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Novel Generalized Additive Information Measure with Coding Theorems and Its Applicability

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Abstract. In this article, a novel additive information measure and a new average code-word length, along with the formulation of noiseless coding theorems for discrete channels is introduced. Additionally, the measures introduced in this study generalize several well-established measures in information theory. Empirical data is used to validate the findings through the implementation of Huffman and Shannon-Fano coding schemes. Furthermore, the key properties of the proposed information measure, including its behavior with respect to uncertainty and biasedness are investigated. Specifically, it is shown that as the measure increases, it captures greater uncertainty and reflects reduced biasedness in the distribution and vice versa and finally, the monotonicity of the proposed measure using a real-life data set is also examined.

1. Introduction

Information theory is a vast and profound mathematical discipline with diverse and impactful applications, particularly in the critical area of coding theory. As a relatively new branch of probability and statistics, it holds immense potential for communication systems. The term "information theory" encompasses the analysis of issues related to systems involving information processing, storage, and decision-making. More specifically, information theory focuses on theoretical challenges associated with transmitting information through communication channels. This involves examining measures of uncertainty and devising practical and cost-effective methods for transmission of information.

The foundation of communication theory can be traced back to Hartley's mathematical work on information transmission (1928) [17] concept of entropy. The concept of information measure, called as entropy was first introduced by Shannon in 1948 as:

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i \tag{1}$$

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Suppose $p_1, p_2, p_3 \ldots, p_n$ represent the probabilities of the code words to be transmitted, and $l_1, l_2, l_3, \ldots, l_n$ denote their corresponding lengths, which satisfy Kraft's (1949) inequality.

$$\sum_{i=1}^{n} D^{-l_i} \leq 1 \tag{2}$$

Where D denotes the size of code alphabet.

Shannon (1948) established his noiseless coding theorem for uniquely decipherable codes, stating that for any code that satisfies Kraft's [13] inequality 2, the minimum possible value of the mean code-word length is

$$L = \sum_{i=1}^{n} p_i l_i \tag{3}$$

lies between $H(P)$ and $H(P) + 1$.

Campbell (1965) [5] considered the more general exponentiated mean code word length as

$$L_\alpha(P) = \frac{\alpha}{1-\alpha} \log_D \left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} \right], \quad \alpha > 0,\ \alpha \neq 1 \tag{4}$$

and demonstrated that, subject to condition 2, the minimum value of expression 4 falls between $R_\alpha(P)$ and $R_\alpha(P) + 1$ where

$$R_\alpha(P) = \frac{1}{1-\alpha} \log_D \left[\sum_{i=1}^{n} p_i^\alpha \right], \quad \alpha > 0,\ \alpha \neq 1 \tag{5}$$

is Renyi's (1961) entropy [16].

Numerous researchers have proposed distinct generalized entropy measures and in relation to these entropy measures have developed numerous generalized code-word lengths along with coding theorems subject to uniquely decipherable. As discussed in Nath's [15] articles on inaccuracy and coding theory. Gurdial and Pessoa [7] obtained a noiseless coding theorem by obtaining the minimum value of another useful average code-word length. Longo [14] derived minimum value of useful mean code-word length in terms of weighted entropy given by Belis and Guiasu [1]. Gurdial and Pessoa [7] also developed the theorem by finding the lower bounds for useful average code-word length of order α; also various authors like Hooda and Bhaker [8], Khan, Bhat and Peerzada [12], Bhat and Baig [2], [3] have also established various generalized coding theorems. In this article, we introduce a new additive informative measure and new average code-word length and exploring these measures from various perspectives.

2. Novel Additive Information Measure

We introduce a novel generalized additive information measure as:

$$H_\beta^\alpha(P) = \frac{\beta}{1-\alpha} \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}} \right], \quad 0 < \alpha < 1,\ \beta > 0,\ p_i \geq 0 \tag{6}$$

$\forall i = 1, 2, \ldots, n,\ \sum_{i=1}^{n} p_i = 1$

2.1. **Particular cases.**

- For $\beta = 1$, 2 reduces to new additive information measure of order α [2] i.e.,

$$\frac{1}{1-\alpha}\sum_{i=1}^{n} p_i^{\alpha}, \quad 0 < \alpha < 1$$

- For $\beta = 1$ and $\alpha \to 1$, 2 reduces to Shannon's [17] entropy, i.e.,

$$H_{\beta}^{\alpha}(P) = -\sum_{i=1}^{n} p_i \log p_i$$

- For $\alpha \to 1$, and $p_i = \frac{1}{n}, \forall i = 1, 2, \ldots, n$, then 2 reduces to maximum entropy i.e.,

$$H\left(\frac{1}{n}\right) = \log n$$

3. Coding Theorems

Additionally, we introduce a novel generalized measure for the average code-word length expressed as:

$$L_{\beta}^{\alpha}(P) = \frac{\beta}{1-\alpha}\left[\sum_{i=1}^{n} p_i D^{-l_i \frac{\alpha-1}{\alpha}}\right]^{\alpha}, \quad 0 < \alpha < 1,\ \beta > 0 \tag{7}$$

$$p_i \geq 0\ \forall i = 1, 2, \ldots, n,\ \sum_{i=1}^{n} p_i = 1$$

Where D represents code alphabet.

3.1. **Particular Cases.**

- For $\beta = 1$, 7 reduces to additive informative measure [2] as

$$L^{\alpha}(P) = \frac{1}{1-\alpha}\left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha}$$

- For $\beta = 1$ and $\alpha \to 1$ and $l_1, l_2, \ldots, l_n = 1$, then 7 reduces to 1, i.e., $L = 1$.
  We determine the lower and upper bounds of the new generalized average code-word length, as defined in equation 7, with respect to the new generalized information measure defined in equation 2, subject to the specified condition.

$$\sum_{i=1}^{n} D^{-l_i} \leq 1 \tag{8}$$

  This is the inequality formulated by Kraft in 1949.

Theorem 3.1. *If the code-word lengths fulfill the Kraft's inequality 2 for all integers such that $D > 1$, the proposed generalized average code-word length in equation 7 will also satisfy the inequality*

$$L_{\beta}^{\alpha}(P) \geq H_{\beta}^{\alpha}(P),\ 0 < \alpha < 1,\ \beta > 0 \tag{9}$$

where the inequality is satisfied if and only if

$$l_i = -\log_D \left[\frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}} \right] \tag{10}$$

Proof. : It is well established that $a_i, b_i > 0, i = 1, 2, 3, \dots, n$ and $\frac{1}{\theta} + \frac{1}{\tau} = 1$, $\theta < 1(\neq 0)$, $\tau < 1(\neq 0), \theta < 0$, then the Holder's Inequality

$$\left(\sum_{i=1}^n a_i^\theta \right)^{\frac{1}{\theta}} \left(\sum_{i=1}^n b_i^\tau \right)^{\frac{1}{\tau}} \leq \sum_{i=1}^n x_i y_i \tag{11}$$

holds, and equality is satisfied in 11 if and only if $\exists$ a positive constant k viz.,

$$a_i^\theta = k b_i^\tau$$

We can proceed with the following substitution,

$$a_i = p_i^{\frac{\alpha}{\alpha-1}} D^{-l_i} \qquad b_i = p_i^{\frac{\alpha}{\beta(\alpha-1)}}$$

$$\theta = \frac{\alpha - 1}{\alpha} \qquad \tau = \alpha - 1$$

Incorporating the above values in 11, we will have

$$\sum_{i=1}^n D^{-l_i} \geq \left[\sum_{i=1}^n p_i D^{-l_i \left(\frac{\alpha-1}{\alpha}\right)} \right]^{\frac{\alpha}{\alpha-1}} \left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}} \right]^{\frac{1}{\alpha-1}} \tag{12}$$

Alternatively, we can write it as

$$\left[\sum_{i=1}^n p_i D^{-l_i \left(\frac{\alpha-1}{\alpha}\right)} \right]^\alpha \leq \left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}} \right] \tag{13}$$

Or,

$$\frac{\beta}{1-\alpha} \left[\sum_{i=1}^n p_i D^{-l_i \frac{\alpha-1}{\alpha}} \right]^\alpha \geq \frac{\beta}{1-\alpha} \left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}} \right]$$

Or,

$$L_\beta^\alpha(P) \geq H_\beta^\alpha(P)$$

The result follows.

Moving on to the equality

$$l_i = -\log_D \left[\frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}} \right]$$

Alternatively,

$$D^{-l_i} = \left[\frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}} \right] \tag{14}$$

After carrying out calculations, we have

$$\sum_{i=1}^n p_i D^{-l_i \left(\frac{\alpha-1}{\alpha}\right)} = \left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}} \right]^{\frac{1}{\alpha}}$$

$$\frac{\beta}{1-\alpha}\left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} = \frac{\beta}{1-\alpha}\left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right]$$

Alternatively, we can write it as

$$L_{\beta}^{\alpha}(P) = H_{\beta}^{\alpha}(P)$$

□

Theorem 3.2. *Given that the code-word length $L = l_1, l_2, l_3, \ldots, l_n$ that fulfill Kraft's Inequality, then $H_{\beta}^{\alpha}(P)$ and $L_{\beta}^{\alpha}(P)$ are connected as shown below:*

$$L_{\beta}^{\alpha}(P) < H_{\beta}^{\alpha}(P) D^{1-\alpha}, \quad 0 < \alpha < 1 \tag{15}$$

Proof. : Based on the above theorem, we have

$$L_{\beta}^{\alpha}(P) = H_{\beta}^{\alpha}(P)$$

is valid if and only if

$$D^{-l_i} = \left[\frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}}\right]; 0 < \alpha < 1$$

Alternatively

$$l_i = -\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right]$$

Given that the code-word length $L = l_1, l_2, l_3, \ldots, l_n$ be chosen such that they satisfy the following inequalities:

$$-\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right] \le l_i < -\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right] + 1 \tag{16}$$

Consider the interval

$$\mu_i = \left[-\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right], -\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right] + 1\right]$$

of length unity. Within every interval μ_i, there is precisely one positive integer l_i, that satisfy the following inequality

$$0 < -\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right] \le l_i < -\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right] + 1 \tag{17}$$

Now from left hand side of the inequality 17, we have

$$-\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right] \le l_i$$

Thereafter,

$$D^{-l_i} = \left[\frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}}\right] \tag{18}$$

Taking the sum over i=1,2,3,...,n, across the inequality 18, we have

$$\sum_{i=1}^{n} D^{-l_i} \leq 1$$

Which is Kraft's (1949) Inequality. From right hand side of inequality 17

$$l_i < -\log_D \left(p_i^{\frac{\alpha}{\beta}} \right) + \log_D \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}} \right] + 1$$

After carrying out some calculations, we will have

$$D^{l_i\left(\frac{1-\alpha}{\alpha}\right)} < \left(\frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}} \right)^{\frac{\alpha-1}{\alpha}} D^{\frac{1-\alpha}{\alpha}}$$

Multiply the entire inequality above by p_i, and summing over i=1,2,3,...,n to the expression obtained and after carrying out some calculations, we will have

$$\left[\sum_{i=1}^{n} p_i D^{l_i\left(\frac{\alpha-1}{\alpha}\right)} \right]^{\alpha} < \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}} \right] D^{1-\alpha}$$

Or,

$$\frac{\beta}{1-\alpha} \left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} \right]^{\alpha} < \frac{\beta}{1-\alpha} \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}} \right] D^{1-\alpha} \tag{19}$$

This leads to the result as

$$L_{\beta}^{\alpha}(P) < H_{\beta}^{\alpha}(P) D^{1-\alpha}$$

for $0 < \alpha < 1$. Therefore, based on the above two theorems, we will have our resultant expression as

$$H_{\beta}^{\alpha}(P) \leq L_{\beta}^{\alpha}(P) < H_{\beta}^{\alpha}(P) D^{1-\alpha}, \quad 0 < \alpha < 1, \beta \geq 1$$

$\square$

4. Huffman and Shannon-Fano Coding

In this portion, we will demonstrate the validity of Theorems 1 and 2 by taking an empirical data from [6] given in table 1. The probability values of p_1, p_2, p_3, p_4, p_5 are given as 0.25, 0.25, 0.2, 0.125, 0.125 respectively. Now by using Huffman-Coding Algorithm and Shannon-Fano-Elias Coding Algorithm, for different values of α and β, the values of $H_{\beta}^{\alpha}(P)$, $L_{\beta}^{\alpha}(P)$, $D^{1-\alpha}$ and $H_{\beta}^{\alpha}(P) D^{1-\alpha}$ are outlined in the subsequent tables:

Table 1: By applying Huffman coding algorithm for different values of α and β, the values of $H_{\beta}^{\alpha}(P)$, $L_{\beta}^{\alpha}(P)$, $D^{1-\alpha}$ and $H_{\beta}^{\alpha}(P) D^{1-\alpha}$ are outlined with D=2, since we are using binary codes here.

TABLE 1. Huffman Coding

Probabilities p_i	Huffman Code-words	l_i	α	β	$H_\alpha^\beta(P)$	$L_\alpha^\beta(P)$	$D^{1-\alpha}$	$H_\alpha^\beta(P)D^{1-\alpha}$
o.25	01	2	0.9	1	11.2170	11.5160	1.0718	12.0224
0.25	10	2	0.5	1	4.4436	4.5606	1.4142	6.2841
0.2	11	2						
0.15	000	3						
0.15	001	3						

Table 2: By applying Shannon- Fano-Elias coding algorithm for different values of α and β, the values of $H_\beta^\alpha(P)$,$L_\beta^\alpha(P)$, $D^{1-\alpha}$ and $H_\beta^\alpha(P)D^{1-\alpha}$ are outlined with D=2, since we are using binary codes here.

TABLE 2. Shannon-Fano Coding

Probabilities p_i	Shannon-Fano Codes	l_i	α	β	$H_\alpha^\beta(P)$	$L_\alpha^\beta(P)$	$D^{1-\alpha}$	$H_\alpha^\beta(P)D^{1-\alpha}$
0.25	001	3	0.9	1	11.2170	11.7340	1.0718	12.0224
0.25	011	3	0.5	1	4.4436	6.0282	1.4142	6.2841
0.2	1001	4						
0.15	1100	4						
0.15	1110	4						

Based on Table **1** and Table **2**, we deduce the following:

(1) The validity of Theorem 1 and Theorem 2 extends in both the cases of Huffman-coding algorithm as well as Shannon-Fano-Elias coding algorithm i.e.,

$$H_\beta^\alpha(P) \leq L_\beta^\alpha(P) < H_\beta^\alpha(P)D^{1-\alpha}, \quad 0 < \alpha < 1, \beta \geq 1$$

(2) The average code-word length $L_\beta^\alpha(P)$ is smaller for Huffman coding algorithm than for Shannon-Fano-Elias coding algorithm. It is inferred that Huffman coding algorithm is more efficient than Shannon-Fano-Elias coding algorithm.

5. Real Life Application

This section evaluates the monotonicity of the proposed measure by taking a real-world representing the Average Wind Speed Data (AWSD), documented in Best et al. (2010) [4]. It comprises 30 daily average wind speeds (measured in km/h) recorded during November 2007 at Elanora Heights, a northeastern suburb of Sydney, Australia. The data are as follows:

2.7	3.2	2.1	4.8	7.6	4.7	4.2	4.0
2.9	2.9	4.6	4.8	4.3	4.6	3.7	2.4
4.9	4.0	7.7	10	5.2	2.6	4.2	3.6
	2.5	3.3	3.1	3.7	2.8	4.0	

Table 3. The behaviour of our proposed measure When α is fixed and β varies for the AWSD Dataset

	β	$H^{\alpha}_{\beta}(P)$		β	$H^{\alpha}_{\beta}(P)$		β	$H^{\alpha}_{\beta}(P)$
	0.25	0.1642		0.35	0.2587		0.45	0.3548
	0.26	0.1736		0.36	0.2683		0.46	0.3644
	0.27	0.1829		0.37	0.2779		0.47	0.3741
	0.28	0.1924		0.38	0.2874		0.48	0.3838
α=0.05	0.29	0.2018	α=0.05	0.39	0.2970	α=0.05	0.49	0.3934
	0.30	0.2112		0.40	0.3067		0.50	0.4031
	0.31	0.2207		0.41	0.3162		0.51	0.4128
	0.32	0.2302		0.42	0.3259		0.52	0.4225
	0.33	0.2397		0.43	0.3355		0.53	0.4322
	0.34	0.2492		0.44	0.3452		0.54	0.4416

Table 4. The behaviour of our proposed measure When β is fixed and α varies for AWSD Dataset

	α	$H^{\alpha}_{\beta}(P)$		α	$H^{\alpha}_{\beta}(P)$		α	$H^{\alpha}_{\beta}(P)$
	0.10	0.8511		0.20	0.7803		0.30	0.7189
	0.11	0.8436		0.21	0.7738		0.31	0.7133
	0.12	0.8361		0.22	0.7673		0.32	0.7077
	0.13	0.8288		0.23	0.7610		0.33	0.7022
β=1	0.14	0.8215	β=1	0.24	0.7547	β=1	0.34	0.6968
	0.15	0.8144		0.25	0.7485		0.35	0.6914
	0.16	0.8074		0.26	0.7425		0.36	0.6861
	0.17	0.8005		0.27	0.7365		0.37	0.6809
	0.18	0.7937		0.28	0.7305		0.38	0.6757
	0.19	0.7869		0.29	0.7247		0.39	0.6706

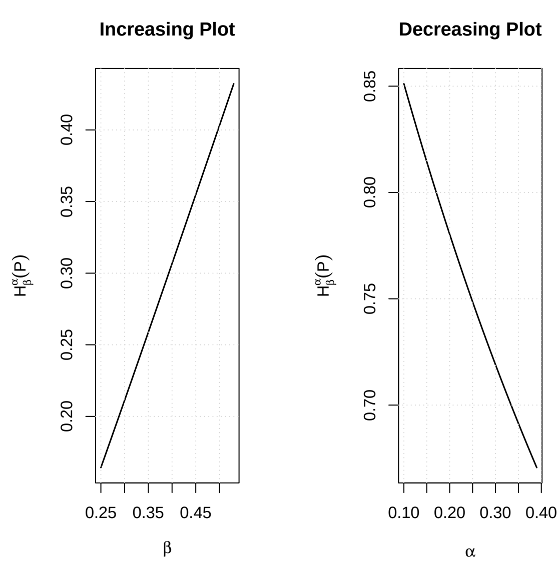

Figure 1. Monotonicity of Our Proposed Measure

Interpretation From Table 3 and 4, we can easily depict that when α is held constant, our proposed measure increases monotonically and when β is held constant our proposed measures decreases monotonically. As the dataset captures the average wind speed measurements recorded during November 2007 So, we can say that from Table 3, when α is held fixed and β varies, our proposed measure increases, which clearly means biasedness decreases and uncertainty increases. In practical terms, tuning β upward could highlight extreme wind speeds, which might be crucial in meteorological assessments, energy generation forecasts, or risk evaluations.
Conversely, when β is held constant and α varies as shown in Table 4, the measure decreases and we can say that biasedness increases and uncertainty decreases. In practical applications, decreasing α may be useful when one wishes to smooth out the effect of outliers or extreme data points, focusing instead on baseline or moderate conditions.The same is shown in Figure 1 where monotonically increasing and decreasing plots are plotted.

6. Conclusion

In this article, we introduced a novel generalized entropy measure and based on this measure, developed a corresponding generalized average codeword length. Additionally, we established bounds for the new generalized average codeword length in terms of the proposed generalized entropy measure. We also demonstrate that the measures introduced in this work are generalizations of several wellknown measures in the fields of coding and information theory. The key properties of the novel proposed entropy measure are also analyzed, including its behavior with respect to uncertainty and biasedness. Specifically, it is observed that as the measure increases, the uncertainty in the system also increases, while the biasedness of the distribution decreases and vice versa. Finally, the bounds derived in this paper for discrete channels have been validated using empirical data through the application of Huffman and Shannon-Fano coding schemes. For illustration purposes, we have used a realworld dataset pertaining to Average Wind Speed Data and presented its results in both tabular and graphical forms.

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