	$0.0266^{\{15\}}$	$0.02149^{\{9\}}$	$0.02561^{\{14\}}$
	D_{abs}	$0.01367^{\{1\}}$	$0.01424^{\{4\}}$	$0.01458^{\{6\}}$	$0.01375^{\{2\}}$	$0.01467^{\{7\}}$	$0.01517^{\{10\}}$	$0.01406^{\{3\}}$	$0.0144^{\{5\}}$	$0.01612^{\{12\}}$	$0.01777^{\{13\}}$	$0.01506^{\{8\}}$	$0.02362^{\{16\}}$	$0.01522^{\{11\}}$	$0.0187^{\{15\}}$	$0.01508^{\{9\}}$	$0.01804^{\{14\}}$
	D_{max}	$0.01964^{\{1\}}$	$0.02045^{\{4\}}$	$0.02099^{\{6\}}$	$0.01975^{\{2\}}$	$0.02108^{\{7\}}$	$0.02181^{\{10\}}$	$0.02019^{\{3\}}$	$0.02071^{\{5\}}$	$0.0231^{\{12\}}$	$0.02556^{\{13\}}$	$0.0216^{\{8\}}$	$0.03412^{\{16\}}$	$0.0219^{\{11\}}$	$0.02689^{\{15\}}$	$0.02169^{\{9\}}$	$0.02588^{\{14\}}$
	ASAE	$0.00961^{\{1\}}$	$0.01004^{\{4\}}$	$0.01013^{\{6\}}$	$0.00967^{\{2\}}$	$0.01012^{\{7\}}$	$0.01056^{\{10\}}$	$0.00966^{\{3\}}$	$0.00979^{\{5\}}$	$0.01079^{\{12\}}$	$0.01155^{\{13\}}$	$0.01087^{\{8\}}$	$0.01563^{\{16\}}$	$0.01057^{\{11\}}$	$0.01275^{\{15\}}$	$0.01059^{\{9\}}$	$0.01276^{\{14\}}$
	$\sum Ranks$	$6^{\{1\}}$	$25^{\{4\}}$	$36^{\{6\}}$	$13^{\{2\}}$	$41^{\{7\}}$	$61^{\{11\}}$	$17^{\{3\}}$	$30^{\{5\}}$	$71^{\{12\}}$	$78^{\{13\}}$	$52^{\{8\}}$	$96^{\{16\}}$	$60^{\{10\}}$	$89^{\{15\}}$	$56^{\{9\}}$	$85^{\{14\}}$
300	BIAS($\hat{\sigma}$)	$0.04182^{\{1\}}$	$0.04706^{\{6\}}$	$0.04905^{\{8\}}$	$0.04324^{\{2\}}$	$0.04715^{\{7\}}$	$0.0531^{\{11\}}$	$0.04569^{\{4\}}$	$0.04358^{\{3\}}$	$0.04977^{\{9\}}$	$0.05501^{\{13\}}$	$0.05324^{\{12\}}$	$0.07531^{\{16\}}$	$0.04645^{\{5\}}$	$0.06085^{\{14\}}$	$0.05227^{\{10\}}$	$0.06189^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.0028^{\{1\}}$	$0.00341^{\{5\}}$	$0.00374^{\{8\}}$	$0.00298^{\{2\}}$	$0.00344^{\{7\}}$	$0.0044^{\{11.5\}}$	$0.00329^{\{4\}}$	$0.00308^{\{3\}}$	$0.00397^{\{9\}}$	$0.00485^{\{13\}}$	$0.0044^{\{11.5\}}$	$0.01006^{\{16\}}$	$0.00342^{\{6\}}$	$0.00576^{\{14\}}$	$0.00431^{\{10\}}$	$0.00608^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.01673^{\{1\}}$	$0.01882^{\{6\}}$	$0.01962^{\{8\}}$	$0.0173^{\{2\}}$	$0.01886^{\{7\}}$	$0.02124^{\{11\}}$	$0.01827^{\{4\}}$	$0.01743^{\{3\}}$	$0.01991^{\{9\}}$	$0.022^{\{13\}}$	$0.02129^{\{12\}}$	$0.03012^{\{16\}}$	$0.01858^{\{5\}}$	$0.02434^{\{14\}}$	$0.02091^{\{10\}}$	$0.02476^{\{15\}}$
	D_{abs}	$0.01181^{\{1\}}$	$0.01326^{\{6\}}$	$0.01384^{\{8\}}$	$0.01216^{\{2\}}$	$0.01328^{\{7\}}$	$0.0148^{\{11\}}$	$0.01288^{\{4\}}$	$0.01229^{\{3\}}$	$0.01402^{\{9\}}$	$0.01546^{\{13\}}$	$0.01501^{\{12\}}$	$0.02143^{\{16\}}$	$0.01309^{\{5\}}$	$0.01714^{\{14\}}$	$0.01469^{\{10\}}$	$0.01739^{\{15\}}$
	D_{max}	$0.01696^{\{1\}}$	$0.01906^{\{6\}}$	$0.01983^{\{8\}}$	$0.01749^{\{2\}}$	$0.01908^{\{7\}}$	$0.02136^{\{11\}}$	$0.01852^{\{4\}}$	$0.01768^{\{3\}}$	$0.02018^{\{9\}}$	$0.02225^{\{13\}}$	$0.02151^{\{12\}}$	$0.03084^{\{16\}}$	$0.01881^{\{5\}}$	$0.02459^{\{14\}}$	$0.02112^{\{10\}}$	$0.02498^{\{15\}}$
	ASAE	$0.00851^{\{1\}}$	$0.00895^{\{6\}}$	$0.00903^{\{8\}}$	$0.00865^{\{2\}}$	$0.00902^{\{7\}}$	$0.00953^{\{11\}}$	$0.00839^{\{4\}}$	$0.00889^{\{3\}}$	$0.0098^{\{9\}}$	$0.01027^{\{13\}}$	$0.00995^{\{12\}}$	$0.01389^{\{16\}}$	$0.0094^{\{5\}}$	$0.01125^{\{14\}}$	$0.00946^{\{10\}}$	$0.01135^{\{15\}}$
	$\sum Ranks$	$7^{\{1\}}$	$34^{\{5.5\}}$	$47^{\{8\}}$	$13^{\{2\}}$	$41^{\{7\}}$	$65.5^{\{11\}}$	$21^{\{4\}}$	$19^{\{3\}}$	$56^{\{9\}}$	$78^{\{13\}}$	$71.5^{\{12\}}$	$96^{\{16\}}$	$34^{\{5.5\}}$	$84^{\{14\}}$	$59^{\{10\}}$	$90^{\{15\}}$
400	BIAS($\hat{\sigma}$)	$0.03902^{\{3\}}$	$0.03893^{\{2\}}$	$0.04089^{\{6\}}$	$0.03749^{\{1\}}$	$0.04096^{\{7\}}$	$0.04449^{\{11\}}$	$0.03906^{\{4\}}$	$0.04032^{\{5\}}$	$0.04428^{\{10\}}$	$0.04769^{\{13\}}$	$0.04472^{\{12\}}$	$0.06684^{\{16\}}$	$0.04349^{\{9\}}$	$0.05398^{\{15\}}$	$0.04163^{\{8\}}$	$0.0537^{\{14\}}$
	MSE($\hat{\sigma}$)	$0.00239^{\{2.5\}}$	$0.00243^{\{4\}}$	$0.0027^{\{7\}}$	$0.00213^{\{1\}}$	$0.00266^{\{6\}}$	$0.00313^{\{11\}}$	$0.00239^{\{2.5\}}$	$0.00252^{\{5\}}$	$0.003^{\{10\}}$	$0.00364^{\{13\}}$	$0.00318^{\{12\}}$	$0.00963^{\{16\}}$	$0.00296^{\{9\}}$	$0.00468^{\{15\}}$	$0.0028^{\{8\}}$	$0.00449^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.01561^{\{3\}}$	$0.01557^{\{2\}}$	$0.01635^{\{6\}}$	$0.015^{\{1\}}$	$0.01638^{\{7\}}$	$0.0178^{\{11\}}$	$0.01563^{\{4\}}$	$0.01613^{\{5\}}$	$0.01771^{\{10\}}$	$0.01908^{\{13\}}$	$0.01789^{\{12\}}$	$0.02673^{\{16\}}$	$0.0174^{\{9\}}$	$0.02159^{\{15\}}$	$0.01665^{\{8\}}$	$0.02148^{\{14\}}$
	D_{abs}	$0.01101^{\{4\}}$	$0.01097^{\{2\}}$	$0.01152^{\{6\}}$	$0.01056^{\{1\}}$	$0.01154^{\{7\}}$	$0.01247^{\{10\}}$	$0.011^{\{3\}}$	$0.01137^{\{5\}}$	$0.01248^{\{11\}}$	$0.01343^{\{13\}}$	$0.01258^{\{12\}}$	$0.01903^{\{16\}}$	$0.01228^{\{9\}}$	$0.01515^{\{15\}}$	$0.01173^{\{8\}}$	$0.01508^{\{14\}}$
	D_{max}	$0.01582^{\{3.5\}}$	$0.01577^{\{2\}}$	$0.01655^{\{6\}}$	$0.01516^{\{1\}}$	$0.0166^{\{7\}}$	$0.01794^{\{10\}}$	$0.01582^{\{3.5\}}$	$0.01634^{\{5\}}$	$0.01795^{\{11\}}$	$0.0193^{\{13\}}$	$0.01806^{\{12\}}$	$0.02747^{\{16\}}$	$0.01763^{\{9\}}$	$0.02181^{\{15\}}$	$0.01684^{\{8\}}$	$0.02168^{\{14\}}$
	ASAE	$0.00711^{\{3.5\}}$	$0.00731^{\{2\}}$	$0.00763^{\{6\}}$	$0.00721^{\{1\}}$	$0.00746^{\{7\}}$	$0.00777^{\{10\}}$	$0.00712^{\{3.5\}}$	$0.00743^{\{5\}}$	$0.00833^{\{11\}}$	$0.00875^{\{13\}}$	$0.00819^{\{12\}}$	$0.01197^{\{16\}}$	$0.00765^{\{9\}}$	$0.00968^{\{15\}}$	$0.00795^{\{8\}}$	$0.00954^{\{14\}}$
	$\sum Ranks$	$17^{\{3\}}$	$16^{\{2\}}$	$38^{\{6\}}$	$8^{\{1\}}$	$40^{\{7\}}$	$62^{\{10\}}$	$19^{\{4\}}$	$30^{\{5\}}$	$64^{\{11\}}$	$78^{\{13\}}$	$71^{\{12\}}$	$96^{\{16\}}$	$53^{\{9\}}$	$90^{\{15\}}$	$50^{\{8\}}$	$84^{\{14\}}$

TABLE 6. Numerical values of simulation measures for $\sigma = 4.0$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSD	MSSLD	MSLND
15	BIAS($\hat{\sigma}$)	$0.31727^{\{2\}}$	$0.31251^{\{1\}}$	$0.33349^{\{4\}}$	$0.34343^{\{7\}}$	$0.3373^{\{5\}}$	$0.37227^{\{12\}}$	$0.34604^{\{8\}}$	$0.34224^{\{6\}}$	$0.37131^{\{11\}}$	$0.37889^{\{13\}}$	$0.36751^{\{10\}}$	$0.48152^{\{16\}}$	$0.32932^{\{3\}}$	$0.44085^{\{14\}}$	$0.34909^{\{9\}}$	$0.44888^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.15796^{\{2\}}$	$0.15702^{\{1\}}$	$0.17917^{\{4\}}$	$0.18441^{\{7\}}$	$0.18072^{\{5\}}$	$0.21043^{\{10\}}$	$0.18381^{\{6\}}$	$0.18676^{\{8\}}$	$0.21436^{\{11\}}$	$0.23211^{\{13\}}$	$0.22176^{\{12\}}$	$0.43498^{\{16\}}$	$0.17626^{\{3\}}$	$0.32393^{\{14\}}$	$0.1883^{\{9\}}$	$0.33125^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.07932^{\{2\}}$	$0.07813^{\{1\}}$	$0.08337^{\{4\}}$	$0.08586^{\{7\}}$	$0.08433^{\{5\}}$	$0.09307^{\{12\}}$	$0.08651^{\{8\}}$	$0.08556^{\{6\}}$	$0.09283^{\{11\}}$	$0.09472^{\{13\}}$	$0.09188^{\{10\}}$	$0.12038^{\{16\}}$	$0.08233^{\{3\}}$	$0.11021^{\{14\}}$	$0.08727^{\{9\}}$	$0.11222^{\{15\}}$
	D_{abs}	$0.05593^{\{2\}}$	$0.05527^{\{1\}}$	$0.05884^{\{4\}}$	$0.05991^{\{6\}}$	$0.05917^{\{5\}}$	$0.06339^{\{10\}}$	$0.06084^{\{8\}}$	$0.06034^{\{7\}}$	$0.06559^{\{12\}}$	$0.06624^{\{13\}}$	$0.06373^{\{11\}}$	$0.08509^{\{16\}}$	$0.05788^{\{3\}}$	$0.07471^{\{14\}}$	$0.0609^{\{9\}}$	$0.07678^{\{15\}}$
	D_{max}	$0.07973^{\{2\}}$	$0.07841^{\{1\}}$	$0.0834^{\{4\}}$	$0.08439^{\{6\}}$	$0.08404^{\{5\}}$	$0.09036^{\{11\}}$	$0.0861^{\{8\}}$	$0.08524^{\{7\}}$	$0.09315^{\{12\}}$	$0.09344^{\{13\}}$	$0.09006^{\{10\}}$	$0.12444^{\{16\}}$	$0.08209^{\{3\}}$	$0.10693^{\{14\}}$	$0.08621^{\{9\}}$	$0.10888^{\{15\}}$
	ASAE	$0.0618^{\{2\}}$	$0.06245^{\{1\}}$	$0.06406^{\{4\}}$	$0.06075^{\{6\}}$	$0.06479^{\{5\}}$	$0.06361^{\{11\}}$	$0.05955^{\{8\}}$	$0.06365^{\{7\}}$	$0.07156^{\{12\}}$	$0.07246^{\{13\}}$	$0.07003^{\{10\}}$	$0.09537^{\{16\}}$	$0.06754^{\{3\}}$	$0.08128^{\{14\}}$	$0.06725^{\{9\}}$	$0.08134^{\{15\}}$
	$\sum Ranks$	$13^{\{2\}}$	$9^{\{1\}}$	$27^{\{4\}}$	$35^{\{6\}}$	$33^{\{5\}}$	$60^{\{10\}}$	$39^{\{7\}}$	$40^{\{8\}}$	$69^{\{12\}}$	$78^{\{13\}}$	$64^{\{11\}}$	$96^{\{16\}}$	$25^{\{3\}}$	$84^{\{14\}}$	$54^{\{9\}}$	$90^{\{15\}}$
45	BIAS($\hat{\sigma}$)	$0.17553^{\{1\}}$	$0.18397^{\{2\}}$	$0.19342^{\{7\}}$	$0.19052^{\{5\}}$	$0.20589^{\{9\}}$	$0.21778^{\{12\}}$	$0.18501^{\{3\}}$	$0.18971^{\{4\}}$	$0.20186^{\{8\}}$	$0.22877^{\{13\}}$	$0.21046^{\{11\}}$	$0.30476^{\{16\}}$	$0.19284^{\{6\}}$	$0.24992^{\{14\}}$	$0.20658^{\{10\}}$	$0.25161^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.04906^{\{1\}}$	$0.0552^{\{3\}}$	$0.06027^{\{7\}}$	$0.05623^{\{4\}}$	$0.06659^{\{9\}}$	$0.07668^{\{12\}}$	$0.05434^{\{2\}}$	$0.05906^{\{6\}}$	$0.06435^{\{8\}}$	$0.08427^{\{13\}}$	$0.07264^{\{11\}}$	$0.15987^{\{16\}}$	$0.05877^{\{5\}}$	$0.0972^{\{14\}}$	$0.06845^{\{10\}}$	$0.10311^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.04388^{\{1\}}$	$0.04599^{\{2\}}$	$0.04836^{\{7\}}$	$0.04763^{\{5\}}$	$0.05147^{\{9\}}$	$0.05444^{\{12\}}$	$0.04625^{\{3\}}$	$0.04743^{\{4\}}$	$0.05047^{\{8\}}$	$0.05719^{\{13\}}$	$0.05261^{\{11\}}$	$0.07619^{\{16\}}$	$0.04821^{\{6\}}$	$0.06248^{\{14\}}$	$0.05164^{\{10\}}$	$0.0629^{\{15\}}$
	D_{abs}	$0.03107^{\{1\}}$	$0.03237^{\{2\}}$	$0.03425^{\{7\}}$	$0.03327^{\{4\}}$	$0.03609^{\{9\}}$	$0.0372^{\{12\}}$	$0.03246^{\{3\}}$	$0.03347^{\{5\}}$	$0.03577^{\{8\}}$	$0.04007^{\{13\}}$	$0.03674^{\{11\}}$	$0.05459^{\{16\}}$	$0.03398^{\{6\}}$	$0.04355^{\{14\}}$	$0.03614^{\{10\}}$	$0.04378^{\{15\}}$
	D_{max}	$0.0444^{\{1\}}$	$0.04626^{\{2\}}$	$0.04893^{\{7\}}$	$0.04775^{\{4\}}$	$0.052^{\{10\}}$	$0.05375^{\{12\}}$	$0.04665^{\{3\}}$	$0.04781^{\{5\}}$	$0.05117^{\{8\}}$	$0.05736^{\{13\}}$	$0.05256^{\{11\}}$	$0.07922^{\{16\}}$	$0.04874^{\{6\}}$	$0.06229^{\{14\}}$	$0.05162^{\{9\}}$	$0.06265^{\{15\}}$
	ASAE	$0.02965^{\{1\}}$	$0.02999^{\{2\}}$	$0.03093^{\{7\}}$	$0.02942^{\{4\}}$	$0.0309^{\{10\}}$	$0.0313^{\{12\}}$	$0.02925^{\{3\}}$	$0.02989^{\{5\}}$	$0.033^{\{8\}}$	$0.03566^{\{13\}}$	$0.0334^{\{11\}}$	$0.04804^{\{16\}}$	$0.03178^{\{6\}}$	$0.03781^{\{14\}}$	$0.03273^{\{9\}}$	$0.03881^{\{15\}}$
	$\sum Ranks$	$8^{\{1\}}$	$16^{\{3\}}$	$42^{\{7\}}$	$24^{\{4\}}$	$52^{\{9\}}$	$68^{\{12\}}$	$15^{\{2\}}$	$28^{\{5\}}$	$51^{\{8\}}$	$78^{\{13\}}$	$67^{\{11\}}$	$96^{\{16\}}$	$38^{\{6\}}$	$84^{\{14\}}$	$59^{\{10\}}$	$90^{\{15\}}$
90	BIAS($\hat{\sigma}$)	$0.12368^{\{1\}}$	$0.13259^{\{3\}}$	$0.13738^{\{6\}}$	$0.13716^{\{5\}}$	$0.1412^{\{8\}}$	$0.15392^{\{12\}}$	$0.13211^{\{2\}}$	$0.13266^{\{4\}}$	$0.1408^{\{7\}}$	$0.1613^{\{13\}}$	$0.1485^{\{9\}}$	$0.22354^{\{16\}}$	$0.14867^{\{10\}}$	$0.1776^{\{14\}}$	$0.14895^{\{11\}}$	$0.18021^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.0248^{\{1\}}$	$0.02738^{\{4\}}$	$0.02947^{\{6\}}$	$0.02901^{\{5\}}$	$0.03108^{\{7\}}$	$0.03833^{\{12\}}$	$0.02709^{\{2\}}$	$0.0273^{\{3\}}$	$0.03172^{\{8\}}$	$0.04115^{\{13\}}$	$0.03508^{\{11\}}$	$0.10641^{\{16\}}$	$0.03398^{\{9\}}$	$0.04946^{\{14\}}$	$0.03473^{\{10\}}$	$0.05287^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.03092^{\{1\}}$	$0.03315^{\{3\}}$	$0.03435^{\{6\}}$	$0.03429^{\{5\}}$	$0.0353^{\{8\}}$	$0.03848^{\{12\}}$	$0.03303^{\{2\}}$	$0.03316^{\{4\}}$	$0.0352^{\{7\}}$	$0.04033^{\{13\}}$	$0.03712^{\{9\}}$	$0.05588^{\{16\}}$	$0.03717^{\{10\}}$	$0.0444^{\{14\}}$	$0.03724^{\{11\}}$	$0.04505^{\{15\}}$
	D_{abs}	$0.02179^{\{1\}}$	$0.02339^{\{4\}}$	$0.02427^{\{6\}}$	$0.0241^{\{5\}}$	$0.02488^{\{8\}}$	$0.0266^{\{12\}}$	$0.02333^{\{2\}}$	$0.02335^{\{3\}}$	$0.02485^{\{7\}}$	$0.02842^{\{13\}}$	$0.02596^{\{9\}}$	$0.04005^{\{16\}}$	$0.0262^{\{11\}}$	$0.03101^{\{14\}}$	$0.02611^{\{10\}}$	$0.0315^{\{15\}}$
	D_{max}	$0.03133^{\{1\}}$	$0.03355^{\{3\}}$	$0.0348^{\{6\}}$	$0.03449^{\{5\}}$	$0.03565^{\{8\}}$	$0.03838^{\{12\}}$	$0.03348^{\{2\}}$	$0.03358^{\{4\}}$	$0.03553^{\{7\}}$	$0.04067^{\{13\}}$	$0.03726^{\{9\}}$	$0.05825^{\{16\}}$	$0.0376^{\{11\}}$	$0.04459^{\{14\}}$	$0.0374^{\{10\}}$	$0.04538^{\{15\}}$
	ASAE	$0.01879^{\{1\}}$	$0.01917^{\{3\}}$	$0.01919^{\{6\}}$	$0.01845^{\{5\}}$	$0.01931^{\{8\}}$	$0.02055^{\{12\}}$	$0.01863^{\{2\}}$	$0.01939^{\{4\}}$	$0.02108^{\{7\}}$	$0.02304^{\{13\}}$	$0.02145^{\{9\}}$	$0.0307^{\{16\}}$	$0.02046^{\{11\}}$	$0.02468^{\{14\}}$	$0.02074^{\{10\}}$	$0.02475^{\{15\}}$
	$\sum Ranks$	$8^{\{1\}}$	$21^{\{3\}}$	$35^{\{6\}}$	$26^{\{5\}}$	$45^{\{7\}}$	$69^{\{12\}}$	$12^{\{2\}}$	$25^{\{4\}}$	$47^{\{8\}}$	$78^{\{13\}}$	$59^{\{9.5\}}$	$96^{\{16\}}$	$59^{\{9.5\}}$	$84^{\{14\}}$	$62^{\{11\}}$	$90^{\{15\}}$
140	BIAS($\hat{\sigma}$)	$0.09976^{\{1\}}$	$0.10696^{\{5\}}$	$0.11683^{\{10\}}$	$0.10226^{\{3\}}$	$0.11338^{\{7\}}$	$0.12534^{\{12\}}$	$0.10118^{\{2\}}$	$0.10646^{\{4\}}$	$0.11927^{\{11\}}$	$0.13249^{\{13\}}$	$0.11682^{\{9\}}$	$0.18911^{\{16\}}$	$0.11221^{\{6\}}$	$0.14522^{\{14\}}$	$0.11352^{\{8\}}$	$0.14523^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.01583^{\{1\}}$	$0.01866^{\{5\}}$	$0.02119^{\{9\}}$	$0.01671^{\{3\}}$	$0.0203^{\{8\}}$	$0.02478^{\{12\}}$	$0.01616^{\{2\}}$	$0.01822^{\{4\}}$	$0.02258^{\{11\}}$	$0.02777^{\{13\}}$	$0.02177^{\{10\}}$	$0.08052^{\{16\}}$	$0.01994^{\{7\}}$	$0.03415^{\{15\}}$	$0.01993^{\{6\}}$	$0.03413^{\{14\}}$
	MRE($\hat{\sigma}$)	$0.02494^{\{1\}}$	$0.02674^{\{5\}}$	$0.02921^{\{10\}}$	$0.02557^{\{3\}}$	$0.02835^{\{7\}}$	$0.03134^{\{12\}}$	$0.02529^{\{2\}}$	$0.02662^{\{4\}}$	$0.02982^{\{11\}}$	$0.03312^{\{13\}}$	$0.0292^{\{9\}}$	$0.04728^{\{16\}}$	$0.02805^{\{6\}}$	$0.0363^{\{14\}}$	$0.02838^{\{8\}}$	$0.03631^{\{15\}}$
	D_{abs}	$0.0176^{\{1\}}$	$0.01882^{\{5\}}$	$0.02058^{\{10\}}$	$0.01794^{\{3\}}$	$0.01991^{\{7.5\}}$	$0.02176^{\{12\}}$	$0.0178^{\{2\}}$	$0.01872^{\{4\}}$	$0.02098^{\{11\}}$	$0.02333^{\{13\}}$	$0.02048^{\{9\}}$	$0.03353^{\{16\}}$	$0.0198^{\{6\}}$	$0.02545^{\{15\}}$	$0.01991^{\{7.5\}}$	$0.02539^{\{14\}}$
	D_{max}	$0.02526^{\{1\}}$	$0.02707^{\{5\}}$	$0.02956^{\{10\}}$	$0.02577^{\{3\}}$	$0.02865^{\{8\}}$	$0.03135^{\{12\}}$	$0.02559^{\{2\}}$	$0.02688^{\{4\}}$	$0.03013^{\{11\}}$	$0.03352^{\{13\}}$	$0.02942^{\{9\}}$	$0.04886^{\{16\}}$	$0.02843^{\{6\}}$	$0.0365^{\{14.5\}}$	$0.02862^{\{7\}}$	$0.0365^{\{14.5\}}$
	ASAE	$0.01381^{\{1\}}$	$0.01421^{\{5\}}$	$0.01461^{\{10\}}$	$0.01427^{\{3\}}$	$0.0144^{\{8\}}$	$0.01516^{\{12\}}$	$0.01382^{\{2\}}$	$0.01443^{\{4\}}$	$0.01571^{\{11\}}$	$0.01731^{\{13\}}$	$0.01601^{\{9\}}$	$0.02386^{\{16\}}$	$0.01521^{\{6\}}$	$0.01826^{\{14.5\}}$	$0.01568^{\{7\}}$	$0.0182^{\{14.5\}}$
	$\sum Ranks$	$6^{\{1\}}$	$28^{\{5\}}$	$56^{\{9\}}$	$19^{\{3\}}$	$42.5^{\{7\}}$	$68^{\{12\}}$	$12^{\{2\}}$	$26^{\{4\}}$	$66^{\{11\}}$	$78^{\{13\}}$	$58^{\{10\}}$	$96^{\{16\}}$	$40^{\{6\}}$	$87.5^{\{15\}}$	$46.5^{\{8\}}$	$86.5^{\{14\}}$
200	BIAS($\hat{\sigma}$)	$0.08859^{\{2\}}$	$0.08992^{\{3\}}$	$0.09638^{\{8\}}$	$0.08661^{\{1\}}$	$0.09384^{\{6\}}$	$0.10206^{\{11\}}$	$0.08999^{\{4\}}$	$0.09128^{\{5\}}$	$0.10149^{\{10\}}$	$0.11174^{\{13\}}$	$0.10536^{\{12\}}$	$0.15862^{\{16\}}$	$0.09537^{\{7\}}$	$0.11805^{\{14\}}$	$0.0989^{\{9\}}$	$0.12274^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.01207^{\{2\}}$	$0.01273^{\{4\}}$	$0.01411^{\{6\}}$	$0.01149^{\{1\}}$	$0.01414^{\{7\}}$	$0.01656^{\{11\}}$	$0.01246^{\{3\}}$	$0.01285^{\{5\}}$	$0.0163^{\{10\}}$	$0.0199^{\{13\}}$	$0.01734^{\{12\}}$	$0.05687^{\{16\}}$	$0.01456^{\{8\}}$	$0.02228^{\{14\}}$	$0.01558^{\{9\}}$	$0.02438^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.02215^{\{2\}}$	$0.02248^{\{3\}}$	$0.02409^{\{8\}}$	$0.02165^{\{1\}}$	$0.02346^{\{6\}}$	$0.02552^{\{11\}}$	$0.0225^{\{4\}}$	$0.02282^{\{5\}}$	$0.02537^{\{10\}}$	$0.02794^{\{13\}}$	$0.02634^{\{12\}}$	$0.03965^{\{16\}}$	$0.02384^{\{7\}}$	$0.02951^{\{14\}}$	$0.02473^{\{9\}}$	$0.03068^{\{15\}}$
	D_{abs}	$0.01561^{\{2\}}$	$0.01588^{\{4\}}$	$0.01697^{\{8\}}$	$0.01527^{\{1\}}$	$0.01655^{\{6\}}$	$0.01782^{\{10\}}$	$0.01583^{\{3\}}$	$0.0161^{\{5\}}$	$0.01787^{\{11\}}$	$0.01959^{\{13\}}$	$0.01851^{\{12\}}$	$0.02825^{\{16\}}$	$0.01683^{\{7\}}$	$0.02069^{\{14\}}$	$0.0173^{\{9\}}$	$0.02153^{\{15\}}$
	D_{max}	$0.02247^{\{2\}}$	$0.02277^{\{4\}}$	$0.0244^{\{8\}}$	$0.0219^{\{1\}}$	$0.02377^{\{6\}}$	$0.02562^{\{10\}}$	$0.02275^{\{3\}}$	$0.02315^{\{5\}}$	$0.02571^{\{11\}}$	$0.0282^{\{13\}}$	$0.02658^{\{12\}}$	$0.04123^{\{16\}}$	$0.02417^{\{7\}}$	$0.02974^{\{14\}}$	$0.02492^{\{9\}}$	$0.03091^{\{15\}}$
	ASAE	$0.01126^{\{2\}}$	$0.01122^{\{4\}}$	$0.01129^{\{8\}}$	$0.01122^{\{1\}}$	$0.01174^{\{6\}}$	$0.01228^{\{10\}}$	$0.01087^{\{3\}}$	$0.01156^{\{5\}}$	$0.01262^{\{11\}}$	$0.01342^{\{13\}}$	$0.01317^{\{12\}}$	$0.01921^{\{16\}}$	$0.01201^{\{7\}}$	$0.01446^{\{14\}}$	$0.01233^{\{9\}}$	$0.01467^{\{15\}}$
	$\sum Ranks$	$14^{\{2\}}$	$21^{\{4\}}$	$43^{\{7\}}$	$7^{\{1\}}$	$38^{\{6\}}$	$62^{\{10\}}$	$18^{\{3\}}$	$31^{\{5\}}$	$63^{\{11\}}$	$78^{\{13\}}$	$72^{\{12\}}$	$96^{\{16\}}$	$44^{\{8\}}$	$84^{\{14\}}$	$55^{\{9\}}$	$90^{\{15\}}$
250	BIAS($\hat{\sigma}$)	$0.07764^{\{1\}}$	$0.08218^{\{5\}}$	$0.08312^{\{7\}}$	$0.0797^{\{3\}}$	$0.08303^{\{6\}}$	$0.09361^{\{11\}}$	$0.07816^{\{2\}}$	$0.08063^{\{4\}}$	$0.086^{\{10\}}$	$0.09889^{\{13\}}$	$0.09589^{\{12\}}$	$0.13588^{\{16\}}$	$0.08453^{\{8\}}$	$0.10928^{\{14\}}$	$0.08558^{\{9\}}$	$0.11031^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00975^{\{2\}}$	$0.01061^{\{5\}}$	$0.01104^{\{7\}}$	$0.01001^{\{3\}}$	$0.01087^{\{6\}}$	$0.0137^{\{11\}}$	$0.00963^{\{1\}}$	$0.01033^{\{4\}}$	$0.01179^{\{9\}}$	$0.01566^{\{13\}}$	$0.01462^{\{12\}}$	$0.05144^{\{16\}}$	$0.01129^{\{8\}}$	$0.0187^{\{14\}}$	$0.01183^{\{10\}}$	$0.01913^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.01941^{\{1\}}$	$0.02054^{\{5\}}$	$0.02078^{\{7\}}$	$0.01992^{\{3\}}$	$0.02076^{\{6\}}$	$0.0234^{\{11\}}$	$0.01954^{\{2\}}$	$0.02016^{\{4\}}$	$0.0215^{\{10\}}$	$0.02472^{\{13\}}$	$0.02397^{\{12\}}$	$0.03397^{\{16\}}$	$0.02113^{\{8\}}$	$0.02732^{\{14\}}$	$0.0214^{\{9\}}$	$0.02758^{\{15\}}$
	D_{abs}	$0.01371^{\{1\}}$	$0.01448^{\{5\}}$	$0.01465^{\{7\}}$	$0.01402^{\{3\}}$	$0.01463^{\{6\}}$	$0.01635^{\{11\}}$	$0.01377^{\{2\}}$	$0.01422^{\{4\}}$	$0.01514^{\{10\}}$	$0.01741^{\{13\}}$	$0.01684^{\{12\}}$	$0.02397^{\{16\}}$	$0.01491^{\{8\}}$	$0.01921^{\{14\}}$	$0.01504^{\{9\}}$	$0.01938^{\{15\}}$
	D_{max}	$0.01967^{\{1\}}$	$0.0208^{\{5\}}$	$0.02107^{\{7\}}$	$0.02012^{\{3\}}$	$0.02102^{\{6\}}$	$0.02354^{\{11\}}$	$0.01978^{\{2\}}$	$0.0204^{\{4\}}$	$0.02181^{\{10\}}$	$0.02498^{\{13\}}$	$0.02416^{\{12\}}$	$0.0351^{\{16\}}$	$0.02143^{\{8\}}$	$0.02757^{\{14\}}$	$0.02162^{\{9\}}$	$0.02788^{\{15\}}$
	ASAE	$0.00961^{\{1\}}$	$0.00981^{\{5\}}$	$0.01006^{\{7\}}$	$0.00987^{\{3\}}$	$0.00992^{\{6\}}$	$0.01066^{\{11\}}$	$0.00943^{\{2\}}$	$0.00996^{\{4\}}$	$0.01083^{\{10\}}$	$0.01183^{\{13\}}$	$0.01124^{\{12\}}$	$0.0154^{\{16\}}$	$0.01045^{\{8\}}$	$0.01288^{\{14\}}$	$0.01072^{\{9\}}$	$0.01302^{\{15\}}$
	$\sum Ranks$	$8^{\{1\}}$	$28^{\{5\}}$	$42^{\{7\}}$	$19^{\{3\}}$	$35^{\{6\}}$	$64^{\{11\}}$	$10^{\{2\}}$	$26^{\{4\}}$	$60^{\{10\}}$	$78^{\{13\}}$	$72^{\{12\}}$	$96^{\{16\}}$	$48^{\{8\}}$	$84^{\{14\}}$	$56^{\{9\}}$	$90^{\{15\}}$
300	BIAS($\hat{\sigma}$)	$0.07552^{\{4\}}$	$0.0712^{\{2\}}$	$0.07706^{\{8\}}$	$0.07584^{\{5\}}$	$0.07686^{\{7\}}$	$0.08476^{\{12\}}$	$0.07296^{\{3\}}$	$0.06965^{\{1\}}$	$0.08329^{\{11\}}$	$0.09057^{\{13\}}$	$0.08042^{\{10\}}$	$0.12692^{\{16\}}$	$0.07655^{\{6\}}$	$0.09663^{\{14\}}$	$0.07872^{\{9\}}$	$0.0995^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00879^{\{4\}}$	$0.00817^{\{2\}}$	$0.00942^{\{8\}}$	$0.00895^{\{5\}}$	$0.00934^{\{7\}}$	$0.01101^{\{12\}}$	$0.0083^{\{3\}}$	$0.00737^{\{1\}}$	$0.01066^{\{11\}}$	$0.01289^{\{13\}}$	$0.01^{\{10\}}$	$0.03647^{\{16\}}$	$0.00914^{\{6\}}$	$0.01443^{\{14\}}$	$0.00982^{\{9\}}$	$0.01579^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.01888^{\{4\}}$	$0.0178^{\{2\}}$	$0.01926^{\{8\}}$	$0.01896^{\{5\}}$	$0.01922^{\{7\}}$	$0.02119^{\{12\}}$	$0.01824^{\{3\}}$	$0.01741^{\{1\}}$	$0.02082^{\{11\}}$	$0.02264^{\{13\}}$	$0.0201^{\{10\}}$	$0.03173^{\{16\}}$	$0.01914^{\{6\}}$	$0.02416^{\{14\}}$	$0.01968^{\{9\}}$	$0.02487^{\{15\}}$
	D_{abs}	$0.01331^{\{4\}}$	$0.01253^{\{2\}}$	$0.01357^{\{8\}}$	$0.01336^{\{5\}}$	$0.01355^{\{7\}}$	$0.01481^{\{12\}}$	$0.01287^{\{3\}}$	$0.01227^{\{1\}}$	$0.01467^{\{11\}}$	$0.01595^{\{13\}}$	$0.01412^{\{10\}}$	$0.02263^{\{16\}}$	$0.01349^{\{6\}}$	$0.01696^{\{14\}}$	$0.01384^{\{9\}}$	$0.01744^{\{15\}}$
	D_{max}	$0.01912^{\{4\}}$	$0.01803^{\{2\}}$	$0.01951^{\{8\}}$	$0.01918^{\{5\}}$	$0.01948^{\{7\}}$	$0.02133^{\{12\}}$	$0.01848^{\{3\}}$	$0.01766^{\{1\}}$	$0.02108^{\{11\}}$	$0.02288^{\{13\}}$	$0.02029^{\{10\}}$	$0.03279^{\{16\}}$	$0.01939^{\{6\}}$	$0.02435^{\{14\}}$	$0.01989^{\{9\}}$	$0.02509^{\{15\}}$
	ASAE	$0.00866^{\{4\}}$	$0.00881^{\{2\}}$	$0.0089^{\{8\}}$	$0.00877^{\{5\}}$	$0.00887^{\{7\}}$	$0.00944^{\{12\}}$	$0.0084^{\{3\}}$	$0.00886^{\{1\}}$	$0.00953^{\{11\}}$	$0.01045^{\{13\}}$	$0.00974^{\{10\}}$	$0.01434^{\{16\}}$	$0.00937^{\{6\}}$	$0.01143^{\{14\}}$	$0.00962^{\{9\}}$	$0.01133^{\{15\}}$
	$\sum Ranks$	$22^{\{4\}}$	$14^{\{2\}}$	$47^{\{8\}}$	$28^{\{5\}}$	$41^{\{7\}}$	$69^{\{12\}}$	$16^{\{3\}}$	$10^{\{1\}}$	$65^{\{11\}}$	$78^{\{13\}}$	$62^{\{10\}}$	$96^{\{16\}}$	$38^{\{6\}}$	$85^{\{14\}}$	$56^{\{9\}}$	$89^{\{15\}}$
400	BIAS($\hat{\sigma}$)	$0.06077^{\{1\}}$	$0.06367^{\{3\}}$	$0.06557^{\{8\}}$	$0.064^{\{6\}}$	$0.06398^{\{5\}}$	$0.0719^{\{11\}}$	$0.06207^{\{2\}}$	$0.06392^{\{4\}}$	$0.07111^{\{10\}}$	$0.07589^{\{13\}}$	$0.07377^{\{12\}}$	$0.10626^{\{16\}}$	$0.06484^{\{7\}}$	$0.0842^{\{14\}}$	$0.06774^{\{9\}}$	$0.08845^{\{15\}}$
	MSE($\hat{\sigma}$)	$0.00569^{\{1\}}$	$0.0063^{\{3\}}$	$0.00673^{\{7.5\}}$	$0.00634^{\{4\}}$	$0.00646^{\{6\}}$	$0.00824^{\{11\}}$	$0.00604^{\{2\}}$	$0.00635^{\{5\}}$	$0.00793^{\{10\}}$	$0.00891^{\{13\}}$	$0.00843^{\{12\}}$	$0.02159^{\{16\}}$	$0.00673^{\{7.5\}}$	$0.011^{\{14\}}$	$0.00733^{\{9\}}$	$0.01247^{\{15\}}$
	MRE($\hat{\sigma}$)	$0.01519^{\{1\}}$	$0.01592^{\{3\}}$	$0.01639^{\{8\}}$	$0.016^{\{5.5\}}$	$0.016^{\{5.5\}}$	$0.01797^{\{11\}}$	$0.01552^{\{2\}}$	$0.01598^{\{4\}}$	$0.01778^{\{10\}}$	$0.01897^{\{13\}}$	$0.01844^{\{12\}}$	$0.02657^{\{16\}}$	$0.01621^{\{7\}}$	$0.02105^{\{14\}}$	$0.01694^{\{9\}}$	$0.02211^{\{15\}}$
	D_{abs}	$0.01071^{\{1\}}$	$0.01121^{\{3\}}$	$0.01156^{\{8\}}$	$0.01126^{\{4\}}$	$0.01129^{\{6\}}$	$0.01257^{\{11\}}$	$0.01095^{\{2\}}$	$0.01127^{\{5\}}$	$0.01254^{\{10\}}$	$0.01338^{\{13\}}$	$0.01298^{\{12\}}$	$0.01888^{\{16\}}$	$0.01144^{\{7\}}$	$0.01478^{\{14\}}$	$0.01193^{\{9\}}$	$0.01555^{\{15\}}$
	D_{max}	$0.01538^{\{1\}}$	$0.01613^{\{3\}}$	$0.0166^{\{8\}}$	$0.0162^{\{5\}}$	$0.01623^{\{6\}}$	$0.01811^{\{11\}}$	$0.01571^{\{2\}}$	$0.01619^{\{4\}}$	$0.01804^{\{10\}}$	$0.01921^{\{13\}}$	$0.01864^{\{12\}}$	$0.02722^{\{16\}}$	$0.01641^{\{7\}}$	$0.02126^{\{14\}}$	$0.01714^{\{9\}}$	$0.02237^{\{15\}}$
	ASAE	$0.00715^{\{1\}}$	$0.00738^{\{3\}}$	$0.00756^{\{8\}}$	$0.00714^{\{5\}}$	$0.00745^{\{6\}}$	$0.0079^{\{11\}}$	$0.00713^{\{2\}}$	$0.00729^{\{4\}}$	$0.00829^{\{10\}}$	$0.00879^{\{13\}}$	$0.00815^{\{12\}}$	$0.01169^{\{16\}}$	$0.00776^{\{7\}}$	$0.00947^{\{14\}}$	$0.00794^{\{9\}}$	$0.00942^{\{15\}}$
	$\sum Ranks$	$8^{\{1\}}$	$20^{\{3\}}$	$46.5^{\{8\}}$	$26.5^{\{5\}}$	$34.5^{\{6\}}$	$64^{\{11\}}$	$11^{\{2\}}$	$26^{\{4\}}$	$62^{\{10\}}$	$78^{\{13\}}$	$71^{\{12\}}$	$96^{\{16\}}$	$43.5^{\{7\}}$	$85^{\{14\}}$	$55^{\{9\}}$	$89^{\{15\}}$

Table 7. Partial and overall ranks for all estimation methods of our proposed model.

Parameter	n	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSD	MSSLD	MSLND
$\sigma = 0.4$	15	2.0	4.0	6.0	3.0	9.0	11.0	1.0	5.0	10.0	13.0	12.0	16.0	7.0	14.0	8.0	15.0
	45	1.0	4.0	9.0	3.0	5.0	11.0	2.0	6.0	10.0	13.0	12.0	16.0	7.0	14.0	8.0	15.0
	90	1.0	2.0	6.0	3.0	7.0	11.0	4.0	5.0	10.0	13.0	12.0	16.0	9.0	14.0	8.0	15.0
	140	2.0	3.0	7.0	1.0	5.0	11.0	4.0	6.0	10.0	13.0	12.0	16.0	9.0	14.0	8.0	15.0
	200	4.0	5.0	9.0	1.0	6.0	8.0	2.0	3.0	11.0	13.0	12.0	16.0	7.0	15.0	10.0	14.0
	250	1.0	3.0	8.0	4.0	5.0	12.0	2.0	6.0	9.0	13.0	11.0	16.0	7.0	14.0	10.0	15.0
	300	3.0	6.0	5.0	1.0	8.0	10.0	4.0	2.0	9.0	12.5	11.0	16.0	7.0	14.0	12.5	15.0
	400	3.0	4.0	8.0	1.0	5.0	10.0	2.0	6.0	11.0	15.0	13.0	16.0	7.0	14.0	9.0	12.0
$\sigma = 0.8$	15	2.0	3.5	7.0	3.5	9.0	12.0	1.0	5.0	11.0	13.0	10.0	16.0	6.0	15.0	8.0	14.0
	45	1.0	2.0	5.0	3.0	4.0	10.5	7.0	6.0	12.0	13.0	10.5	16.0	8.0	15.0	9.0	14.0
	90	3.0	1.0	7.0	8.0	4.0	11.0	2.0	5.0	10.0	13.0	12.0	16.0	6.0	14.0	9.0	15.0
	140	1.0	6.0	7.5	4.0	5.0	11.0	2.0	3.0	10.0	13.0	12.0	16.0	7.5	14.0	9.0	15.0
	200	3.0	4.0	6.0	2.0	8.0	10.0	1.0	5.0	11.0	13.0	12.0	16.0	7.0	14.0	9.0	15.0
	250	2.0	4.0	7.0	1.0	6.0	11.0	3.0	5.0	12.0	13.0	10.0	16.0	8.0	14.0	9.0	15.0
	300	1.0	6.0	5.0	4.0	7.0	12.0	3.0	2.0	11.0	13.0	9.0	16.0	8.0	15.0	10.0	14.0
	400	4.5	2.0	7.0	3.0	4.5	10.5	1.0	6.0	12.0	13.0	10.5	16.0	8.0	15.0	9.0	14.0
$\sigma = 1.5$	15	3.0	2.0	4.0	5.0	7.0	11.0	1.0	10.0	12.0	13.0	8.0	16.0	9.0	15.0	6.0	14.0
	45	1.0	2.0	5.0	3.0	7.0	10.0	4.0	6.0	12.0	13.0	11.0	16.0	9.0	15.0	8.0	14.0
	90	1.0	4.0	5.0	2.0	6.0	11.0	3.0	7.0	8.0	13.0	10.0	16.0	9.0	15.0	12.0	14.0
	140	1.0	4.0	5.0	3.0	8.0	10.0	2.0	6.0	11.0	13.0	12.0	16.0	7.0	14.0	9.0	15.0
	200	1.0	3.0	6.0	4.0	7.0	12.0	5.0	2.0	10.0	13.0	11.0	16.0	9.0	14.0	8.0	15.0
	250	1.0	3.0	4.0	6.0	7.0	12.0	5.0	2.0	11.0	13.0	10.0	16.0	8.0	15.0	9.0	14.0
	300	1.0	4.0	6.0	3.0	7.5	12.0	2.0	5.0	11.0	13.0	9.0	16.0	7.5	15.0	10.0	14.0
	400	2.0	5.0	7.0	1.0	3.0	10.0	4.0	6.0	11.0	13.0	12.0	16.0	8.0	15.0	9.0	14.0
$\sigma = 2.0$	15	2.0	4.0	7.0	5.0	9.0	11.0	1.0	3.0	10.0	13.0	12.0	16.0	6.0	15.0	8.0	14.0
	45	1.0	3.5	7.0	5.0	6.0	12.0	3.5	2.0	10.0	13.0	11.0	16.0	9.0	15.0	8.0	14.0
	90	1.0	2.0	8.0	3.0	6.0	10.0	5.0	4.0	11.0	13.0	12.0	16.0	7.0	14.0	9.0	15.0
	140	1.0	4.0	8.0	3.0	6.0	9.0	2.0	5.0	11.0	13.0	12.0	16.0	7.0	14.0	10.0	15.0
	200	1.0	7.0	5.0	2.5	2.5	11.0	4.0	6.0	9.0	13.0	12.0	16.0	10.0	14.0	8.0	15.0
	250	1.0	6.0	5.0	3.0	8.0	11.0	2.0	4.0	9.0	13.0	12.0	16.0	7.0	14.0	10.0	15.0
	300	1.0	5.0	7.0	3.0	6.0	11.0	4.0	2.0	10.0	13.0	12.0	16.0	8.0	15.0	9.0	14.0
	400	1.0	7.0	8.0	2.0	5.0	12.0	4.0	3.0	9.0	13.0	11.0	16.0	6.0	15.0	10.0	14.0
$\sigma = 2.5$	15	1.0	2.0	6.0	3.0	7.0	9.0	5.0	4.0	11.0	13.0	12.0	16.0	10.0	14.0	8.0	15.0
	45	2.0	6.0	4.0	3.0	7.0	8.0	1.0	5.0	9.0	13.0	12.0	16.0	10.0	14.0	11.0	15.0
	90	1.5	7.0	5.0	3.0	9.0	12.0	4.0	1.5	10.5	13.0	10.5	16.0	8.0	15.0	6.0	14.0
	140	1.0	4.0	7.0	3.0	6.0	8.0	2.0	5.0	11.0	13.0	10.0	16.0	9.0	14.0	12.0	15.0
	200	1.0	6.0	5.0	3.0	8.0	12.0	4.0	2.0	9.0	13.0	11.0	16.0	7.0	14.0	10.0	15.0
	250	1.0	4.0	6.0	2.0	7.0	11.0	3.0	5.0	12.0	13.0	8.0	16.0	10.0	15.0	9.0	14.0
	300	1.0	5.5	8.0	2.0	7.0	11.0	4.0	3.0	9.0	13.0	12.0	16.0	5.5	14.0	10.0	15.0
	400	3.0	2.0	6.0	1.0	7.0	10.0	4.0	5.0	11.0	13.0	12.0	16.0	9.0	15.0	8.0	14.0
$\sigma = 4.0$	15	2.0	1.0	4.0	6.0	5.0	10.0	7.0	8.0	12.0	13.0	11.0	16.0	3.0	14.0	9.0	15.0
	45	1.0	3.0	7.0	4.0	9.0	12.0	2.0	5.0	8.0	13.0	11.0	16.0	6.0	14.0	10.0	15.0
	90	1.0	3.0	6.0	5.0	7.0	12.0	2.0	4.0	8.0	13.0	9.5	16.0	9.5	14.0	11.0	15.0
	140	1.0	5.0	9.0	3.0	7.0	12.0	2.0	4.0	11.0	13.0	10.0	16.0	6.0	15.0	8.0	14.0
	200	2.0	4.0	7.0	1.0	6.0	10.0	3.0	5.0	11.0	13.0	12.0	16.0	8.0	14.0	9.0	15.0
	250	1.0	5.0	7.0	3.0	6.0	11.0	2.0	4.0	10.0	13.0	12.0	16.0	8.0	14.0	9.0	15.0
	300	4.0	2.0	8.0	5.0	7.0	12.0	3.0	1.0	11.0	13.0	10.0	16.0	6.0	14.0	9.0	15.0
	400	1.0	3.0	8.0	5.0	6.0	11.0	2.0	4.0	10.0	13.0	12.0	16.0	7.0	14.0	9.0	15.0
$\sum$ Ranks		79.0	187.5	309.5	151.0	309.5	518.0	142.5	214.5	497.5	625.5	533.0	768.0	367.0	691.0	436.5	698.0
Overall Rank		1.0	4.0	6.5	3.0	6.5	11.0	2.0	5.0	10.0	13.0	12.0	16.0	8.0	14.0	9.0	15.0

6. Modeling of Electrical Engineering and Meteorology Data

In this section, we apply our newly proposed model to two real data sets: one from Electrical Engineering and the other from Meteorology.

The first data set, obtained from [22], represents the failure times of electrical insulation subjected to continuously increasing voltage stress. The observations are as follows:

21.8	70.7	24.4	138.6	151.9	12.3	95.5	98.1	43.2	28.6	46.9

The second data set consists of thirty observations for the rainfall (in inches) in March in Minneapolis/St. Paul [23]. The values are as follows:

0.77	1.74	0.81	1.20	1.95	1.20	0.47	1.43	3.37	2.20
3.00	3.09	1.51	2.10	0.52	1.62	1.31	0.32	0.59	0.81
2.81	1.87	1.18	1.35	4.75	2.48	0.96	1.89	0.90	2.05

Before applying our model to the two real-life datasets, we conducted an exploratory data analysis to determine if they align with the requirements of our newly proposed model, which exhibits positive skewness. The outcomes of this analysis are presented in Tables 8 and 9, while the visual representations are shown in Figures 7 and 8. The analysis reveals that both datasets exhibit positive skewness. Specifically, the electrical engineering dataset shows moderate positive skewness with a skewness value of 0.604, whereas the meteorological dataset demonstrates high positive skewness with a skewness value of 1.14. The visual representations of the data analysis further support the findings of the descriptive statistics.

Table 8. Descriptive Statistics of the Electrical Engineering dataset

n	Min	Max	Q1	Median	Q3	Mean	Kurtosis	Skewness
11	12.3	151.90	24.4	46.9	98.1	66.5	2.007	0.604

Table 9. Descriptive Statistics of the Meteorology dataset

n	Min	Max	Q1	Median	Q3	Mean	Kurtosis	Skewness
30	0.320	4.75	0.878	1.470	2.125	1.675	1.67	1.14

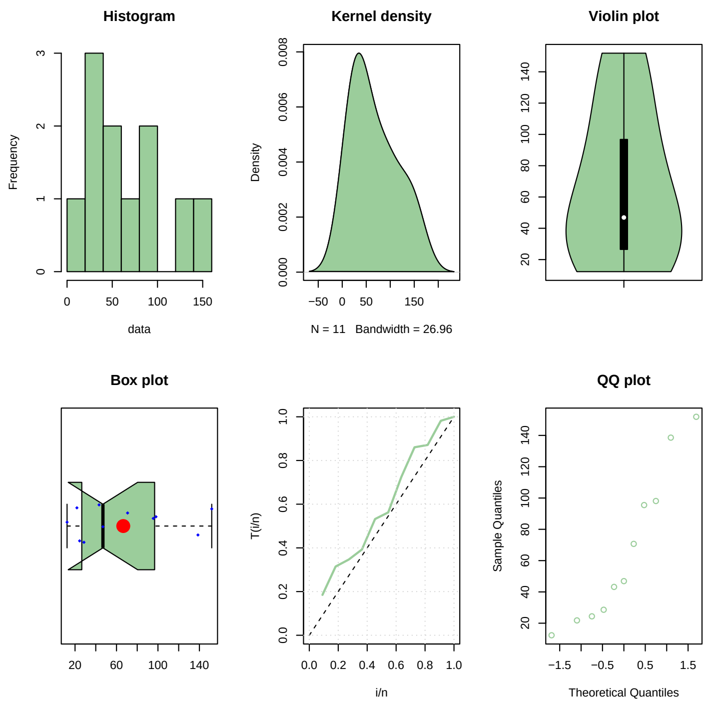

FIGURE 7. Data Analysis Visualization of Electrical Engineering Data

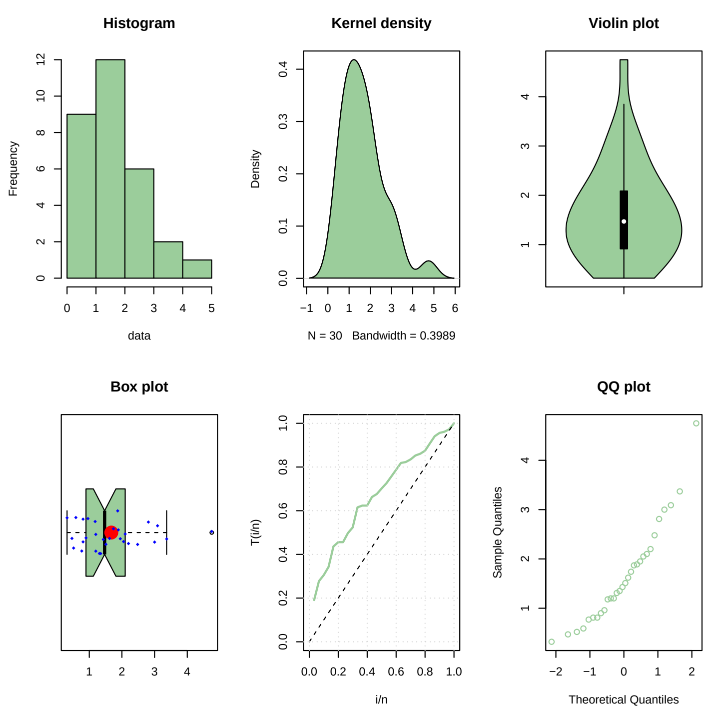

FIGURE 8. Data Analysis Visualization of Electrical Engineering Data

The goodness of fit of the newly proposed model is compared to five extensions of the Rayleigh distribution found in the literature. The PDFs and references for these extensions are shown in Table 10.

TABLE 10. Competing Models

Sn	Model	PDF	Reference
1	Rayleigh Distribution (R)	$f(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x}{2\sigma^2}\right); x, \sigma > 0$	[11]
2	Inverse Rayleigh (IR)	$f(x) = \frac{2\sigma^2}{x^3} e^{-(\sigma/x)^2}; x, \sigma > 0$	[24]
3	Modified Inverse Rayleigh (MIR)	$f(x) = \left(\alpha + \frac{2\theta}{x}\right)\left(\frac{1}{x}\right)^2 \exp\left\{-\frac{\alpha}{x} - \theta\left(\frac{1}{x}\right)^2\right\}; x, \alpha, \theta > 0$	[25]
4	Half Logistic Inverse Rayleigh (HLIR)	$f(x) = \frac{4\lambda\alpha^2 \exp\left(-\left(\frac{\alpha}{x}\right)^2\right)\left(1-\exp\left(-\left(\frac{\alpha}{x}\right)^2\right)\right)^{\lambda-1}}{x^3\left(1+\left(1-\exp\left(-\left(\frac{\alpha}{x}\right)^2\right)\right)^{\lambda}\right)^2}; x, \lambda, \alpha > 0$	[26]
5	Type II Topp Leone Generalized Inverse Rayleigh (TIITGIR)	$f(x) = 4\theta\gamma\alpha^2 x^{-3} \exp\left(-2\gamma\left(\frac{\alpha}{x}\right)^2\right)\left(1 - \exp\left(-2\gamma\left(\frac{\alpha}{x}\right)^2\right)\right)^{\theta-1}; x, \theta, \gamma, \lambda > 0$	[27]

To determine the best fit among the models, we utilized the following metrics: Negative Log Likelihood (NLL), Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Corrected Akaike Information Criterion (CAIC), Hannan-Quinn Information Criterion (HQIC), Kolmogorov-Smirnov test (KS) with the p-value, Anderson-Darling test (AD) with the p-value, and Cramér-von Mises test (CVM) with the p-value. The outcomes of the goodness-of-fit tests for the two datasets are presented in Tables 11 and 12. The CR model provides a superior fit among the comparative models, as evidenced by its lowest NLL, AIC, BIC, CAIC, HQIC, KS, AD, and CVM values. Additionally, the p-values for the KS, AD, and CVM tests are the highest among the competing models, further supporting the superiority of the proposed model over the competing models. The CDF, SF, and PP plots of the CR model for the two datasets are shown in Figures 9 and 10.

TABLE 11. Model Fit Metrics for the Electrical Engineering Data

Model	Estimate	NLL	AIC	BIC	CAIC	HQIC	KS (p-value)	AD (p-value)	CVM (p-value)
CR	$\hat{\sigma}$=4.9281	5.2867	12.5734	12.9713	13.0178	12.3226	0.1455 (0.9484)	0.2415 (0.9748)	0.0367 (0.9566)
R	$\hat{\sigma}$=5.7688	6.4403	14.8808	15.2787	15.3252	14.6300	0.1884 (0.7647)	0.4825 (0.7611)	0.0690 (0.7680)
IR	$\hat{\sigma}$=28.9625	58.6603	119.3207	119.7186	119.7651	119.0699	0.3001 (0.2251)	1.7179 (0.1329)	0.2457 (0.1945)
MIR	$\hat{\alpha}$=25.6165 $\hat{\theta}$=260.043	57.1307	118.2616	119.0574	119.7616	117.7600	0.1647 (0.8806)	0.3211 (0.9201)	0.0410 (0.9362)
HLIR	$\hat{\alpha}$=21.3486 $\hat{\lambda}$=0.7126	56.6861	117.3723	118.1681	118.8723	116.8706	0.1556 (0.9164)	0.33317 (0.9096)	0.0487 (0.8934)
TIITGIR	$\hat{\alpha}$=7.3656 $\hat{\gamma}$=5.0570 $\hat{\theta}$=0.5548	57.178	120.3574	121.5511	123.786	119.6050	0.1698 (0.8578)	0.4302 (0.8151)	0.0662 (0.7853)

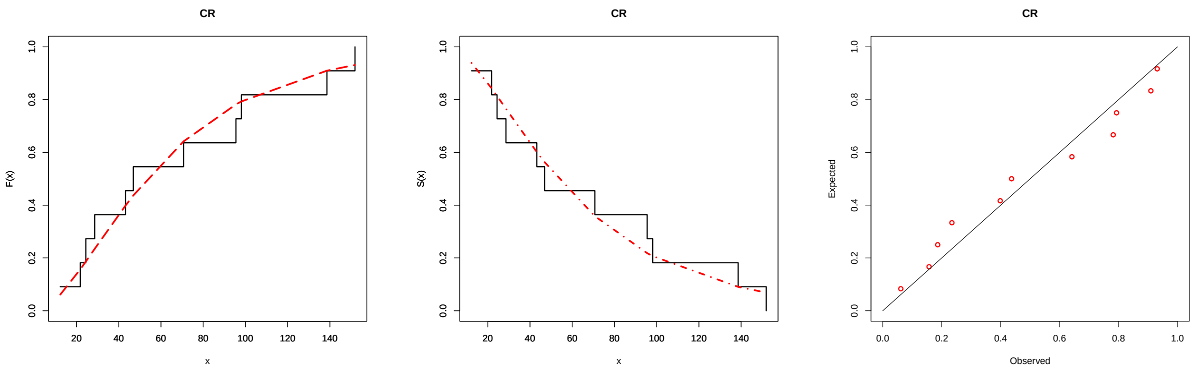

FIGURE 9. CDF, SF, and PP plots showing the model fit to the Electrical Engineering data

TABLE 12. Model Fit Metrics for the Meteorological Data

Model	Estimate	NLL	AIC	BIC	CAIC	HQIC	KS (p-value)	AD (p-value)	CVM (p-value)
CR	$\hat{\sigma}$=0.7919	8.9823	19.9646	21.9646	21.3659	20.1076	0.1246 (0.7401)	0.5718 (0.6736)	0.0844 (0.6701)
R	$\hat{\sigma}$=0.9151	14.5587	31.1174	32.5186	31.2603	31.5657	0.2352 (0.0723)	2.5142 (0.0491)	0.4540 (0.0515)
IR	$\hat{\sigma}$=0.9267	44.1365	90.2730	91.6742	90.4159	90.7213	0.2397 (0.0636)	2.1829 (0.0735)	0.4309 (0.0593)
MIR	$\hat{\theta}$=0.3595 $\hat{\lambda}$=0.5882	43.6299	91.2598	94.0622	91.7043	92.1564	0.1833 (0.2658)	1.0976 (0.3094)	0.1896 (0.2896)
HLIR	$\hat{\alpha}$=0.7663 $\hat{\lambda}$=0.9486	41.3120	86.6241	89.4264	87.0685	87.5205	0.1667 (0.3745)	0.8004 (0.4796)	0.1461 (0.4034)
TIITGIR	$\hat{\alpha}$=0.5503 $\hat{\gamma}$=1.1336 $\hat{\theta}$=0.7315	43.2011	92.4023	96.6059	93.3254	93.7471	0.1984 (0.1882)	1.2099 (0.2635)	0.2278 (0.2198)

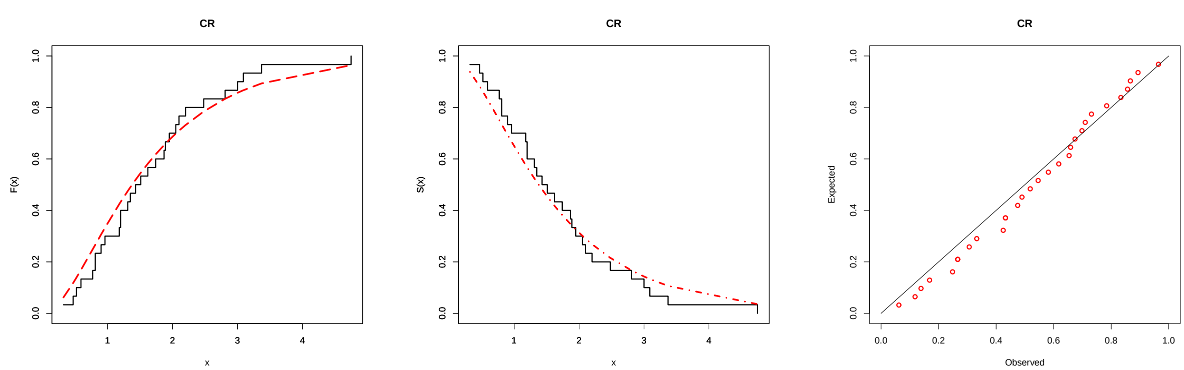

FIGURE 10. CDF, SF, and PP plots showing the model fit to the Meteorological data

7. Conclusion

In this study, a novel extension of the Rayleigh distribution, termed the Cosine Rayleigh (CR) distribution, was introduced to enhance the flexibility of the classical Rayleigh model for modeling real-life datasets. Several statistical properties of the CR distribution were derived, including the survival function, hazard function, moments, moment-generating function, entropy, and order statistics. The model parameter was estimated using 16 different methods, and a simulation study was conducted to assess the consistency of the estimators. The results indicated that all 16 methods produced consistent estimates. To demonstrate the practical utility of the proposed model, it was applied to two real-life datasets from engineering and meteorology. Comparisons with competing models showed that the CR distribution outperformed the existing alternatives.

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References

[1] R.A.R. Bantan, F. Jamal, C. Chesneau, M. Elgarhy, Truncated Inverted Kumaraswamy Generated Family of Distributions with Applications, Entropy 21 (2019), 1089. https://doi.org/10.3390/e21111089.

[2] Z.M. Nofal, E. Altun, A.Z. Afify, M. Ahsanullah, The Generalized Kumaraswamy-G Family of Distributions, J. Stat. Theory Appl. 18 (2019), 329. https://doi.org/10.2991/jsta.d.191030.001.

[3] M.A. Khaleel, P.E. Oguntunde, J.N.A. Abbasi, N.A. Ibrahim, M.H. AbuJarad, The Marshall-Olkin Topp Leone-G Family of Distributions: a Family for Generalizing Probability Models, Sci. Afr. 8 (2020), e00470. https://doi.org/10.1016/j.sciaf.2020.e00470.

[4] R.A.R. Bantan, F. Jamal, C. Chesneau, M. Elgarhy, Type Ii Power Topp-Leone Generated Family of Distributions with Statistical Inference and Applications, Symmetry 12 (2020), 75. https://doi.org/10.3390/sym12010075.

[5] A. Ahmad, N. Alsadat, M.N. Atchadé, S. Qurat ul Ain, A.M. Gemeay, M.A. Meraou, E.M. Almetwally, M.M. Hossain, E. Hussam, New Hyperbolic Sine-Generator with an Example of Rayleigh Distribution: Simulation and Data Analysis in Industry, Alex. Eng. J. 73 (2023), 415–426. https://doi.org/10.1016/j.aej.2023.04.048.

[6] R.M. Mandouh, M.R. Mahmoud, R.E. Abdelatty, A New (T-X^{θ}) Family of Distributions: Properties, Discretization and Estimation with Applications, Sci. Rep. 14 (2024), 1613. https://doi.org/10.1038/s41598-023-49425-2.

[7] A.F. Hashem, M.A. Abdelkawy, A.H. Muse, H.M. Yousof, A Novel Generalized Weibull Poisson G Class of Continuous Probabilistic Distributions with Some Copulas, Properties and Applications to Real-Life Datasets, Sci. Rep. 14 (2024), 1741. https://doi.org/10.1038/s41598-023-49873-w.

[8] M.N. Atchadé, M.A. N'bouké, A.M. Djibril, A.A. Mutairi, M.S. Mustafa, E. Hussam, H. Alsuhabi, S.G. Nassr, A New Topp-Leone Kumaraswamy Marshall-Olkin Generated Family of Distributions with Applications, Heliyon 10 (2024), e24001. https://doi.org/10.1016/j.heliyon.2024.e24001.

[9] S.O. Bashiru, A.M. Isa, A.A. Khalaf, M.A. Khaleel, K.C. Arum, C.L. Anioke, A Hybrid Cosine Inverse Lomax-G Family of Distributions with Applications in Medical and Engineering Data, Niger. J. Technol. Dev. 22 (2025), 260–277.

[10] L. Souza, New Trigonometric Classes of Probabilistic Distributions, Ph.D. Thesis, Universidade Federal Rural de Pernambuco, Recife, Brazil, 2015.

[11] L. Rayleigh, On the Resultant of a Large Number of Vibrations of the Same Pitch and of Arbitrary Phase, Lond. Edinb. Dublin Philos. Mag. J. Sci. 10 (1880), 73–78. https://doi.org/10.1080/14786448008626893.

[12] M. Siddiqui, Some Problems Connected with Rayleigh Distributions, J. Res. Natl. Bur. Stand. Sect. D: Radio Propag. 66D (1962), 167–174. https://doi.org/10.6028/jres.066d.020.

[13] K. Hirano, Rayleigh Distribution, in: S. Kotz, N.L. Johnson, C.B. Read (Eds.), Encyclopedia of Statistical Sciences, Wiley, New York, pp. 647–649, 1986.

[14] H. Howlader, A. Hossain, On Bayesian Estimation and Prediction From Rayleigh Based on Type II Censored Data, Commun. Stat. - Theory Methods 24 (1995), 2251–2259. https://doi.org/10.1080/03610929508831614.

[15] S.O. Bashiru, O.Y. Halid, On the Properties and Applications of Topp-Leone Gompertz Inverse Rayleigh Distribution, Reliab. Theory Appl. 18 (2023), 1032–1045. https://doi.org/10.24412/1932-2321-2023-476-1032-1045.

[16] S.O. Bashiru, I.I. Itopa, A.M. Isa, On the Properties of Generalized Rayleigh Distribution With Applications, Reliab. Theory Appl. 18 (2023), 374–386. https://doi.org/10.24412/1932-2321-2023-374-374-386.

[17] S.O. Bashiru, A.A. Khalaf, A.M. Isa, Topp-Leone Exponentiated Gompertz Inverse Rayleigh Distribution: Properties and Applications, Reliab. Theory Appl. 19 (2024), 59–77. https://doi.org/10.24412/1932-2321-2024-379-59-77.

[18] H.O. Yusuf, S.O. Bashiru, A Classical and Bayesian Estimation Techniques for Gompertz Inverse Rayleigh Distribution: Properties and Application, Pak. J. Stat. 38 (2022), 49–76.

[19] K.H. Habib, M.A. Khaleel, H. Al-Mofleh, P.E. Oguntunde, S.J. Adeyeye, Parameters Estimation for the [0, 1] Truncated Nadarajah Haghighi Rayleigh Distribution, Sci. Afr. 23 (2024), e02105. https://doi.org/10.1016/j.sciaf.2024.e02105.

[20] H.M. Almongy, E.M. Almetwally, H.M. Aljohani, A.S. Alghamdi, E. Hafez, A New Extended Rayleigh Distribution with Applications of COVID-19 Data, Results Phys. 23 (2021), 104012. https://doi.org/10.1016/j.rinp.2021.104012.

[21] M.A. Khaleel, A.M. Abdulwahab, A.M. Gaftan, M.K. Abdal-hammed, A New [0, 1] Truncated Inverse Weibull Rayleigh Distribution: Properties with Application to COVID-19, Int. J. Nonlinear Anal. Appl. 13 (2022), 2933–2946. https://doi.org/10.22075/ijnaa.2022.6026.

[22] M. Yahya, M. Shaaban, Estimation of Stress-Strength Reliability From Exponentiated Inverse Rayleigh Distribution Based on Neoteric Ranked Set Sampling Approach, Pak. J. Stat. 38 (2022), 491–511.

[23] M. Ali, A. Khalil, M. Ijaz, N. Saeed, Alpha-power Exponentiated Inverse Rayleigh Distribution and Its Applications to Real and Simulated Data, PLOS ONE 16 (2021), e0245253. https://doi.org/10.1371/journal.pone.0245253.

[24] M.K. Gharraph, Comparison of Estimators of Location Measures of an Inverse Rayleigh Distribution, Egypt. Stat. J. 37 (1993), 295–309.

[25] M. ShuaibKhan, Modified Inverse Rayleigh Distribution, Int. J. Comput. Appl. 87 (2014), 28–33. https://doi.org/10.5120/15270-3868.

[26] A.M. Almarashi, M.M. Badr, M. Elgarhy, F. Jamal, C. Chesneau, Statistical Inference of the Half-Logistic Inverse Rayleigh Distribution, Entropy 22 (2020), 449. https://doi.org/10.3390/e22040449.

[27] N. Yahia, H.F. Mohammed, The Type II Topp Leone Generalized Inverse Rayleigh Distribution, Int. J. Contemp. Math. Sci. 14 (2019), 113–122. https://doi.org/10.12988/ijcms.2019.912.