

Performance of Pearson, Spearman, and Kendall Correlation Coefficients Under Bivariate Count Distributions

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ABSTRACT. The study aimed to investigate the impact of count data on bivariate correlation coefficients, including Pearson's correlation coefficient, Spearman's rank correlation coefficient, and Kendall's correlation coefficient, particularly when the data follows bivariate binomial and Poisson distributions. The study considered sample sizes of 10, 20, 30, 50, and 100, with correlation levels of 0, 0.2, 0.6, and 0.8. The study compared the performance of the correlation coefficients based on their ability to control the probability of type I error and power at a significance level of 0.05. The simulation results indicated that when the data adhere to bivariate binomial and Poisson distributions, the Pearson correlation coefficient, Spearman's rank correlation coefficient, and Kendall's tau correlation coefficient effectively control the probability of a type I error. Furthermore, when assessing the power of a test at correlation levels of 0.6 and 0.8, the Pearson's correlation test statistics demonstrated the highest power across all scenarios, even with small sample sizes.

1. INTRODUCTION

The correlation coefficient is a statistical measure used to assess the relationship between two or more quantitative variables. It indicates the strength and direction of the relationship between the variables. Represented by the symbol ρ , the correlation coefficient ranges from -1 to +1. A value close to -1 or +1 signifies a strong linear relationship, while a value close to 0 indicates a weak or nonexistent linear relationship. The sign of the correlation coefficient also carries important information. A positive correlation coefficient suggests that the variables move in the same direction, whereas a negative correlation coefficient indicates that they move in opposite directions. The most well-known correlation coefficients include the Pearson correlation coefficient, Spearman's rank correlation coefficient, and Kendall's rank correlation coefficient. Each of these coefficients has specific applications depending on the nature of the data and the assumptions underlying their use.

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The Pearson correlation coefficient is widely used to measure the linear relationship between two quantitative variables, assuming that the data follows a normal distribution and is at least on an interval scale. In contrast, Spearman's rank correlation coefficient and Kendall's correlation coefficient are non-parametric statistics suitable for ordinal data and do not require the assumption of normality. Several studies have examined the effectiveness and robustness of these correlation coefficients, particularly in the presence of outliers and non-normal data. For example, Pimchan et al. (2007) [1] studied the impact of outliers on the robustness of different correlation coefficients. Their findings indicated that with no outliers and a small sample size ($n = 20$), the robust correlation coefficient was the Spearman rank correlation coefficient. However, with a larger sample size, the Pearson product-moment correlation coefficient proved to be more robust. Furthermore, when outliers made up less than 10% of the sample size, the Biweight Midcorrelation and Spearman rank correlation coefficient exhibited similar robustness properties. In instances where outliers constituted more than 10% of the sample, the Spearman rank correlation coefficient was deemed the most robust. Winter et al. (2016) [2] compared the efficiency of Pearson and Spearman correlation coefficients (denoted as r_p and r_s , respectively) across various distributions and sample sizes. Their research demonstrated that opting for r_s over r_p could lower variability in terms of standard deviations by approximately 20%. Additionally, r_p was found to be suitable for light-tailed distributions, while r_s was preferred for heavy-tailed distributions or when outliers were present. Schober et al. (2018) [3] asserted that the Pearson correlation should be employed to measure linear relationships between two normally distributed random variables, while the Spearman rank correlation is more appropriate for non-normally distributed continuous data, ordinal data, and situations involving outliers. Pruekpramool et al. (2020) [4] investigated the efficiency of Pearson, Spearman, and Kendall's correlation coefficients in non-normally distributed data. Their results indicated that Pearson's correlation coefficients typically provided the most accurate estimates with the lowest relative bias, whereas Kendall's correlation coefficients tended to have the highest relative bias. Areechart et al. (2017) [5] explored the robustness of rank correlation coefficients in the presence of outliers. The findings revealed that the Hoeffding correlation coefficient was the most robust across almost all sample sizes, particularly when the data contained outliers, and the two variables were uncorrelated. Conversely, when two variables were highly correlated, the Kendall tau correlation coefficient and the absolute bias of the Spearman and Bland correlation coefficients exhibited the smallest mean square error for all sample sizes. Moreover, the mean square error of the Spearman correlation coefficient was lowest when the correlation between the two variables was between 0.6 and 0.8 across all sample sizes. Lastly, Raadt et al. (2021) [6] examined reliability coefficients for ordinal rating scale data. Their results indicated that the differences between quadratic kappa and the Pearson and intraclass correlations increased as the level of agreement among raters rose. Furthermore, the differences between the three coefficients (Cohen's kappa, linearly

weighted kappa, and quadratically weighted kappa) were generally minimal when the differences in means and variances among raters were small.

This research examines and compares the performance of three correlation coefficients: the Pearson correlation coefficient, Spearman's rank correlation coefficient, and Kendall's rank correlation coefficient. It specifically focuses on their effectiveness when applied to bivariate binomial and Poisson data. The study evaluates the efficiency of these statistics based on two criteria: their ability to control the probability of type I error and their statistical power. The primary aim of this research is to assist researchers in choosing the appropriate correlation coefficient for their data, as selecting the wrong one can lead to incorrect conclusions [7].

2. BIVARIATE DISTRIBUTION

Bivariate distributions are statistical models that describe the probability behavior of two random variables simultaneously. These distributions are crucial for understanding the relationship between these two variables, offering insights into their joint behavior, correlation, and dependence structure. This section presents two important types of bivariate distributions: the bivariate binomial distribution and the bivariate Poisson distribution.

2.1. Bivariate Binomial distribution. The bivariate binomial distribution expands on the binomial distribution by considering two related binary outcomes. It is used to analyze the combined likelihood of two events, both of which follow a binomial distribution and are somewhat correlated. This distribution is beneficial in fields such as genetics, epidemiology, and quality control, where outcomes are frequently binary and interdependent.

Let (X, Y) is a pair of random variables with a joint distribution as follow: $P(X = 0, Y = 0) = p_{00}$, $P(X = 1, Y = 0) = p_{10}$, $P(X = 0, Y = 1) = p_{01}$, $P(X = 1, Y = 1) = p_{11}$, where $p_{00} + p_{10} + p_{01} + p_{11} = 1$. Therefore, this distribution has bivariate Bernoulli law.

Consider the distribution of the sum of n mutually independent random vectors which have the same bivariate Bernoulli distribution law: (X_1, Y_1) , (X_2, Y_2) , ..., (X_n, Y_n) . We can calculate the probabilities $P(\sum_{i=1}^n X_i = k, \sum_{i=1}^n Y_i = l)$ for all k and l satisfying $0 \leq k \leq n, 0 \leq l \leq n$. Let α, β, γ and δ are numbers of the events $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$ occur respectively, where $\alpha + \beta + \gamma + \delta = n$. Then the sum of pairs $(\sum_{i=1}^n X_i, \sum_{i=1}^n Y_i)$ is equal to $(\beta + \delta, \gamma + \delta)$. By the notion of multinomial distribution, the probability of the event described above is then given by

$$P_{\alpha\beta\gamma\delta} = \frac{n!}{\alpha!\beta!\gamma!\delta!} p_{00}^\alpha p_{10}^\beta p_{01}^\gamma p_{11}^\delta. \quad (1)$$

Then the probability $P(\sum_{i=1}^n X_i = k, \sum_{i=1}^n Y_i = l)$ is given by the sum of the probabilities $P_{\alpha\beta\gamma\delta}$, where α, β, γ and δ take all over the values of non-negative integral values satisfying the conditions

$\beta + \delta = k$, $\gamma + \delta = l$ and $\alpha + \beta + \gamma + \delta = n$, and it is given by

$$P\left(\sum_{i=1}^n X_i = k, \sum_{i=1}^n Y_i = l\right) = \sum_{\substack{\beta+\delta=k \\ \gamma+\delta=l \\ \alpha+\beta+\gamma+\delta=n}} \frac{n!}{\alpha! \beta! \gamma! \delta!} p_{00}^\alpha p_{10}^\beta p_{01}^\gamma p_{11}^\delta, \quad (2)$$

where k and l are non-negative integers satisfying $0 \leq k, l \leq n$ [8].

2.2. Bivariate Poisson distribution. The bivariate Poisson distribution expands the Poisson distribution to account for two correlated count variables. It predicts the likelihood of pairs of events occurring simultaneously, with each event following a Poisson distribution and exhibiting a certain level of correlation between them. This distribution is commonly used in fields such as insurance, traffic flow analysis, and epidemiology, where events occur randomly over time or space, and the number of occurrences is the primary focus of the study.

Consider the bivariate binomial distribution when the probabilities are expressed as $p_{10} = \lambda_{10}/n$, $p_{01} = \lambda_{01}/n$ and $p_{11} = \lambda_{11}/n$, we have the joint distribution of the sum vector of n independent vectors of bivariate Bernoulli law is given by

$$P\left(\sum_{i=1}^n X_i = k, \sum_{i=1}^n Y_i = l\right) = \sum_{\delta=\max(k+l-n, 0)}^{\min(k, l)} \frac{n!}{(n - (k + l) + \delta)!(k - \delta)!(l - \delta)!\delta!} \left(1 - \frac{\lambda_{10} + \lambda_{01} + \lambda_{11}}{n}\right)^{n - (k + l) + \delta} \left(\frac{\lambda_{10}}{n}\right)^{k - \delta} \left(\frac{\lambda_{01}}{n}\right)^{l - \delta} \left(\frac{\lambda_{11}}{n}\right)^\delta. \quad (3)$$

Consider the limiting distribution of the bivariate binomial distribution above as $n \rightarrow \infty$, the term of the right side converges to

$$\frac{\lambda_{10}^{k-\delta} \lambda_{01}^{l-\delta} \lambda_{11}^\delta}{(k - \delta)!(l - \delta)!\delta!} e^{-(\lambda_{10} + \lambda_{01} + \lambda_{11})} \quad (4)$$

[9], and the sum of the right side becomes to δ varying $0, 1, 2, \dots, \min(k, l)$ as n increases to infinity. Then the limiting distribution of the sum vector (X, Y) of the n vectors is given by the form

$$P(X = k, Y = l) = \sum_{\delta=0}^{\min(k, l)} \frac{\lambda_{10}^{k-\delta} \lambda_{01}^{l-\delta} \lambda_{11}^\delta}{(k - \delta)!(l - \delta)!\delta!} e^{-(\lambda_{10} + \lambda_{01} + \lambda_{11})} \quad (5)$$

[8].

3. CORRELATION COEFFICIENT

3.1. Pearson's correlation coefficient. The Pearson correlation coefficient is a widely used statistical measure that evaluates the linear relationship between two quantitative variables. This statistic assumes that the data is normally distributed and is measured on at least an interval scale. The correlation coefficient can be calculated by dividing the sample covariance of the two

variables by the product of their sample standard deviations or by using the specific formula for this calculation [11]:

$$r_p = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{\sqrt{\left(n \sum_{i=1}^n x_i^2 - \left[\sum_{i=1}^n x_i \right]^2 \right) \left(n \sum_{i=1}^n y_i^2 - \left[\sum_{i=1}^n y_i \right]^2 \right)}}, \quad (6)$$

where $\sum_{i=1}^n x_i$ and $\sum_{i=1}^n y_i$ are the sum of the data measured by the X variable and the Y variable, respectively, $\sum_{i=1}^n x_i y_i$ is the sum of the products of the X and Y variables, and $\sum_{i=1}^n x_i^2$ and $\sum_{i=1}^n y_i^2$ are the sum of the squares of the data measured by the variable X and the Y variable, respectively.

3.2. Spearman's rank correlation coefficient. Spearman's rank correlation coefficient, also known as Spearman's rho, is a practical non-parametric statistic. It is used to measure the relationship between ordered pairs of variables by using the rank values of the data. In simpler terms, it is a straightforward tool for measuring the relationship between two sets of data that have been ranked in order, such as when dealing with ordinal data [12] [13] [14]. However, its practicality extends to both continuous and discrete variables, as claimed by Lehman (2005) [15].

When calculating Spearman's rank correlation coefficient, it is essential to note that the underlying assumption differs from that of the Pearson correlation coefficient. The data is assumed to be from a random sample and is at least on an ordinal scale. The coefficient can be calculated using the following formula [16]:

$$r_s = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{\sqrt{\left(n \sum_{i=1}^n x_i^2 - \left[\sum_{i=1}^n x_i \right]^2 \right) \left(n \sum_{i=1}^n y_i^2 - \left[\sum_{i=1}^n y_i \right]^2 \right)}}. \quad (7)$$

The formula for calculating Spearman's rank correlation coefficient is similar to the formula for calculating Pearson's correlation coefficient. However, it is calculated using the ranks of the values of each of the two variables instead of their actual values. Additionally, there is an alternative formula to obtain Spearman's rank correlation coefficient as provided below [17] [18]:

$$r_s = 1 - \frac{6 \sum_{i=1}^n (a_i - b_i)^2}{n^3 - n}, \quad (8)$$

where $\sum_{i=1}^n (a_i - b_i)^2$ is the sum of the squares of the difference in rank in each pair, and n is the sample size.

3.3. Kendall's correlation coefficient. Kendall's correlation coefficient or Kendall's tau [9] [19] is similar to Spearman's rank correlation coefficient. It is a non-parametric statistic used to measure the relationship between two variables on an ordinal scale without assuming normality. Kendall's correlation coefficient can be calculated using the following formula [20]:

$$\tau = \frac{N_c - N_d}{(n^2 - n)/2}, \quad (9)$$

where N_c is the number of corresponding ranks, N_d is the number of inconsistent ranks, and n is the sample size.

4. CRITERIA USED TO COMPARE THE PERFORMANCE OF PROPOSED CORRELATION COEFFICIENTS

When evaluating the performance of proposed correlation coefficients, it is crucial to establish clear and objective criteria that accurately measure their effectiveness. Two key criteria commonly employed for this purpose are the empirical probability of Type I error and empirical power. These criteria help evaluate the accuracy and reliability of correlation coefficients across different statistical tests and practical applications.

4.1. Empirical probability of type I error. The empirical probability of a type I error, often referred to as the false positive rate, indicates the likelihood that a statistical test incorrectly rejects a true null hypothesis (H_0). When applied to correlation coefficients, this criterion evaluates how often the proposed coefficient indicates a significant correlation when, in reality, no such correlation exists.

In this study, the empirical probability of type I error ($\hat{\alpha}$) is calculated as the number of times the null hypothesis is rejected when it is true, divided by the total number of iterations. If any test statistic gives the empirical probability of a type I error corresponding to Bradley's criteria [22], that is, the empirical probability of a type I error is in the range [0.025, 0.075] at the 0.05 significance level, it will be concluded that the test statistic can control the probability of a type I error.

4.2. Empirical power of a test. Empirical power of a test is the probability that a statistical test correctly rejects a false null hypothesis, thereby identifying a true effect [21]. For correlation coefficients, empirical power assesses the ability of the proposed measure to detect actual correlations between variables.

In this study, the empirical power ($1 - \hat{\beta}$) is calculated from the number of times the null hypothesis is rejected when the null hypothesis is not true divided by the number of iterations. If any test statistic can control the probability of type I error and gives the highest power, it will be concluded that such test statistic is the best.

5. SIMULATION STUDY

This research is a simulation study using RStudio software. It aims to compare the efficiency of the three correlation coefficients, including Pearson's correlation coefficient (P), Spearman's rank correlation coefficient (S), and Kendall's correlation coefficient (K), when data follow bivariate binomial and Poisson distributions.

5.1. **Research scopes.** The scopes of this study were defined as follows:

1) This study covers a range of sample sizes, including small (10, 20), medium (30), and large (50 and 100).

2) The distributions of data considered in this study were the bivariate binomial distribution with different parameter combinations: $(n_1, p_1) = (n_2, p_2) = (n, p) = (10, 0.3), (10, 0.5), (10, 0.7), (20, 0.3), (20, 0.5), (20, 0.7), (30, 0.3), (30, 0.5), (30, 0.7), (50, 0.3), (50, 0.5), (50, 0.7), (100, 0.3), (100, 0.5), (100, 0.7)$, and the bivariate Poisson distribution with parameters $(\lambda_1, \lambda_2) = (5, 5), (10, 10), (18, 18)$ and $(30, 30)$.

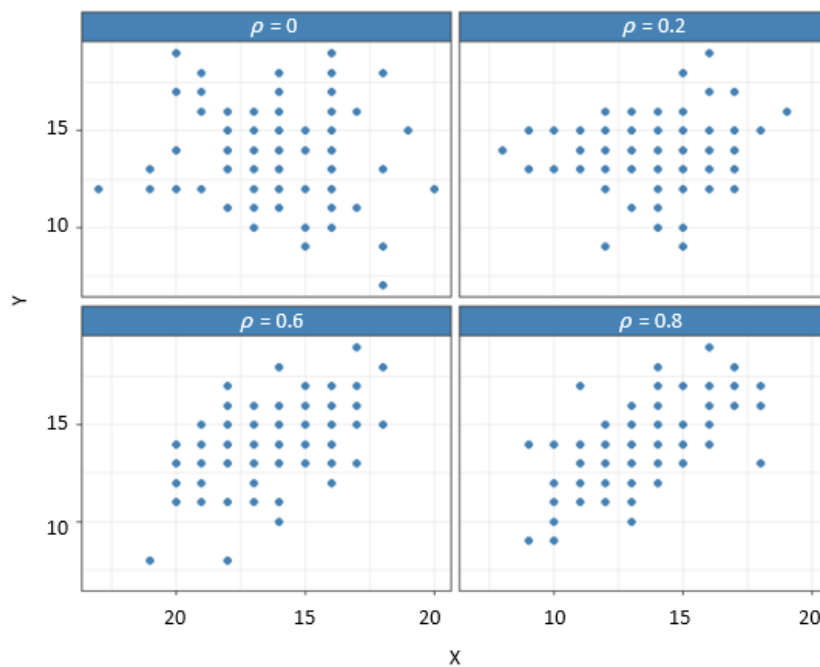


FIGURE 1. Bivariate binomial data with parameters $(n_1, p_1) = (20, 0.7)$ and $(n_2, p_2) = (20, 0.7)$, and $n = 100$

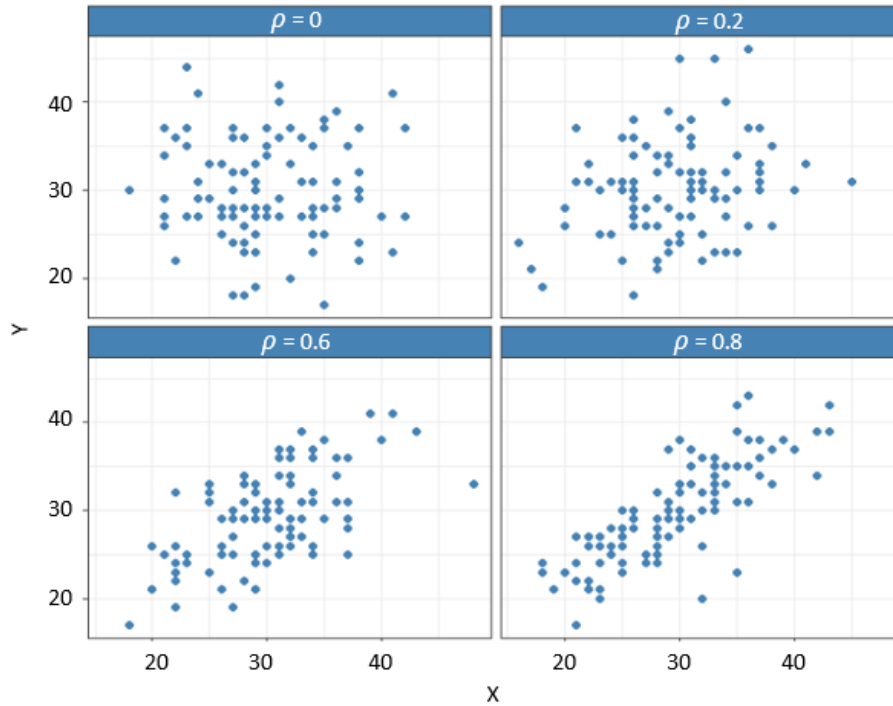


FIGURE 2. Bivariate Poisson data with parameter $(\lambda_1, \lambda_2) = (30, 30)$ and $n = 100$

3) The population correlation (ρ) were considered at 0.0, 0.2, 0.6, and 0.8 levels to encompass all relationship strengths. As depicted in Figure 1-2, ρ values of 0.0, 0.2, 0.6, and 0.8 represent no relationship, weak relationship, moderate relationship, and strong relationship between two variables, respectively.

4) The significance level (α) for the test was set at 0.05.

5) The number of iterations was 10,000 times for each situation.

5.2. Research procedures. The research procedures are divided into two studies as follows:

5.2.1. The study of the probability of type I error. Processes to study the probability of type I error are shown below.

Step 1: Simulate the data within the research scope and under the null hypothesis (H_0) that is set to be true. The hypothesis of the test is stated as follows:

$$H_0 : \rho = 0 \text{ versus } H_1 : \rho \neq 0.$$

Step 2: Calculated the test statistics based on the Pearson's correlation coefficient, Spearman's rank correlation coefficient, and Kendall's correlation coefficient.

Step 3: Consider a result of the hypothesis test whether to accept or reject the null hypothesis (H_0).

Step 4: Repeat Step 1-3 for 10,000 times.

Step 5: Count the number of rejections of the null hypothesis.

Step 6: Calculate the empirical probability of type I error.

Step 7: Compare the empirical probability of type I error with Bradley's criterion [22]. If any test statistic gives the empirical probability of type I error is in the range [0.025, 0.075] at the 0.05 significance level, it will be concluded that the test statistic can control the probability of type I error.

5.2.2. *The study of the power of a test.* Processes to study the power of a test are presented below.

Step 1: Simulate the data within the research scope and under the alternative hypothesis (H_0) that is set to be true. The hypothesis of the test is stated as follows:

$$H_0 : \rho = 0 \text{ versus } H_1 : \rho = \rho_0,$$

where $\rho_0 = 0.2, 0.6, 0.8$.

Step 2: Calculated the test statistics based on the Pearson's correlation coefficient, Spearman's rank correlation coefficient, and Kendall's correlation coefficient.

Step 3: Consider a result of the hypothesis test whether to accept or reject the null hypothesis (H_0).

Step 4: Repeat Step 1-3 for 10,000 times.

Step 5: Count the number of rejections of the null hypothesis.

Step 6: Calculate the empirical power of the test.

Step 7: Compare the empirical power of the three test statistics. If any test statistic can control the probability of type I error and gives the highest power, it will be concluded that such test statistic is the best.

6. RESULTS

The aim of this study was to compare the effectiveness of three different correlation coefficients, Pearson's, Spearman's rank, and Kendall's, when dealing with data that follows bivariate binomial and Poisson distributions. The criteria used for the comparison were the ability to control the probability of type I error and the power. The detailed research results are presented in the following subsections.

6.1. Results of the ability to control the probability of type I error. In Table 1-2 and Figure 3-4, it is shown that when analyzing data that adhere to bivariate binomial and Poisson distributions, test statistics derived from Pearson's correlation coefficient, Spearman's rank correlation coefficient, and Kendall's correlation coefficient can effectively control the probability of a type I error. According to Bradley's criterion [22], at a significance level of 0.05, the empirical probabilities of a type I error typically range from 0.025 to 0.075. Furthermore, it has been observed that the test statistic based on Kendall's correlation coefficient consistently produces the lowest probability of a type I error compared to the other methods.

TABLE 1. Empirical probabilities of type I error ($\hat{\alpha}$) for testing $H_0 : \rho = 0$ versus $H_1 : \rho \neq 0$ when data follows bivariate binomial distribution

Sample sizes	Parameter		Correlation coefficient		
	n	p	P	S	K
10	10	0.3	0.0480	0.0492	0.0447
		0.5	0.0501	0.0522	0.0467
		0.7	0.0541	0.0577	0.0512
	20	0.3	0.0515	0.0569	0.0500
		0.5	0.0514	0.0522	0.0479
		0.7	0.0447	0.0506	0.0464
	30	0.3	0.0532	0.0548	0.0492
		0.5	0.0485	0.0511	0.0471
		0.7	0.0507	0.0539	0.0465
20	10	0.3	0.0475	0.0512	0.0484
		0.5	0.0487	0.0511	0.0492
		0.7	0.0428	0.0458	0.0422
	20	0.3	0.0469	0.0496	0.0471
		0.5	0.0519	0.0538	0.0502
		0.7	0.0503	0.0512	0.0490
	30	0.3	0.0494	0.0534	0.0503
		0.5	0.0518	0.0546	0.0528
		0.7	0.0494	0.0509	0.0503
30	10	0.3	0.0512	0.0522	0.0504
		0.5	0.0490	0.0492	0.0478
		0.7	0.0521	0.0527	0.0509
	20	0.3	0.0448	0.0458	0.0446
		0.5	0.0486	0.0479	0.0478
		0.7	0.0498	0.0474	0.0461
	30	0.3	0.0494	0.0488	0.0472
		0.5	0.0527	0.0511	0.0508
		0.7	0.0503	0.0520	0.0494
50	10	0.3	0.0520	0.0484	0.0487
		0.5	0.0483	0.0514	0.0516
		0.7	0.0479	0.0479	0.0479
	20	0.3	0.0507	0.0509	0.0509
		0.5	0.0451	0.0480	0.0467
		0.7	0.0497	0.0492	0.0472
	30	0.3	0.0489	0.0493	0.0490
		0.5	0.0505	0.0505	0.0507
		0.7	0.0545	0.0517	0.0520
100	10	0.3	0.0469	0.0473	0.0487
		0.5	0.0513	0.0506	0.0505
		0.7	0.0524	0.0521	0.0518
	20	0.3	0.0483	0.0493	0.0485
		0.5	0.0474	0.0495	0.0494
		0.7	0.0464	0.0501	0.0497
	30	0.3	0.0509	0.0512	0.0507
		0.5	0.0510	0.0498	0.0492
		0.7	0.0509	0.0501	0.0501

TABLE 2. Empirical probabilities of type I error ($\hat{\alpha}$) for testing $H_0 : \rho = 0$ versus $H_1 : \rho \neq 0$ when data follows the bivariate Poisson distribution

Sample size	λ	Correlation coefficient		
		P	S	K
10	5	0.0502	0.0532	0.0479
	10	0.0464	0.0499	0.0435
	18	0.0476	0.0486	0.0423
	30	0.0511	0.0543	0.0482
20	5	0.0494	0.0504	0.0472
	10	0.0510	0.0506	0.0482
	18	0.0492	0.0492	0.0455
	30	0.0497	0.0502	0.0499
30	5	0.0458	0.0480	0.0454
	10	0.0463	0.0463	0.0451
	18	0.0537	0.0535	0.0532
	30	0.0504	0.0513	0.0516
50	5	0.0518	0.0557	0.0547
	10	0.0466	0.0492	0.0485
	18	0.0488	0.0488	0.0497
	30	0.0503	0.0513	0.0496
100	5	0.0552	0.0516	0.0520
	10	0.0473	0.0490	0.0482
	18	0.0525	0.0513	0.0512
	30	0.0506	0.0525	0.0513

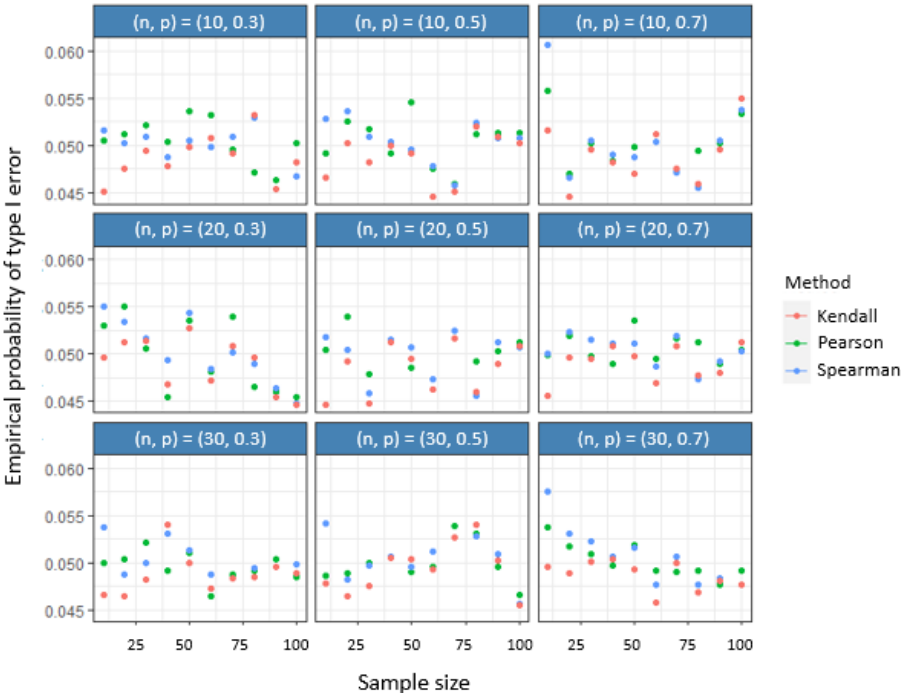


FIGURE 3. Scatterplot of sample size versus empirical probability of type I error for bivariate binomial distribution.

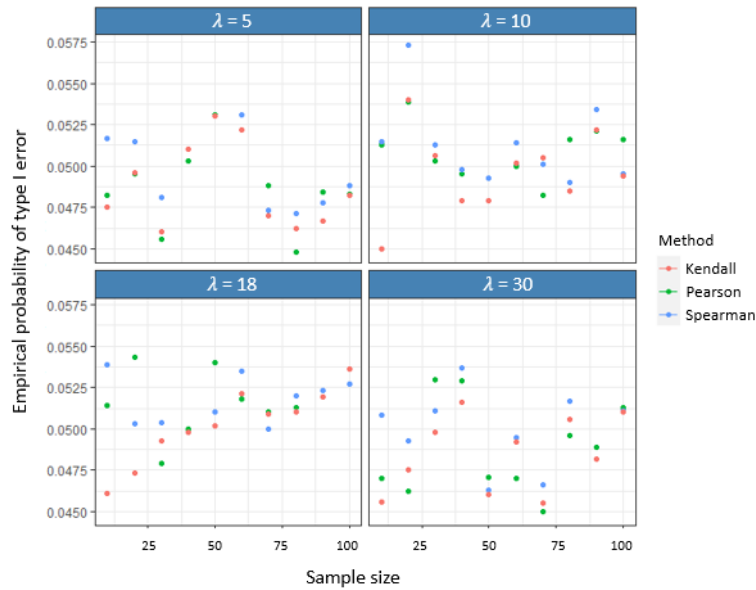


FIGURE 4. Scatterplot of sample size versus empirical probability of type I error for bivariate Poisson distribution.

6.2. Empirical power. Regarding Table 3, the results show that for data following a bivariate binomial distribution with a correlation of 0.2 ($\rho = 0.2$) and a sample size of 10, the empirical power of the test statistics based on Spearman's rank correlation coefficient and Kendall's correlation coefficient did not differ from that of the test statistics based on Pearson's correlation coefficient. However, as the sample size increased to 20, 30, 50, and 100, the test statistic based on Kendall's correlation coefficient proved to be the most effective, demonstrating the highest power (see Figure 5). Furthermore, it was observed that at correlation levels of 0.6 and 0.8 ($\rho = 0.6$ and 0.8), the test statistic based on Pearson's correlation coefficient outperformed the others in all scenarios, even with small sample sizes (see Figure 6 as an example).

For data that follows a Poisson distribution, Table 4 and Figure 7 illustrate that with a correlation of 0.2 ($\rho = 0.2$), the test statistics based on Spearman's rank correlation and Kendall's correlation coefficients showed similar performance, as indicated by their comparable empirical power. However, when the correlation level increased to 0.6 and 0.8 ($\rho = 0.6$ and 0.8), the test statistic based on Pearson's correlation coefficient consistently demonstrated the highest power across all scenarios, even with small sample sizes.

TABLE 3. Empirical power $(1 - \hat{\beta})$ for hypothesis testing $H_0 : \rho = 0$ versus $H_1 : \rho = \rho_0$ when data follows the bivariate binomial distribution

Sample size	Parameter n	p	$\rho_0 = 0.2$			$\rho_0 = 0.6$			$\rho_0 = 0.8$		
			P	S	K	P	S	K	P	S	K
10	10	0.3	0.087	0.107	0.113	0.484	0.415	0.371	0.870	0.818	0.821
		0.5	0.081	0.095	0.100	0.482	0.410	0.369	0.874	0.826	0.820
		0.7	0.085	0.105	0.110	0.486	0.406	0.362	0.871	0.819	0.821
	20	0.3	0.081	0.093	0.093	0.485	0.414	0.383	0.869	0.805	0.796
		0.5	0.085	0.093	0.089	0.483	0.415	0.382	0.871	0.799	0.790
		0.7	0.083	0.095	0.093	0.482	0.420	0.384	0.870	0.797	0.791
	30	0.3	0.086	0.096	0.092	0.491	0.423	0.395	0.872	0.801	0.791
		0.5	0.087	0.094	0.088	0.481	0.418	0.384	0.871	0.796	0.787
		0.7	0.081	0.087	0.080	0.482	0.417	0.388	0.871	0.798	0.784
20	10	0.3	0.133	0.153	0.189	0.835	0.761	0.720	0.996	0.992	0.993
		0.5	0.139	0.151	0.176	0.840	0.771	0.736	0.997	0.993	0.993
		0.7	0.130	0.152	0.184	0.831	0.765	0.725	0.996	0.991	0.993
	20	0.3	0.128	0.135	0.150	0.836	0.770	0.747	0.995	0.990	0.991
		0.5	0.132	0.140	0.151	0.837	0.772	0.753	0.996	0.993	0.993
		0.7	0.138	0.146	0.157	0.832	0.781	0.754	0.995	0.991	0.991
	30	0.3	0.132	0.137	0.141	0.829	0.765	0.749	0.995	0.989	0.990
		0.5	0.132	0.136	0.141	0.838	0.779	0.763	0.996	0.990	0.990
		0.7	0.134	0.142	0.148	0.835	0.772	0.754	0.997	0.991	0.990
30	10	0.3	0.187	0.210	0.264	0.959	0.919	0.895	1.000	1.000	1.000
		0.5	0.181	0.195	0.241	0.960	0.921	0.900	1.000	1.000	1.000
		0.7	0.181	0.204	0.259	0.956	0.918	0.896	1.000	1.000	1.000
	20	0.3	0.184	0.194	0.222	0.952	0.919	0.907	1.000	1.000	1.000
		0.5	0.191	0.199	0.220	0.958	0.926	0.914	1.000	1.000	1.000
		0.7	0.186	0.193	0.222	0.957	0.929	0.916	1.000	1.000	1.000
	30	0.3	0.185	0.187	0.204	0.955	0.920	0.912	1.000	1.000	1.000
		0.5	0.188	0.191	0.201	0.956	0.925	0.919	1.000	0.999	1.000
		0.7	0.187	0.191	0.208	0.954	0.923	0.915	1.000	1.000	1.000
50	10	0.3	0.285	0.321	0.416	0.998	0.993	0.989	1.000	1.000	1.000
		0.5	0.288	0.302	0.384	0.998	0.994	0.990	1.000	1.000	1.000
		0.7	0.288	0.320	0.413	0.997	0.993	0.989	1.000	1.000	1.000
	20	0.3	0.279	0.292	0.339	0.998	0.995	0.993	1.000	1.000	1.000
		0.5	0.290	0.296	0.331	0.997	0.993	0.991	1.000	1.000	1.000
		0.7	0.288	0.299	0.346	0.997	0.994	0.991	1.000	1.000	1.000
	30	0.3	0.297	0.304	0.333	0.998	0.995	0.994	1.000	1.000	1.000
		0.5	0.288	0.292	0.318	0.998	0.995	0.994	1.000	1.000	1.000
		0.7	0.284	0.294	0.323	0.998	0.994	0.993	1.000	1.000	1.000
100	10	0.3	0.516	0.569	0.712	1.000	1.000	1.000	1.000	1.000	1.000
		0.5	0.514	0.543	0.668	1.000	1.000	1.000	1.000	1.000	1.000
		0.7	0.522	0.570	0.713	1.000	1.000	1.000	1.000	1.000	1.000
	20	0.3	0.521	0.541	0.621	1.000	1.000	1.000	1.000	1.000	1.000
		0.5	0.523	0.538	0.600	1.000	1.000	1.000	1.000	1.000	1.000
		0.7	0.508	0.529	0.605	1.000	1.000	1.000	1.000	1.000	1.000
	30	0.3	0.524	0.536	0.584	1.000	1.000	1.000	1.000	1.000	1.000
		0.5	0.514	0.526	0.572	1.000	1.000	1.000	1.000	1.000	1.000
		0.7	0.514	0.531	0.583	1.000	1.000	1.000	1.000	1.000	1.000

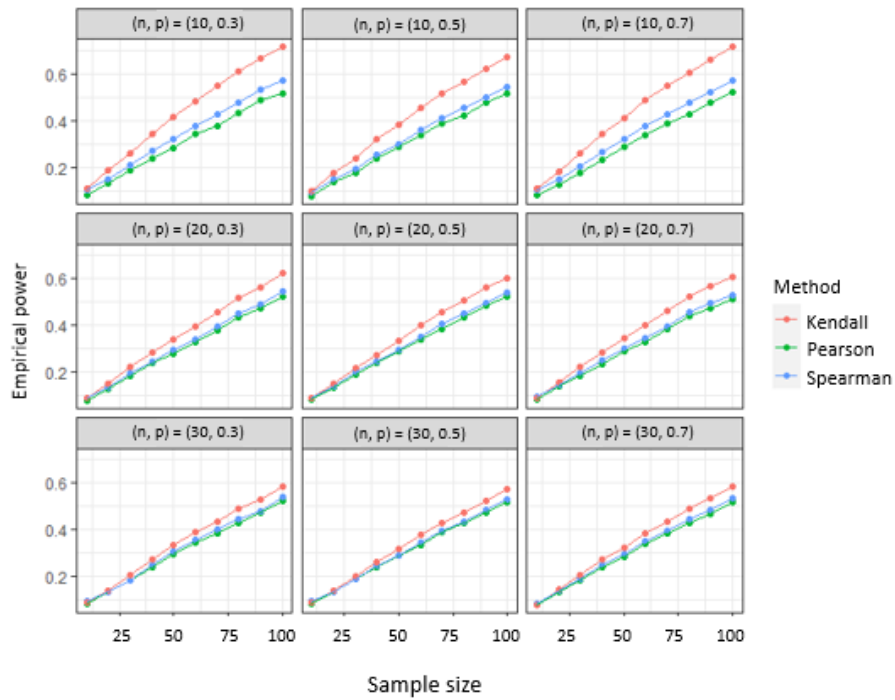


FIGURE 5. Empirical power from bivariate binomial distribution for testing $H_0: \rho = 0$ versus $H_1: \rho = 0.2$.

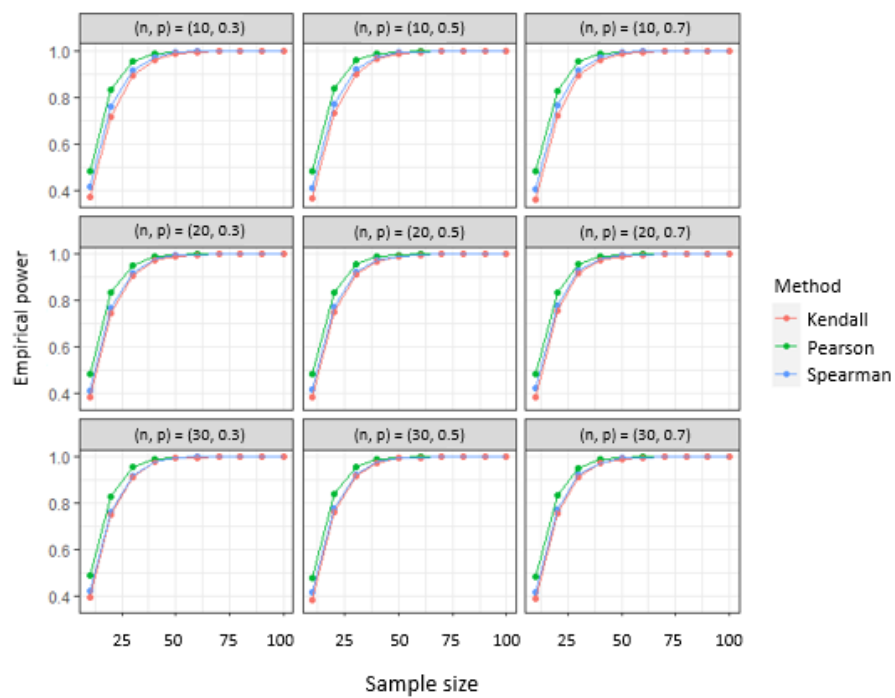


FIGURE 6. Empirical power from bivariate binomial distribution for testing $H_0: \rho = 0$ versus $H_1: \rho = 0.6$.

TABLE 4. Empirical power $(1 - \hat{\beta})$ for hypothesis testing $H_0 : \rho = 0$ versus $H_1 : \rho = \rho_0$ when data follows the bivariate Poisson distribution

Sample sizes	λ	$\rho_0 = 0.2$			$\rho_0 = 0.6$			$\rho_0 = 0.8$		
		P	S	K	P	S	K	P	S	K
10	5	0.090	0.092	0.090	0.485	0.407	0.376	0.865	0.809	0.802
	10	0.088	0.090	0.086	0.477	0.409	0.384	0.868	0.792	0.780
	18	0.093	0.094	0.091	0.483	0.412	0.389	0.865	0.784	0.773
	30	0.087	0.088	0.085	0.485	0.414	0.398	0.869	0.785	0.772
20	5	0.138	0.142	0.154	0.836	0.771	0.754	0.995	0.991	0.991
	10	0.132	0.132	0.134	0.833	0.769	0.757	0.995	0.988	0.989
	18	0.138	0.139	0.137	0.837	0.777	0.767	0.995	0.989	0.989
	30	0.140	0.129	0.129	0.832	0.770	0.757	0.996	0.989	0.989
30	5	0.186	0.193	0.216	0.958	0.926	0.916	1.000	1.000	1.000
	10	0.191	0.191	0.200	0.956	0.926	0.921	1.000	1.000	1.000
	18	0.180	0.174	0.178	0.955	0.925	0.922	1.000	1.000	1.000
	30	0.183	0.175	0.179	0.954	0.927	0.923	1.000	1.000	1.000
50	5	0.283	0.299	0.339	0.997	0.995	0.993	1.000	1.000	1.000
	10	0.286	0.290	0.309	0.996	0.993	0.993	1.000	1.000	1.000
	18	0.294	0.287	0.298	0.998	0.994	0.993	1.000	1.000	1.000
	30	0.292	0.279	0.286	0.998	0.994	0.993	1.000	1.000	1.000
100	5	0.517	0.543	0.615	1.000	1.000	1.000	1.000	1.000	1.000
	10	0.514	0.524	0.558	1.000	1.000	1.000	1.000	1.000	1.000
	18	0.520	0.516	0.539	1.000	1.000	1.000	1.000	1.000	1.000
	30	0.523	0.505	0.522	1.000	1.000	1.000	1.000	1.000	1.000

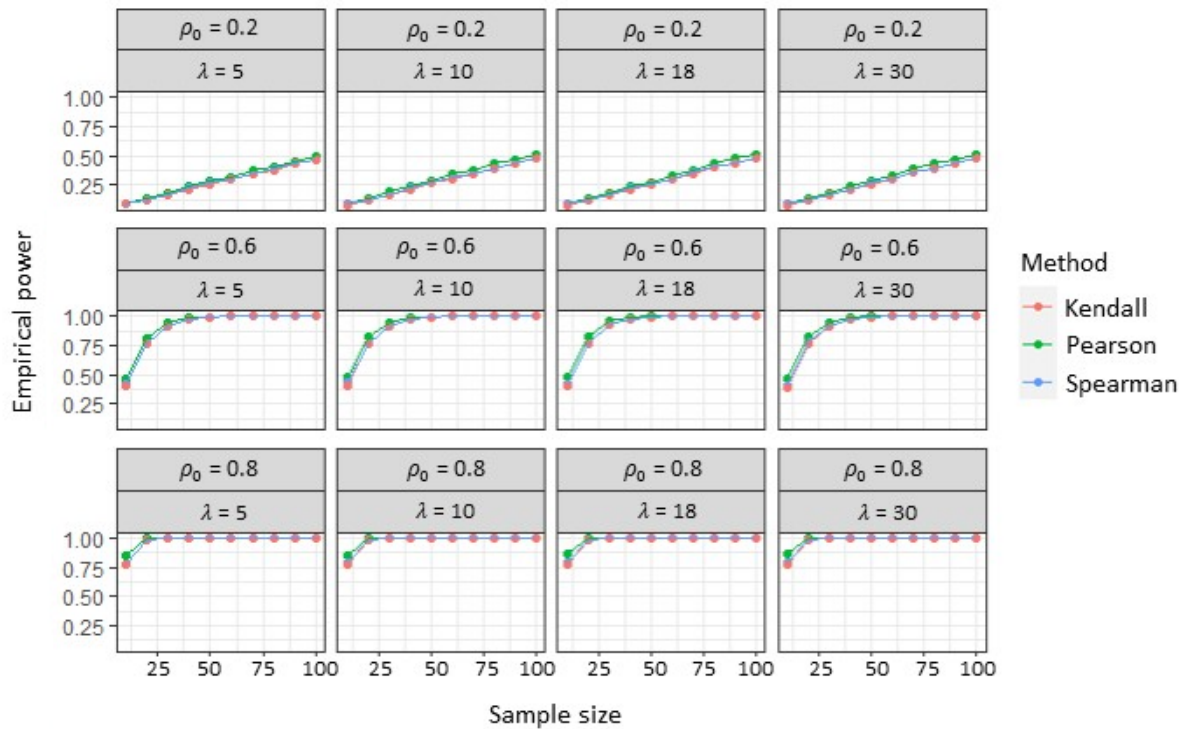


FIGURE 7. Empirical power from bivariate Poisson distribution.

7. CONCLUSION AND DISCUSSION

Table 1–2 demonstrates that when dealing with bivariate binomial and Poisson distributions, the test statistics derived from Pearson's, Spearman's rank, and Kendall's correlation coefficients effectively control the probability of Type I error in all scenarios. The empirical probability of Type I error falls within the range of [0.025, 0.075], as per Bradley's criterion [22], at a significance level of 0.05. Furthermore, it was observed that the test statistic based on Kendall's correlation coefficient consistently yielded the lowest probability of Type I error compared to the other methods across nearly all cases.

Regarding the power, The findings indicated that when the correlation levels were 0.6 and 0.8, the test statistic based on Pearson's correlation coefficient consistently yielded the highest power across all scenarios, including cases with small sample sizes. These results align with the findings of Kaewkun (1996), which suggested that the test statistic based on Spearman's rank correlation coefficient effectively controlled the probability of Type I error at both 0.05 and 0.01 significance levels for all sample sizes.

Additionally, our results support those of Pruekpramool et al. (2020) [4], indicating that Pearson's correlation coefficient generally provides the highest test power. Furthermore, the test statistic based on Pearson's correlation coefficient was found to be suitable for data with moderate to high correlation and large sample sizes. This conclusion is consistent with the results of Bonett (2000) [23], Pimchan et al. (2007) [1], and Mukaka (2012) [24], all of which demonstrated that Pearson's correlation coefficient offers the highest power when sample sizes are large; this can be attributed to the nature of larger datasets, which tend to converge to a normal distribution, a requisite for using Pearson's correlation coefficient. When the correlation between variables was high, it was observed that the correlation coefficients from the three methods demonstrated no significant difference in power.

Competing interests: The authors declare that there is no conflict of interest regarding the publication of this paper.

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